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Assessing tipping-point risk in carbon dioxide removal with a network-based framework

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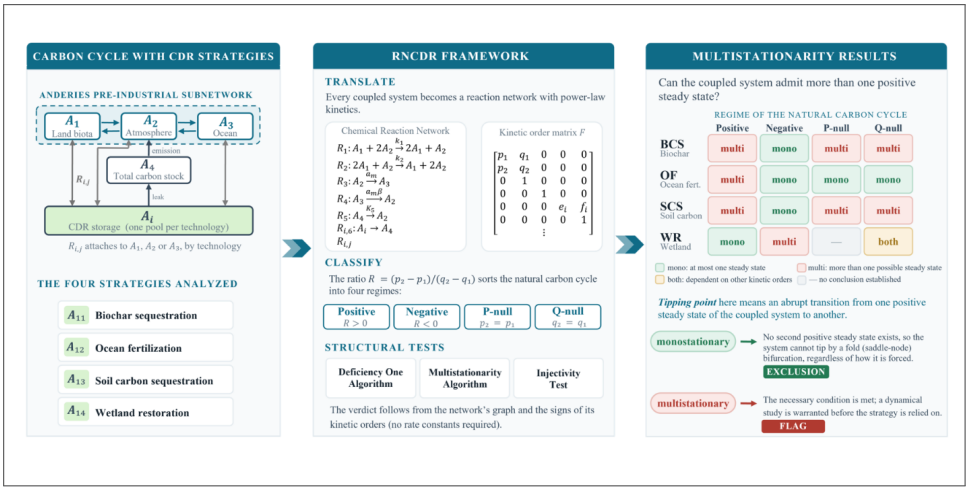

Abstract

Carbon dioxide removal (CDR) strategies are central to long-term plans for reaching carbon neutrality, and projections of their mitigation potential underpin national and global carbon accounting. How reliably a strategy delivers that potential depends on how its technology interacts with the natural carbon cycle, and in particular on whether the coupled system can settle into more than one steady state. Throughout, a tipping point means an abrupt transition from one positive steady state of the coupled system to a different one. Multiple steady states are a necessary condition for such a transition since a system with only one has no second state to move to. Such a shift would leave the strategy removing far less CO_2 than projected, invalidating the carbon accounting that justified it, and recovery may be slow. Using Chemical Reaction Network Theory (CRNT), we represent four CDR strategies (biochar sequestration, ocean fertilization, soil carbon sequestration, and wetland restoration) as networks of carbon pools and the transfers between them. From the graphical and kinetic structure of each network alone, without exact rate values, we identify the conditions under which multiple steady states can exist and those under which they are excluded. For ocean fertilization, multiple steady states are guaranteed in one regime of the natural carbon cycle, provided the technology's own reaction sensitivities also meet additional conditions;

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outside this identified regime, the framework does not rule multistationarity out. Biochar sequestration and soil carbon sequestration share the broadest reach across the regimes in which multiple steady states can occur, reflecting shared features of their networks. Wetland restoration departs most from the others, admitting at most one steady state in a regime where the other three admit multiple. The network-level structure of a CDR system is therefore itself an assessment criterion for the reliability of projected carbon removal, and CRNT offers a parameter-minimal first screen for tipping-point risk, complementing conceptual and numerical carbon-cycle models.

INTRODUCTION

The global carbon cycle can settle into more than one steady state. This possibility, supported by a growing body of modeling evidence[1,2], is consequential because it implies the existence of climate "tipping points": thresholds beyond which the system may shift abruptly and irreversibly into a warmer equilibrium from which recovery is practically impossible[3,4,5]. Human-induced climate change may be pushing the Earth system closer to such thresholds[5,6], and the mechanisms governing these transitions remain poorly understood[7,8].

This uncertainty has direct implications for carbon dioxide removal (CDR), the family of strategies aimed at actively removing CO_2 from the atmosphere as part of long-term carbon neutrality planning[9,10,11]. Projections of how much CO_2 each strategy can remove over a given horizon (i.e., its mitigation potential) are a core input to national carbon budgets, Intergovernmental Panel on Climate Change (IPCC) scenario pathways, and the carbon accounting that guides long-term climate policy[9,12,13]. The existence of multiple steady states in a CDR system could have significant consequences for whether that projected potential is actually delivered. If a CDR system settles into a steady state that removes far less CO_2 than projected, the carbon accounting behind its deployment no longer holds, and the broader mitigation pathway built around it may fail to meet its emissions targets. Moreover, abrupt transitions between steady states would make this risk especially difficult to anticipate or quickly recover from, since the shift could occur faster than monitoring, reporting, and verification systems can detect. Yet a critical gap remains in our understanding of when and how CDR systems exhibit multiple steady states under varying environmental conditions[10,14].

Current approaches to CDR risk assessment address related but different questions. Earth system models project how the climate responds to a given deployment over time, and multi-model intercomparisons have shown that the carbon cycle does not simply retrace its path when emissions turn negative[15,16]. Integrated assessment models ask which mix of technologies is affordable under land and energy constraints, taking the removal rates themselves as given[9,17]. Life-cycle, permanence and monitoring frameworks ask whether the carbon stored at a particular site will stay there, and whether that can be verified[14,18,19]. Deployment-impact studies ask what scaling would cost in resources; recent work finds that removal at the gigatonne scale could strain energy, water and land systems across whole regions[20,21]. None of them asks a prior question: whether the coupled system of technology and carbon cycle can settle into more than one steady state at all.

Earth system models would seem the natural place to look for multiple steady states, but they are not designed for the task. They are indispensable for projecting climate trajectories and human impacts[1,22], and they achieve this by integrating the system forward in time under prescribed forcing rather than by locating the states it can settle into. Those states are seldom computed, and the slow processes that would determine them are among the least constrained by observation[2,16]. The number of steady states a coupled CDR–carbon-cycle system admits is therefore a question posed at the level of smaller and simpler models, where the steady states are well defined and open to direct study[3,7,8]. Even at that level, locating them by simulation requires sweeping a parameter space that is large and poorly constrained. A more tractable approach is needed, one that can

characterize multistationarity, the property of admitting more than one positive steady state, from the structure of the system rather than from exhaustive numerical simulation.

Chemical Reaction Network Theory (CRNT) provides a framework well-suited to this purpose. Originally developed in the context of chemical kinetics, CRNT represents system components as interacting species and transfers between them as reactions governed by rate laws. When formulated with power-law kinetics, it admits a parameter-minimal analysis that relies primarily on the graphical and kinetic structure of the network, rather than precise parameter values, to identify sufficient conditions under which multiple steady states may arise. This makes the framework well-suited to systems in which parameter uncertainty is high. The carbon-cycle box models considered here are such systems. Reaction network modeling has been successfully applied to biochemical networks[23,24], ecological systems[25,26], and epidemiology[27], and its application to the carbon cycle is an active and growing area[28,29,30,31].

In this study, we apply the Reaction Network Carbon Dioxide Removal (RNCDR) framework[29] to analyze steady-state multiplicity in four CDR strategies: biochar sequestration, ocean fertilization, soil carbon sequestration, and wetland restoration. Each system is translated into a reaction network with power-law kinetics, in which carbon pools, namely land biota, atmosphere, ocean, total carbon stock, and CDR storage, play the role of interacting species, and carbon transfers between them are modeled as reactions. The resulting network structure is analyzed to identify parameter conditions under which the system may exhibit multistationarity or is guaranteed to be monostationary, admitting at most one positive steady state. This distinction is crucial: a multistationary CDR system could settle into a steady state characterized by low net CO_2 removal performance, undermining its reliability as a climate mitigation tool, while a monostationary system offers more predictable and recoverable dynamics. This approach provides a practical, structure-based way to screen each of the four CDR strategies for the possibility of multistable behavior. The screen comes before any commitment to detailed numerical modeling of a specific deployment.

A screen of this kind establishes less than the language of tipping points suggests, and the distinction is worth drawing at the outset. We use tipping point throughout in a specific sense: an abrupt transition from one positive steady state of the coupled system to a different one. Thus, multiple steady states are necessary since a system admitting only one has no second state to move to. They are not sufficient. A transition also requires that at least two of those states be attractors, that the one currently occupied cease to be available as conditions drift, and that a disturbance of realistic magnitude carry the system across the basin between them. Abrupt behavior of other kinds, which does not end in a second steady state, falls outside this definition and outside what we analyze; Section "Multistationarity and tipping points: what this framework does and does not establish" provides the distinction. Whether that occurs depends on perturbation magnitude, timescale and prevailing conditions, none of which is resolved here, and the other mechanisms by which a system may tip lie outside the analysis altogether.

The two kinds of results reported below therefore differ in force. A monostationarity result is an exclusion: a system admitting a unique positive steady state across a parameter range has no second state with which one could merge and vanish, and so cannot tip by that route however it is forced. A multistationarity result is a flag rather than a diagnosis: the structural precondition is present, and the configuration warrants dynamical study before the strategy is relied upon. We retain the language of tipping-point risk throughout, since it is the concern that makes the structural question worth asking, but no result in this paper establishes that any particular deployment will tip. Section "Multistationarity and tipping points: what this framework does and does not establish" develops the distinction and states its limits.

The remainder of the paper is structured as follows. Section "THE RNCDR FRAMEWORK" presents the RNCDR framework. Section "FOUR CDR STRATEGIES AND THEIR REACTION NETWORK REPRESEN-

TATIONS" describes the four CDR systems and their reaction network representations. Section "CONDITIONS FOR MULTISTATIONARITY IN CDR REACTION NETWORKS" establishes the multistationarity and monostationarity conditions for each system. Section "COMPARATIVE ANALYSIS AND IMPLICATIONS" presents a comparative analysis of the four systems and discusses implications for CDR risk assessment. Section "SUMMARY, CONCLUSION, AND FUTURE RESEARCH" concludes with a summary of the key findings, their broader significance for Earth system science and climate policy, and directions for future research.

THE RNCDR FRAMEWORK

The RNCDR framework, formally introduced by Fortun *et al.*[29], builds on and extends their earlier work[31,32]. This approach applies CRNT to model the Earth's carbon cycle and, in the most recent studies[28,30], to two carbon removal technologies, direct air capture (DAC) and direct ocean capture (DOC). The present study expands this line of work by applying the RNCDR framework to four additional CDR strategies, namely biochar sequestration, ocean fertilization, soil carbon sequestration, and wetland restoration, and by carrying out a comparative steady-state analysis across them to identify structural conditions of multistationarity that are common to, or distinctive of, each strategy.

A chemical reaction network (CRN) is a way of describing a system in terms of three ingredients: the species (the things that interact, in our case the carbon pools), the complexes (the combinations of species that appear on either side of an arrow, such as land plus atmosphere), and the reactions (the directional transfers from one complex to another, each describing a way carbon moves through the system). Together, these define the structure of the network. Each reaction is assigned a rate function describing how fast it proceeds, and the collection of these rates gives the dynamics of the system. In this study, the rate functions take the form of power-law kinetics, a flexible class of rate laws that can approximate a wide range of biogeochemical processes. Precise definitions are provided in Supplementary Section 1.

In this framework, the global carbon cycle is modeled as a CRN in which carbon moves among five interconnected pools: land biota (A_1), representing carbon stored in forests, soil, and living organisms; atmosphere (A_2), representing carbon as atmospheric CO_2; ocean (A_3), representing carbon dissolved in seawater; total carbon stock (A_4), which comprises the geological reserve, including fossil fuel, together with the inorganic carbon pool. It is the reservoir on which the emission reaction R_5 draws, and it is a species distinct from the CDR storage pools A_i, which exchange carbon with it through the leak reactions $R_{i,6}$ rather than forming part of it; and CDR storage (A_i, $i = 8, \ldots, 17$), representing carbon captured and stored by a specific negative emissions technology (NET).

Every RNCDR system is assembled from three components: the Anderies pre-industrial subnetwork, the fossil fuel emission reaction, and the CDR carbon storage. Figure 1 provides an overview of how these three components combine to form an RNCDR system. The Anderies pre-industrial subnetwork (Component 1) tracks the natural carbon cycle among land biota, the atmosphere, and the ocean. The fossil fuel emission reaction (Component 2) introduces the total carbon stock and the transfer of carbon from this stock to the atmosphere. The CDR carbon storage (Component 3) introduces a storage pool A_i associated with a specific NET, together with at least one reaction returning carbon to the total carbon stock and one or more technology-specific reactions. Together, these three components are assembled into a single chemical reaction network with power-law kinetics, with five species (A_1, A_2, A_3, A_4, and A_i) and at least seven reactions. The remainder of this section describes each component in detail.

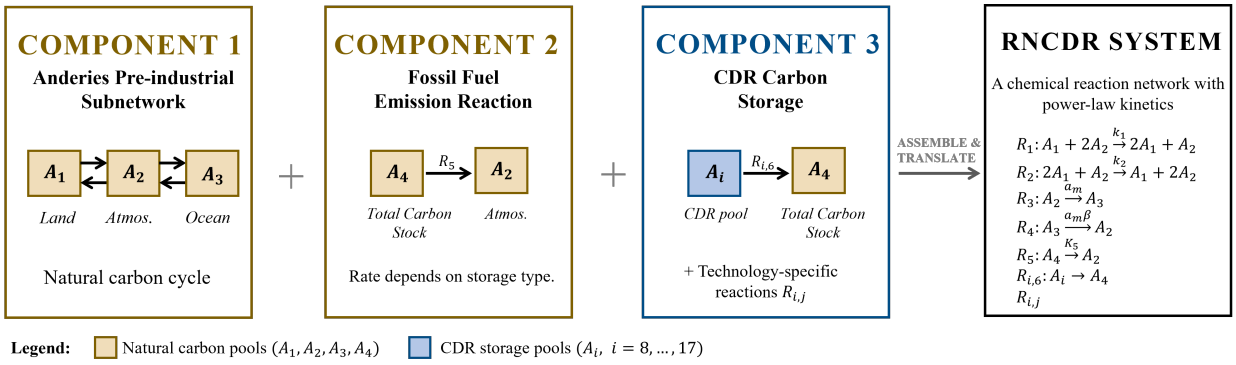

Figure 1. Structure of an RNCDR system. Every RNCDR model is assembled from three components: the Anderies pre-industrial subnetwork representing the natural carbon cycle among land biota (A_1), atmosphere (A_2), and ocean (A_3); the fossil fuel emission reaction connecting the total carbon stock (A_4) to the atmosphere; and the CDR carbon storage pool (A_i) representing a specific NET. The three components combine into a single chemical reaction network with power-law kinetics.

Building an RNCDR system from three components

The foundation of every RNCDR model is the Anderies pre-industrial subnetwork, which captures the natural carbon cycle among land (A_1), atmosphere (A_2), and ocean (A_3). This subnetwork is based on the carbon cycle model of Anderies *et al.*[3] in The Topology of Non-linear Global Carbon Dynamics: From Tipping Points to Planetary Boundaries, which serves as the formal basis of the influential work of Steffen *et al.*[33].

The model tracks carbon transfers among three pools, namely land (A_1), atmosphere (A_2), and ocean (A_3). Solid arrows represent direct carbon transfers between pools, while dashed arrows indicate biogeochemical feedbacks (fertilization and temperature) that modulate the transfer rates; this structure recurs, highlighted, within each technology-specific system introduced in Section "FOUR CDR STRATEGIES AND THEIR REACTION NETWORK REPRESENTATIONS". The total carbon stock is absent from the pre-industrial model since fossil fuel use is not considered.

The model is formulated as a generalized mass action (GMA) system[34,35,36]. This is an ordinary differential equation (ODE) system in which each carbon transfer is individually approximated by a power-law term via Taylor linearization in logarithmic coordinates, with a positive sign for incoming transfers and a negative sign for outgoing transfers. The resulting CRN representation is dynamically equivalent to the original ODE system. The power-law approximation is summarized in Table 1, and the resulting ODE system, due to the carbon-cycle model of Anderies *et al.*[3] in the power-law representation of Fortun *et al.*[32], is:

$$\dot{A}_1 = k_1 A_1^{p_1} A_2^{q_1} - k_2 A_1^{p_2} A_2^{q_2}, \tag{1}$$

$$\dot{A}_2 = -k_1 A_1^{p_1} A_2^{q_1} + k_2 A_1^{p_2} A_2^{q_2} - a_m A_2 + a_m \beta A_3, \tag{2}$$

$$\dot{A}_3 = a_m A_2 - a_m \beta A_3, \tag{3}$$

where the kinetic orders p_1, p_2, q_1, q_2 encode the influence of each carbon transfer on the sizes of the land biota and the atmosphere pools, respectively.

The CRN representation of the subnetwork consists of four reactions:

Table 1. Power-law kinetics of the Anderies pre-industrial model [3], in the representation of [32]

Carbon transfer	Rate constant	Power-law kinetic
$A_2 \to A_1$	k_1	$k_1 A_1^{p_1} A_2^{q_1}$
$A_1 \to A_2$	k_2	$k_2 A_1^{p_2} A_2^{q_2}$
$A_2 \to A_3$	a_m	$a_m A_2$
$A_3 \to A_2$	$a_m \beta$	$a_m \beta A_3$

$$R_1 : A_1 + 2A_2 \xrightarrow{k_1} 2A_1 + A_2 \quad \text{(atmosphere-to-land)}$$
$$R_2 : 2A_1 + A_2 \xrightarrow{k_2} A_1 + 2A_2 \quad \text{(land-to-atmosphere)}$$
$$R_3 : A_2 \xrightarrow{a_m} A_3 \quad \text{(atmosphere-to-ocean)}$$
$$R_4 : A_3 \xrightarrow{a_m \beta} A_2 \quad \text{(ocean-to-atmosphere)}$$

Reactions R_1 and R_2 represent photosynthesis and respiration, respectively. The unusual-looking stoichiometry (e.g., $A_1 + 2A_2 \to 2A_1 + A_2$ rather than simply $A_2 \to A_1$) arises from the power-law kinetic representation of the original nonlinear dynamics; it encodes how each carbon pool influences the carbon transfer. Reactions R_3 and R_4 are simpler: they represent the exchange of CO_2 between the atmosphere and the ocean.

The industrial carbon cycle introduces the total carbon stock A_4, representing the geological and inorganic carbon reserves. Fossil fuel combustion is modeled by:

$$R_5 : A_4 \xrightarrow{K_5} A_2, \tag{4}$$

which transfers carbon from the total carbon stock directly into the atmosphere through the combustion of its geological component. The rate function K_5 depends on the CDR technology in use, as described in Section "Connecting storage to the atmosphere through the emission reaction".

Each CDR technology introduces one additional storage pool A_i and at least two reactions:

$$R_{i,6} : A_i \to A_4 \quad \text{(CDR storage to total carbon stock)},$$
$$R_{i,j} \quad \text{(CDR-specific reactions)},$$

where $i = 8, 9, \ldots, 17$ for both, and $j \geq 7$ for the CDR-specific reactions. The reaction $R_{i,6}$ models the possibility of a "leak" [37,38]: even after carbon is captured, some fraction may eventually return to the total carbon stock, from which it can be re-emitted. The notation for the CDR storage species is summarized in Table 2. Including this leak allows the framework to evaluate CDR performance even under imperfect storage conditions. The reactions $R_{i,j}$ are CDR-specific and represent the additional interactions derived from each particular CDR technology.

Characterizing the storage with λ_i and μ_i

Not all CDR technologies store carbon in the same way. Two parameters characterize the output of each technology:

Table 2. Notation for CDR-specific storage species

Symbol	CDR technology
A_8	Bioenergy with Carbon Capture and Storage (BECCS) [29]
A_9	Direct Air Capture (DAC) [30]
A_{10}	Enhanced Weathering (EW) [39]
A_{11}	Biochar [40]
A_{12}	Ocean Fertilization (OF) [41]
A_{13}	Soil Carbon Sequestration (SCS) [42]
A_{14}	Wetland Restoration (WR) [43]
A_{15}	Afforestation/Reforestation (AR) [44]
A_{16}	Ocean Alkalinization (OA) [45]
A_{17}	Direct Ocean Capture (DOC) [28]

- $\lambda_i \in [0,1]$: the fraction of captured carbon that ultimately enters the geological reserve within the total carbon stock A_4, whether directly or through the leak reaction $R_{i,6}$, and is therefore available for subsequent release by fossil fuel combustion. It is not a statement about where the carbon is held while it remains in the CDR pool A_i, which is what the physical storage column of Table 3 records. Carbon in the geological reserve is not thereby permanent: it is precisely the carbon from which the emission reaction $R_5 : A_4 \to A_2$ draws. A value of $\lambda_i = 1$ means all captured carbon enters the geological reserve; $\lambda_i = 0$ means none of it does.

- $\mu_i \in [0,1]$: the fraction of the long-term stored carbon that is inorganic. Inorganic carbon (e.g., carbonate minerals) behaves differently from organic carbon (e.g., biomass) and does not contribute to fossil fuel emissions.

Table 3. Parameter values λ_i and μ_i for each CDR technology in the RNCDR framework. The derived quantity $c_i = (1-\lambda_i)+\mu_i\lambda_i$, the fraction of CDR storage withheld from emission, is the only combination of the two that enters the analysis; see Supplementary Section 2

Technology	Physical storage	λ_i	μ_i	c_i
BECCS (A_8)	CO_2 injected into geological stock	1	0	0
DAC (A_9)	CO_2 injected into geological stock	1	0	0
EW (A_{10})	Rock spread on beaches/fields	1	1	1
Biochar (A_{11})	Biocharcoal in soil (0.5), biofuel in geostock (0.5)	1	0.5	0.5
OF (A_{12})	Deep ocean	1	0	0
SCS (A_{13})	Soil (microbial enhancement)	0.01	0	0.99
WR (A_{14})	Wetland floor	0.01	0.9	0.999
AR (A_{15})	Sequence of trees and soil	0.5	0	0.5
OA (A_{16})	Deep ocean	1	1	1
DOC (A_{17})	CO_2 injected into geological stock	1	0	0

Only organic carbon held in the geological reserve can be released by fossil fuel combustion. The emittable fraction of captured carbon is therefore $\lambda_i(1-\mu_i)$, and the fraction withheld from emission is $1-\lambda_i(1-\mu_i) = (1-\lambda_i)+\mu_i\lambda_i$, which is exactly the bracketed term in the emission rate function (5) below. The leak reaction $R_{i,6} : A_i \to A_4$ and that bracketed term represent distinct mechanisms: the former is a physical transfer of carbon from the CDR pool into the total carbon stock, whereas the latter weights the availability of stock carbon for combustion.

These parameters (see Table 3) are best understood as relative estimates rather than fixed empirical quantities; they allow the model to explore system behavior under different assumptions and can be recalibrated as real-world data become available.

Because these values are nominal rather than measured, it is worth being explicit about how far the results depend on them. The two parameters enter the analysis only through the single combination

$$c_i = (1 - \lambda_i) + \mu_i \lambda_i,$$

the fraction of CDR storage withheld from emission, which, together with the positive steady state about which (5) is linearized, determines the emission kinetic orders e_i and f_i. The storage parameters alone do not fix e_i and f_i; Supplementary Section 2 gives the relation and the operating-point dependence explicitly. Conditions in Section "CONDITIONS FOR MULTISTATIONARITY IN CDR REACTION NETWORKS" that require only $e_i \geq 1$ and $f_i \leq 0$ hold for any admissible pair (λ_i, μ_i) and are unaffected by recalibration. Conditions that turn on a strict inequality, or on the exact value $c_i = 0$, are not: two of the results below depend on the particular values in Table 3, and we identify each where it arises. Supplementary Section 2 gives the derivation and a proposition-by-proposition account.

Connecting storage to the atmosphere through the emission reaction

The emission reaction $R_5 : A_4 \to A_2$, which transfers carbon from the total carbon stock into the atmosphere by combustion of its geological component, is a defining feature of the RNCDR framework[29]. Its rate function K_5, introduced in that work, depends on both the total carbon stock A_4 and the CDR storage A_i:

$$K_5(A_4, A_i) = k_5(A_4 - [(1 - \lambda_i) + \mu_i \lambda_i] A_i), \tag{5}$$

where the term $[(1 - \lambda_i) + \mu_i \lambda_i] A_i$ represents the portion of CDR storage that cannot contribute to fossil fuel emissions. In other words, the emission rate is proportional to the "available" geological carbon after subtracting what has been effectively locked away by CDR. Since (5) is not in power-law form, a power-law approximation around a positive steady state gives

$$K_5 \approx k_5 A_4^{e_i} A_i^{f_i}, \tag{6}$$

where the kinetic orders satisfy

$$e_i \geq 1, \qquad f_i \leq 0, \qquad e_i + f_i = 1. \tag{7}$$

These three conditions have clear physical interpretations: $e_i \geq 1$ means emission is at least as sensitive to the total carbon stock as in a basic linear model; $f_i \leq 0$ means more CDR storage reduces emission; and $e_i + f_i = 1$ reflects the complementarity of the two effects. When $\lambda_i = 1$ and $\mu_i = 0$ (as in BECCS[29] and DAC[30]), these reduce to $e_i = 1$, $f_i = 0$, and $K_5 = k_5 A_4$, recovering linear mass action.

Encoding the kinetics in the kinetic order matrix

The power-law kinetics of the entire RNCDR system is encoded in the kinetic order matrix F, whose entry F_{ij} gives the kinetic order of the j-th species in the i-th reaction. For a single-CDR system, the first six rows of F are common to all RNCDR models:

$$F = \begin{array}{c} \begin{matrix} A_1 & A_2 & A_3 & A_4 & A_i \end{matrix} \\ \begin{bmatrix} p_1 & q_1 & 0 & 0 & 0 \\ p_2 & q_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & e_i & f_i \\ 0 & 0 & 0 & 0 & 1 \\ & & \vdots & & \end{bmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_{i,6} \\ R_{i,j},\ j \geq 7 \end{array} \tag{8}$$

The rows for R_1 and R_2 encode the kinetic influence of land biota (p_1, p_2) and atmosphere (q_1, q_2) on the natural carbon cycle. The row for R_5 encodes the coupled dependence of emission on both the total carbon stock and the CDR storage, with kinetic orders satisfying $e_i \geq 1$, $f_i \leq 0$, and $e_i + f_i = 1$ as established in (7). The row for $R_{i,6}$ reflects the common storage-to-total carbon stock reaction, with mass-action kinetics of order 1 in A_i. The remaining rows, indicated by the ellipsis, correspond to the CDR-specific reactions $R_{i,j}$ for $j \geq 7$, whose number and kinetic orders vary by technology and are specified for each CDR system in Section "FOUR CDR STRATEGIES AND THEIR REACTION NETWORK REPRESENTATIONS".

Classifying RNCDR systems

The dynamic behavior of an RNCDR system is strongly shaped by the properties of its Anderies subnetwork. Following [31], we classify Anderies systems – and by extension, RNCDR systems [29] – according to the ratio

$$R = \frac{p_2 - p_1}{q_2 - q_1}, \qquad q_2 - q_1 \neq 0, \tag{9}$$

which compares how much respiration and photosynthesis differ in their influence on land biota versus atmosphere. The kinetic orders p_1, p_2 encode the degree to which photosynthesis and respiration are influenced by the size of the land biota pool, while q_1, q_2 encode the degree to which they are influenced by the level of atmospheric CO_2. A value of $p_2 > p_1$ means that respiration responds more strongly to land biota than photosynthesis does; in other words, as forests grow, they release carbon faster than they absorb it. Similarly, $q_2 > q_1$ means respiration is more strongly driven by atmospheric CO_2 concentration than photosynthesis is. When both inequalities hold simultaneously, these asymmetries reinforce each other, creating the nonlinear feedback conditions under which multiple equilibria can arise. Four primary classes arise from the ratio R, each with distinct dynamic consequences:

- **Positive class** ($R > 0$): both $p_2 > p_1$ and $q_2 > q_1$ (or both reversed), so the same process dominates the feedback in both pools: respiration when $p_2 > p_1$ and $q_2 > q_1$, photosynthesis when both inequalities are reversed. What makes R positive is that the two inequalities point in the same direction, not the identity of the dominant process; the Anderies subnetwork may admit multiple steady states, meaning the Earth could settle into more than one equilibrium carbon distribution.

- **Negative class** ($R < 0$): photosynthesis dominates one feedback while respiration dominates the other, leading to more stabilizing dynamics; the Anderies subnetwork contains monostationary subsets (unique steady states under appropriate kinetic order conditions).

- **P-null class** ($R = 0$): the Anderies subnetwork is monostationary.

- **Q-null class** ($q_2-q_1 = 0,\ p_2-p_1 \neq 0$): defined for the case where the ratio R is undefined but $Q = (q_2-q_1)/(p_2-p_1) = 0$. This class shares the monostationarity of the P-null class in the Anderies subnetwork.

These classes characterize the dynamic behavior of the Anderies subnetwork in isolation. Once the Anderies subnetwork is embedded in an RNCDR system, the addition of the fossil fuel emission reaction and the CDR-specific reactions can change the steady-state behavior: a class that is monostationary at the Anderies level need not remain so for the full RNCDR system, and the propositions in Section "CONDITIONS FOR MULTISTATIONARITY IN CDR REACTION NETWORKS" make this explicit for each of the four CDR strategies analyzed.

Note the asymmetry in these definitions: the P-null class arises when the numerator of R vanishes (i.e., $p_2 = p_1$), while the Q-null class arises when the denominator vanishes (i.e., $q_2 = q_1$) with a nonzero numerator. The two classes are therefore structurally distinct, and as Propositions 3 and 4 demonstrate, they can exhibit different multistationarity behavior within the same CDR system.

Therefore, an RNCDR kinetic system for a single CDR technology consists of five species (A_1, A_2, A_3, A_4, and one CDR storage species A_i) and at least seven reactions. The latter include the five reactions R_1–R_5 that are common to all RNCDR models, the common CDR storage reaction $R_{i,6} : A_i \to A_4$, and at least one CDR-specific reaction $R_{i,j}$ for $j \geq 7$, whose number and form depend on the particular CDR technology deployed. The dynamics of the system follow power-law kinetics encoded by the matrix F in (8), with the emission kinetic orders e_i and f_i determined by the storage parameters λ_i and μ_i via (5).

Figure 2 summarizes how the RNCDR framework is applied in practice. The analysis proceeds in five steps: (1) selecting a CDR strategy; (2) parametrizing its storage through λ_i and μ_i; (3) representing the system as a chemical reaction network together with its kinetic order matrix F; (4) classifying the underlying Anderies subnetwork through the ratio $R = (p_2 - p_1)/(q_2 - q_1)$; and (5) testing for multistationarity or monostationarity through the Deficiency One Algorithm, the Multistationarity Algorithm, and the injectivity test (discussed in Section "CONDITIONS FOR MULTISTATIONARITY IN CDR REACTION NETWORKS"). The outcome of this workflow is a determination of whether a given CDR system can admit multiple steady states, and therefore whether it may be vulnerable to tipping-point behavior under varying conditions. Notably, this characterization is obtained from the graphical and kinetic structure of the network alone, without requiring precise knowledge of rate constants. The four CDR strategies analyzed in this study, presented in Section "FOUR CDR STRATEGIES AND THEIR REACTION NETWORK REPRESENTATIONS", all follow this same workflow, with the differences between them arising from the specific reactions and kinetic orders introduced at steps 2 and 3.

The key question the framework is designed to answer is: *Under what conditions can a CDR system settle into more than one steady state, and therefore become vulnerable to a tipping point that would compromise the carbon removal it was designed to deliver?* As explored in the subsequent sections, the answer depends critically on the class (positive, negative, or null) of the underlying Anderies subnetwork, and on the structural features of the CDR-specific reactions, including their kinetic orders and the resulting network deficiency. This provides a basis for screening CDR strategies for carbon-accounting risk before their projected mitigation potential is committed to in deployment plans or long-term climate policy.

Multistationarity and tipping points: what this framework does and does not establish

Throughout this paper, multistationarity means the existence of more than one positive steady state, and monostationarity means that at most one exists. Relating these to tipping requires care, because tipping is not a single phenomenon. We take a tipping point to be an abrupt transition from one positive steady state to a different one, and on that definition multistationarity is necessary: the destination state must exist. The definition is

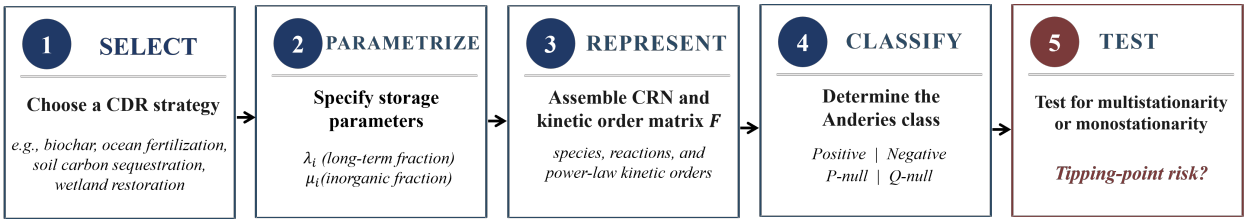

Figure 2. The RNCDR analysis workflow. Each CDR strategy is screened for tipping-point risk through five sequential steps: selecting the strategy, parametrizing its storage, representing it as a chemical reaction network, classifying the Anderies subnetwork, and testing for multistationarity or monostationarity. The framework relies on the graphical and kinetic structure of the network rather than precise rate constants.

narrower than the term as used in the wider literature. In the classification of Ashwin *et al.*[46], now standard in this field[47], a system may tip through a bifurcation of a quasi-static attractor as a control parameter drifts (B-tipping), through noise carrying it out of a basin of attraction (N-tipping), or through failing to track a moving attractor when that parameter changes too quickly (R-tipping). Each can end somewhere other than a second steady state, in a limit cycle or an excitable excursion, and R-tipping in particular can occur in systems whose steady state is unique and globally attracting[46,48]. Those cases lie outside our definition, and outside what the framework addresses.

Within that definition, what the framework establishes can be stated exactly. A system admitting at most one positive steady state throughout a parameter range has no second state to move to, and so cannot undergo a transition of this kind however it is forced. That is a conclusion about the dynamics obtained from the structure of the network alone, and it holds whichever mechanism might otherwise have driven the transition. The best studied of these is the fold, or saddle-node, bifurcation, in which two steady states merge and annihilate as a slowly varying control passes a threshold; monostationarity excludes it directly, since the merging requires two states to begin with.

Nor does the argument require the system to be at rest. A system need not settle into a steady state for the number of steady states to matter, since the steady states and their stability determine which transitions are available to it. The results below should accordingly be read asymmetrically. A monostationarity result is an exclusion: under the stated kinetic-order conditions the model cannot support multiple positive steady states, and so cannot support a transition between them at all. A multistationarity result is a flag rather than a diagnosis: the necessary condition is met, and a dynamical study of the configuration is warranted before the strategy is relied upon. Establishing that a transition would actually occur requires stability, basin geometry, perturbation magnitude and timescale, none of which is determined here, and we retain the language of tipping-point risk only because it is the concern that makes the structural question worth asking.

FOUR CDR STRATEGIES AND THEIR REACTION NETWORK REPRESENTATIONS

This section presents the reaction network representations of the four CDR strategies analyzed in this study — biochar sequestration, ocean fertilization, soil carbon sequestration, and wetland restoration — together with the box models used to represent each system, their corresponding RNCDR reaction network translation, and the kinetic order matrices encoding their dynamics. Each system extends the RNCDR framework of Section "THE RNCDR FRAMEWORK" by specifying the CDR storage species A_i, the storage parameters λ_i and μ_i, and the CDR-specific reactions $R_{i,6}$ and $R_{i,j}$ for $j \geq 7$. Casting all four strategies in a common reaction network form makes their structural differences explicit, and provides the basis for the comparative analysis of multistationarity in Sections "CONDITIONS FOR MULTISTATIONARITY IN CDR REACTION NETWORKS" and "COMPARATIVE ANALYSIS AND IMPLICATIONS". This is one way in which a reaction network approach

can complement existing assessments of CDR options: by exposing structural features that may be relevant to choosing among strategies, but that may not be apparent from numerical simulation alone. The full ODE systems for each model are provided in Supplementary Section 3 for reference.

The four strategies were chosen to span the ways in which CDR systems physically hold carbon and the settings in which they are deployed, rather than to sample a single class of technology. Biochar sequestration stores carbon as a solid, thermochemically stabilized product applied to agricultural soil. Ocean fertilization stores it in the deep ocean by way of the marine biological pump. Soil carbon sequestration stores it as soil organic matter maintained by land management practice. Wetland restoration stores it in waterlogged sediment, where anoxia suppresses decomposition[43]. The four therefore differ in the medium that holds the carbon, in how long it is held, and in how it is lost: biochar by slow oxidation of the applied char[40,49], ocean carbon by overturning circulation returning it to the surface, soil carbon by tillage or a reversal of land use, and wetland carbon by drainage[19].

They differ likewise in deployment setting and maturity. Biochar is applied in a distributed way on agricultural land and is already produced commercially. Soil carbon sequestration operates over large areas of existing farmland at low cost per hectare, but its gains saturate as soils approach a new equilibrium and are difficult to verify[42,50]. Wetland restoration is undertaken at the scale of individual coastal and inland sites, usually alongside biodiversity and flood-protection objectives. Ocean fertilization requires no land at all, but its efficacy is contested and it is restricted under international marine agreements. The set thus ranges from a commercially delivered engineered product, through two forms of land and ecosystem management, to an open-ocean intervention that remains experimental.

This spread is what makes the four informative within the RNCDR framework, since permanence and reversibility enter the model directly through λ_i, μ_i and the leak reaction $R_{i,6}$. As a consequence the four occupy near-extremal positions in the (λ_i, μ_i) plane, namely $(1, 0.5)$, $(1, 0)$, $(0.01, 0)$ and $(0.01, 0.9)$; they cover both deficiency classes, with biochar and soil carbon sequestration admitting the Deficiency One Algorithm and ocean fertilization and wetland restoration requiring the Multistationarity Algorithm; and they cover the range of coupling to the natural carbon cycle, from CDR reactions that leave R_1–R_4 structurally unchanged to explicit bidirectional atmosphere–storage exchange.

The remaining technologies in Table 2 are excluded for specific reasons. Direct air capture, direct ocean capture and bioenergy with carbon capture and storage have already been analyzed within this framework[28,29,30], and all three store captured CO_2 by injection into geological formations, so including them here would duplicate both the published analysis and a storage mechanism already covered. Enhanced weathering and ocean alkalinization store carbon geochemically, as dissolved bicarbonate and carbonate rather than as organic matter, and the carbon follows a long multi-step path from mineral grain through soil solution and rivers to the ocean; representing that path faithfully would require pools beyond the five used here, and we therefore treat the two together as the natural next case. Afforestation and reforestation store carbon in standing biomass and soil on a route close to that of biochar, and would add no new coverage of the storage-parameter space, since $\lambda_{15} = 0.5$ and $\mu_{15} = 0$ give $c_{15} = 0.5$, the same withheld fraction as biochar; they are the subject of a separate study in preparation by a group that includes several of the present authors.

Biochar sequestration

Biochar sequestration, hereafter referred to as the BCS system, involves incorporating carbon-rich biochar into soils to enhance long-term carbon storage[40]. Biochar is produced from biomass through pyrolysis, a process of burning organic material in an oxygen-limited environment, creating a stable, carbon-rich material that resists decomposition. In the RNCDR framework, biochar storage is represented by A_{11}, with parameters $\lambda_{11} = 1$ and

$\mu_{11} = 0.5$: all captured carbon enters long-term storage, half as inorganic biocharcoal in soil and half as organic biofuel in the geological stock.

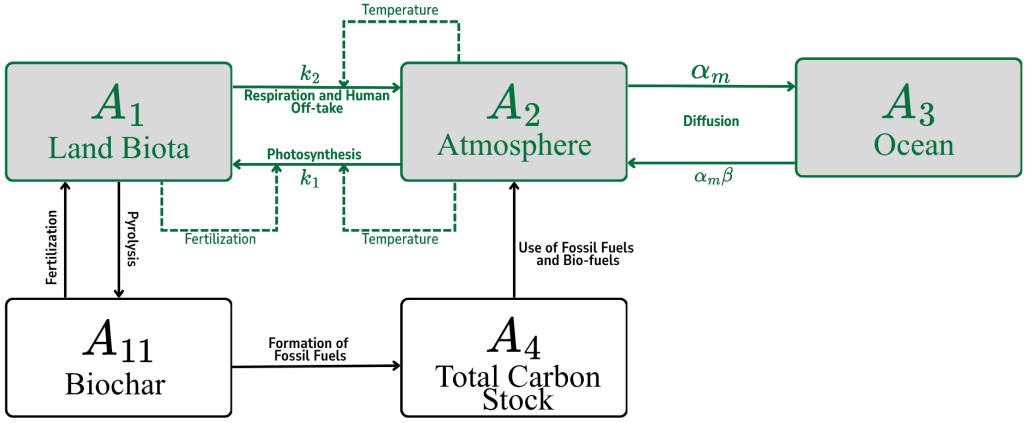

Figure 3. Box model of the biochar sequestration system. The green-outlined pools and reactions (A_1, A_2, A_3, and the transfers among them) reproduce the pre-industrial Anderies subnetwork; the black-outlined pools and reactions are specific to biochar and to the fossil-fuel accounting that links the total carbon stock to the atmosphere. The same convention is used in the box models of the remaining CDR systems.

The box model is shown in Figure 3. The BCS system involves five carbon pools: land biota (A_1), atmosphere (A_2), ocean (A_3), total carbon stock (A_4), and biochar storage (A_{11}). Biomass is converted into biochar through pyrolysis ($A_1 \to A_{11}$). When applied to soil, biochar stimulates plant growth and photosynthesis, facilitating further carbon capture ($A_{11} \to A_1$). Biochar may also undergo fossilization into the total carbon stock ($A_{11} \to A_4$), from which carbon can be re-emitted to the atmosphere through fossil fuel combustion ($A_4 \to A_2$). The CDR-specific reactions are:

$$
\begin{aligned}
R_{11,6} &: A_{11} \to A_4 && \text{(biochar to total carbon stock)},\\
R_{11,7} &: A_1 \to A_{11} && \text{(land-to-biochar capture)},\\
R_{11,8} &: A_{11} \to A_1 && \text{(biochar-to-land transfer)}.
\end{aligned}
$$

Together with the four Anderies reactions R_1–R_4 and the emission reaction R_5, this gives a BCS system with 5 species and 8 reactions. The dynamics of this system are encoded in the kinetic order matrix:

$$
F_{BCS} = \begin{array}{c} \begin{matrix} A_1 & A_2 & A_3 & A_4 & A_{11} \end{matrix} \\ \begin{bmatrix} p_1 & q_1 & 0 & 0 & 0 \\ p_2 & q_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & e_{11} & f_{11} \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_{11,6} \\ R_{11,7} \\ R_{11,8} \end{matrix}
$$

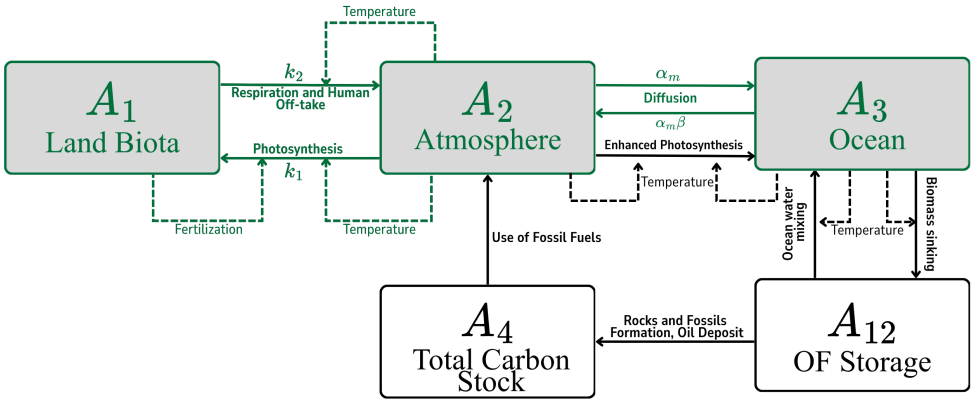

Figure 4. Box model of the OF system. As in Figure 3, the green-outlined portion reproduces the Anderies subnetwork.

The row for R_5 encodes the coupled dependence of emission on both the total carbon stock and biochar storage, with $e_{11} \geq 1$, $f_{11} \leq 0$, and $e_{11} + f_{11} = 1$. The rows for $R_{11,6}$, $R_{11,7}$, and $R_{11,8}$ reflect mass-action kinetics for the biochar-specific transfers.

Ocean fertilization

Ocean fertilization, hereafter referred to as the OF system, seeks to stimulate the growth of marine phytoplankton to enhance the oceanic uptake of CO_2 [41]. The process introduces nutrients such as iron to the ocean surface, inducing phytoplankton blooms that absorb atmospheric carbon into their biomass. When the phytoplankton die, their biomass sinks to the deep ocean where the carbon is stored for a long period of time. The efficacy of ocean fertilization is contested. Field experiments have generally found that only a small and highly variable fraction of the carbon fixed in a bloom reaches the deep ocean, and estimates of the cost per tonne removed span roughly two orders of magnitude, driven largely by uncertainty in that export efficiency [41,51]. Deployment beyond legitimate scientific research has been restricted since 2008 by resolution of the parties to the London Convention and Protocol, an approach codified in a 2013 amendment to the London Protocol that has not yet entered into force [41]. We retain it here as a modeling case rather than as an endorsement: it continues to appear in CDR portfolio inventories, and it is the analytically distinctive member of our set, being the only one of the four whose emission reaction reduces exactly to linear mass action. In the RNCDR framework, OF storage is represented by the separate pool A_{12}, which holds carbon in the deep ocean and is distinct from the total carbon stock A_4. Carbon leaves A_{12} for A_4 only through the leak reaction $R_{12,6}$, which represents fossilization on geological timescales. The parameters $\lambda_{12} = 1$ and $\mu_{12} = 0$ describe the disposition of that transferred carbon, namely that it is organic and therefore available for subsequent emission, rather than the physical location of the OF store itself. Since $\lambda_{12} = 1$ and $\mu_{12} = 0$, the emission kinetics of the OF system reduce to linear mass action: $K_5 = k_5 A_4$.

The box model is shown in Figure 4. The OF system involves five carbon pools: land biota (A_1), atmosphere (A_2), ocean (A_3), total carbon stock (A_4), and OF storage (A_{12}). The OF process introduces an enhanced photosynthesis pathway from the atmosphere to the ocean, a transfer from the ocean to OF storage representing biomass sinking to the deep ocean ($A_3 \rightarrow A_{12}$), and a reverse transfer from OF storage back to the ocean representing ocean water mixing ($A_{12} \rightarrow A_3$). The CDR-specific reactions are:

$$
\begin{aligned}
&R_{12,6} : A_{12} \to A_4 && \text{(OF to total carbon stock)},\\
&R_{12,7} : 2A_2 + A_3 \to A_2 + 2A_3 && \text{(atmosphere-to-ocean photosynthesis)},\\
&R_{12,8} : 2A_3 \to A_3 + A_{12} && \text{(ocean-to-OF capture)},\\
&R_{12,9} : A_3 + A_{12} \to 2A_3 && \text{(OF-to-ocean release)}.
\end{aligned}
$$

Together with R_1–R_4 and R_5, this gives the OF system with 5 species and 9 reactions. The dynamics of the OF system are encoded in the kinetic order matrix:

$$
F_{OF} = \begin{array}{c} \begin{matrix} A_1 & A_2 & A_3 & A_4 & A_{12} \end{matrix} \\ \begin{bmatrix} p_1 & q_1 & 0 & 0 & 0 \\ p_2 & q_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & p_3 & q_3 & 0 & 0 \\ 0 & 0 & p_4 & 0 & 0 \\ 0 & 0 & p_5 & 0 & q_5 \end{bmatrix} \end{array} \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_{12,6} \\ R_{12,7} \\ R_{12,8} \\ R_{12,9} \end{matrix}
$$

The row for R_5 reflects linear mass action kinetics ($e_{12} = 1$, $f_{12} = 0$) since $\lambda_{12} = 1$ and $\mu_{12} = 0$. The rows for $R_{12,7}$, $R_{12,8}$, and $R_{12,9}$ introduce additional kinetic orders p_3, q_3, p_4, p_5, and q_5 that encode the influence of the atmosphere and ocean pools on the OF processes.

Soil carbon sequestration

Soil carbon sequestration, hereafter referred to as the SCS system, aims to increase the organic carbon content of agricultural and natural soils through various land management practices[42]. Carbon from the atmosphere is absorbed by plants through photosynthesis and stored in the soil as soil organic carbon. Practices such as cover cropping, crop rotation, and reduced tillage are used to increase the rate at which carbon is added to the soil while decreasing the rate at which it is lost through decomposition. In the RNCDR framework, SCS storage is represented by A_{13}, with parameters $\lambda_{13} = 0.01$ and $\mu_{13} = 0$: only a very small fraction of sequestered carbon enters long-term geological storage, and this fraction is entirely organic.

The box model is shown in Figure 5. The SCS system involves five carbon pools: land biota (A_1), atmosphere (A_2), ocean (A_3), total carbon stock (A_4), and SCS storage (A_{13}). Carbon decays from land biota into SCS storage ($A_1 \to A_{13}$), from which it may enter the total carbon stock through fossilization ($A_{13} \to A_4$). From there, carbon can be re-emitted to the atmosphere through fossil fuel combustion ($A_4 \to A_2$). The CDR-specific reactions are:

$$
\begin{aligned}
&R_{13,6} : A_{13} \to A_4 && \text{(SCS to total carbon stock)},\\
&R_{13,7} : A_1 \to A_{13} && \text{(land-to-SCS capture)}.
\end{aligned}
$$

Together with R_1–R_4 and R_5, this gives the SCS system with 5 species and 7 reactions. The dynamics of the SCS system are encoded in the kinetic order matrix:

$$F_{SCS} = \begin{array}{c} \begin{matrix} A_1 & A_2 & A_3 & A_4 & A_{13} \end{matrix} \\ \begin{bmatrix} p_1 & q_1 & 0 & 0 & 0 \\ p_2 & q_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & e_{13} & f_{13} \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_{13,6} \\ R_{13,7} \end{matrix}$$

The row for R_5 encodes the coupled dependence of emission on both the total carbon stock and SCS storage, with $e_{13} \geq 1$, $f_{13} \leq 0$, and $e_{13} + f_{13} = 1$. The rows for $R_{13,6}$ and $R_{13,7}$ reflect mass-action kinetics for the SCS-specific transfers. As with the BCS system, the CDR storage species A_{13} does not appear in the rows corresponding to the natural carbon cycle reactions R_1–R_4, so the SCS-specific reactions extend the natural carbon cycle without directly modulating its rates.

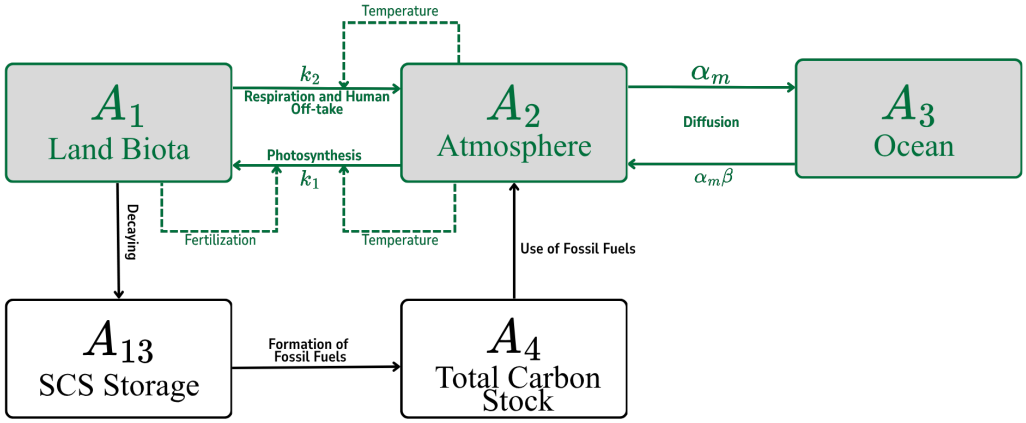

Figure 5. Box model of the SCS system. As in Figure 3, the green-outlined portion reproduces the Anderies subnetwork.

Wetland restoration

Wetland restoration, hereafter referred to as the WR system, leverages the natural carbon sequestration capacity of wetland ecosystems[43]. Wetlands are ecosystems where plants and soil are submerged in water; they can capture large amounts of CO_2 from the atmosphere and store it in soils for hundreds or even thousands of years. In the RNCDR framework, WR storage is represented by A_{14}, with parameters $\lambda_{14} = 0.01$ and $\mu_{14} = 0.9$: only a very small fraction of sequestered carbon enters long-term geological storage, and most of this fraction is inorganic.

The box model is shown in Figure 6. The WR system involves five carbon pools: land biota (A_1), atmosphere (A_2), ocean (A_3), total carbon stock (A_4), and WR storage (A_{14}). The WR system is the most complex of the four, with carbon transfers between the atmosphere and WR storage in both directions, mediated by respiration and photosynthesis processes. Carbon also transfers between land biota and WR storage in both directions ($A_1 \to A_{14}$ and $A_{14} \to A_1$), reflecting the interaction between land biota and wetland ecosystems. The CDR-specific reactions are:

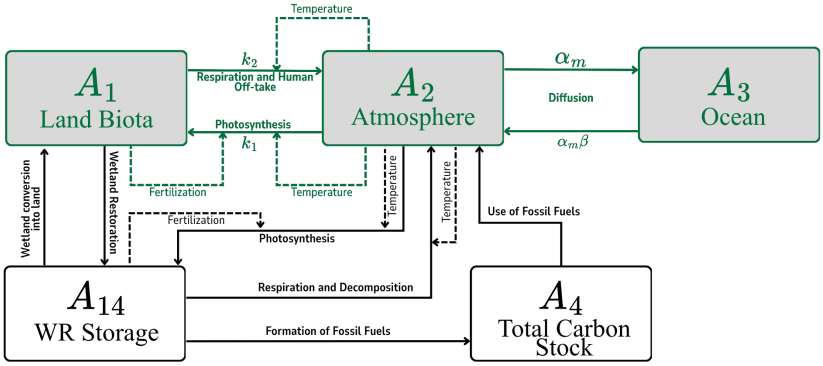

Figure 6. Box model of the WR system. As in Figure 3, the green-outlined portion reproduces the Anderies subnetwork.

$$
\begin{aligned}
R_{14,6} &: A_{14} \to A_4 && \text{(wetland to total carbon stock)},\\
R_{14,7} &: A_2 + 2A_{14} \to 2A_2 + A_{14} && \text{(wetland respiration)},\\
R_{14,8} &: 2A_2 + A_{14} \to A_2 + 2A_{14} && \text{(wetland photosynthesis)},\\
R_{14,9} &: A_1 \to A_{14} && \text{(land-to-wetland capture)},\\
R_{14,10} &: A_{14} \to A_1 && \text{(wetland-to-land transfer)}.
\end{aligned}
$$

Together with R_1–R_4 and R_5, this gives the WR system with 5 species and 10 reactions. The dynamics of the WR system are encoded in the kinetic order matrix:

$$
F_{WR} = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_{14,6} \\ R_{14,7} \\ R_{14,8} \\ R_{14,9} \\ R_{14,10} \end{array}
\begin{array}{c}
\begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_{14} \end{array} \\
\begin{bmatrix}
p_1 & q_1 & 0 & 0 & 0 \\
p_2 & q_2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & e_{14} & f_{14} \\
0 & 0 & 0 & 0 & 1 \\
0 & q_4 & 0 & 0 & p_4 \\
0 & q_5 & 0 & 0 & p_5 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

The WR system has the largest kinetic order matrix among the four systems, reflecting its greater complexity. The rows for $R_{14,7}$ and $R_{14,8}$ introduce additional kinetic orders q_4, p_4, q_5, and p_5 that encode the influence of both the atmosphere (A_2) and WR storage (A_{14}) on the respiration and photosynthesis processes within the wetland ecosystem. This bidirectional coupling between A_2 and A_{14} is a structural feature unique to the WR system.

CONDITIONS FOR MULTISTATIONARITY IN CDR REACTION NETWORKS

Having established the RNCDR representations of the four CDR systems, we now present the conditions under which each system may exhibit multistationarity, the capacity to admit more than one positive steady state, or is guaranteed to be monostationary, settling into at most one. Recall from Section "THE RNCDR FRAMEWORK"

that the dynamic behavior of the underlying Anderies subnetwork is organized by the ratio $R = (p_2 - p_1)/(q_2 - q_1)$. It was shown in [31] that a positive class ($R > 0$) is associated with potential multistationarity at the level of the Anderies subnetwork, while negative, P-null, and Q-null classes are associated with monostationarity under appropriate conditions. When the Anderies subnetwork is embedded in a full RNCDR system, however, the addition of CDR-specific reactions can either preserve or change this baseline behavior. The propositions below establish, for each of the four CDR systems, the precise conditions under which the full RNCDR system inherits, departs from, or qualitatively rewrites the Anderies-level behavior.

The analysis proceeds by applying two complementary tools to each CRN representation. A key structural index that determines which tools are applicable is the deficiency of the network. The deficiency is a non-negative measure that quantifies the degree of "linear independence" among a network's reactions [52]. A deficiency of zero represents the highest level of linear independence consistent with the network's directed graph structure. Conversely, a higher deficiency indicates a lower degree of linear independence, indicating greater complexity in the network structure. As shown in Supplementary Section 3, the BCS and SCS systems have deficiency 1, while the OF and WR systems have deficiency 2.

To establish monostationarity, we apply the injectivity test of Wiuf and Feliu [53,54], which uses a computational approach and Maple script to determine whether the determinant of the matrix M^* has all positive or all negative coefficients; if so, the network is injective and hence monostationary. To establish multistationarity, we apply either the Deficiency One Algorithm (DOA) for power-law kinetic systems [32] for the BCS and SCS systems (deficiency 1), or the Multistationarity Algorithm (MSA) of Hernandez *et al.* [55] for the OF and WR systems (deficiency 2). Both the DOA and MSA translate the question of whether multiple positive steady states can exist, a nonlinear problem, into a tractable linear system of equations and inequalities, whose solvability is equivalent to the existence of multiple steady states. The detailed computations and proofs are provided in the Supplementary Materials; here we state and interpret the results.

Proposition 1 (Biochar Sequestration) *The following hold for the BCS system:*

(i) ***Positive class.*** *If $p_2 > p_1$ and $q_2 > q_1$, or $p_1 > p_2$ and $q_1 > q_2$, then the system is multistationary.*

(ii) ***Negative class.*** *If $p_1 < 0$, $p_2 > 0$, $q_1 > 0$, $q_2 < 0$, $e_{11} \geq 1$, and $f_{11} \leq 0$, then the system is monostationary.*

(iii) ***P-null class.*** *The system is multistationary.*

(iv) ***Q-null class.*** *The system is multistationary.*

The conditions $p_2 > p_1$ and $q_2 > q_1$ mean that respiration is more strongly influenced by both land biota and atmosphere than photosynthesis is. When these inequalities hold, the nonlinear feedback between the carbon pools is strong enough to sustain multiple equilibria. Physically, this corresponds to a scenario in which the Earth's natural carbon cycle is itself in a potentially multistationary regime, and BCS does not eliminate this underlying instability. Beyond the positive class, multistationary subsets have also been identified in the P-null and Q-null classes, indicating that BCS can admit multiple steady states even when the Anderies subnetwork is in a structurally degenerate regime (i.e., when the kinetic-order ratio R is zero or undefined). The negative class remains the one Anderies regime in which the BCS system is guaranteed to be monostationary, and therefore the one in which the carbon dynamics admit at most one positive equilibrium distribution, a condition for predictable and recoverable long-term behavior.

Proposition 2 (Ocean Fertilization) *The following hold for the OF system:*

(i) ***Positive class.*** *If* $q_1 > q_2 > 0$ *and* $p_1 > p_2 > 0$ *(or* $q_2 > q_1 > 0$ *and* $p_2 > p_1 > 0$*), together with* $p_3, q_3, p_4, p_5, q_5 > 0$ *and* $p_3\mu_{A_2} > M_3$*, then the system is multistationary.*

(ii) ***Negative class.*** *If* $q_2 > q_1 > 0$ *and* $p_1 > 0 > p_2$*, together with* $p_3, q_3, p_5, q_5 > 0$*,* $p_4 < 0$ *and* $p_3\mu_{A_2} > M_3$*, then the system is monostationary.*

(iii) ***Q-null class.*** *If* $q_2 = q_1$ *and* $p_1 > 0 > p_2$*, together with* $p_3, q_3, p_5, q_5 > 0$*,* $p_4 < 0$ *and* $p_3\mu_{A_2} > M_3$*, then the system is monostationary.*

(iv) ***P-null class.*** *If* $p_2 = p_1$*,* $q_2 \neq q_1$*, and* $q_3 > 0 > p_3$*, then the system is monostationary.*

A notable feature of the OF system is that, since $\lambda_{12} = 1$ and $\mu_{12} = 0$, the emission reaction reduces to linear mass action ($K_5 = k_5 A_4$), the simplest possible form. This is the one system of the four for which the withheld fraction c_i vanishes, $c_{12} = 0$, and the reduction is contingent on that: any departure from these values, meaning $\lambda_{12} < 1$ or $\mu_{12} > 0$, would give $f_{12} < 0$ and move the system off mass action. This means the total carbon stock contributes to atmospheric CO_2 at a rate directly proportional to its size, with no modulating effect from the OF storage pool. Despite this simplification, the OF system retains the same potential for multistationarity as the more complex systems. The additional kinetic orders p_3, q_3, p_4, p_5, and q_5 introduced by the OF-specific reactions must also satisfy appropriate sign conditions for multistationarity to arise, reflecting the richer kinetic structure of the OF system relative to the other three. In addition to these sign conditions, the positive, negative, and Q-null cases each carry the requirement $p_3\mu_{A_2} > M_3$, where μ_{A_2} is the equilibrium value of the atmospheric pool A_2 and M_3 is a parameter arising from the multistationarity analysis (see Supplementary Materials). Intuitively, this requirement constrains how strongly the OF-mediated atmosphere-to-ocean photosynthesis pathway depends on the atmospheric pool relative to the rest of the network; it must be satisfied for the parametric conditions identified in Proposition 2 to apply. In real-world terms, this means that the steady-state behavior of an OF deployment depends not only on which Anderies class the natural carbon cycle is in, but also on whether the enhanced photosynthesis pathway is sensitive enough to atmospheric CO_2 for the framework's sufficient conditions to be in effect.

Proposition 3 (Soil Carbon Sequestration) *The following hold for the SCS system:*

(i) ***Positive class.*** *If* $p_2 > p_1 > 0$ *and* $q_2 > q_1 > 0$*, or* $p_1 > p_2 > 0$ *and* $q_1 > q_2 > 0$*, then the system is multistationary.*

(ii) ***Negative class.*** *If* $p_1, q_2 < 0$ *and* $p_2, q_1 > 0$*, together with* $e_{13} \geq 1$ *and* $f_{13} \leq 0$*, then the system is monostationary.*

(iii) ***Q-null class.*** *If* $q_2 = q_1$ *and* $p_1 > p_2 > 0$*, then the system is multistationary.*

(iv) ***P-null class.*** *If* $p_2 = p_1$ *and* $q_1 \neq q_2$ *(with* $q_1 \neq 0$*), then the system is multistationary.*

Despite differences in their underlying physical processes, the SCS and BCS systems exhibit the same multistationarity profile across Anderies classes: both admit multistationary subsets in the positive, P-null, and Q-null classes, with monostationarity guaranteed only in the negative class under appropriate sign conditions. The two systems therefore share the broadest reach of multistationarity across Anderies classes among the four strategies analyzed.

Both BCS and SCS admit the DOA, which identifies parameter conditions under which multiple steady states exist in the positive, P-null, and Q-null classes for these networks. The DOA's applicability follows from the deficiency-one structure of both networks together with the specific kinetic-order pattern in which the CDR storage species (A_{13} for SCS, A_{11} for BCS) appears only in rows corresponding to CDR-specific reactions in the kinetic order matrix, leaving the natural carbon cycle reactions R_1–R_4 structurally unchanged. Under this combined structural setting, multistationarity can arise in the P-null and Q-null classes, classes that are monostationary at the level of the Anderies subnetwork in isolation. From a policy standpoint, this means that SCS deployments may face tipping-point risks not only when the natural carbon cycle is in a regime that would produce them on its own, but also under regimes that would otherwise be classified as structurally degenerate (i.e., when the kinetic-order ratio R is zero or undefined). Analyses that classify CDR risk solely on the basis of the Anderies regime of the natural carbon cycle therefore risk missing the additional multistationarity pathways that deficiency-one CDR systems of the kind shared by BCS and SCS can introduce.

Proposition 4 (Wetland Restoration) *The following hold for the WR system:*

(i) ***Positive class.*** *If $p_1 \neq p_2$ or $p_4 \neq p_5$, together with $e_{14} > 0$ and $f_{14} < 0$, then the system is monostationary.*

(ii) ***Negative class.*** *If $p_1 \neq p_2$ or $p_4 \neq p_5$, then the system is multistationary.*

(iii) ***Q-null class.***
- *(a) If $(q_4 - q_5)/(p_5 - p_4) > 0$ and $p_4 \neq p_5$, then the system is monostationary.*
- *(b) If $(q_4 - q_5)/(p_5 - p_4) < 0$ and $p_4 \neq p_5$, then the system is multistationary.*

The WR system exhibits the most nuanced multistationarity behavior among the four systems. Unlike the BCS, OF, and SCS systems, where multistationarity arises in the positive class, the WR system is monostationary in the positive class and multistationary in the negative class. This is a consequence of the richer network structure of the WR system, which involves bidirectional transfers between the atmosphere and WR storage mediated by additional kinetic orders q_4, p_4, q_5, and p_5. These additional parameters introduce a new ratio $(q_4 - q_5)/(p_5 - p_4)$ that governs the behavior of the Q-null class, independent of the Anderies subnetwork class. This makes the WR system the most structurally complex of the four systems, with multistationarity behavior that depends on the largest number of kinetic parameters among the four systems analyzed.

This result also depends on the storage parameterization in a way the others do not. Proposition 4(i) requires $f_{14} < 0$ strictly, which holds only when $c_{14} \neq 0$. Table 3 assigns wetland restoration $\lambda_{14} = 0.01$ and $\mu_{14} = 0.9$, giving $c_{14} = 0.999$, so the condition is comfortably met; under a parameterization with $\lambda_{14} = 1$ and $\mu_{14} = 0$ one would have $f_{14} = 0$ and Proposition 4(i) would not apply. Monostationarity of the wetland system in the positive Anderies class is therefore contingent on those values, and Supplementary Section 2 sets out the dependence in full.

This reversal of the positive- and negative-class behavior carries an implication for how WR might be represented in larger climate models. Models that capture the natural carbon cycle but treat the wetland–atmosphere interaction as a one-way flux to a storage pool would correspond, in the RNCDR framework, to a system without the bidirectional coupling encoded in q_4, p_4, q_5, and p_5, and therefore to a deficiency-one system without atmosphere–storage interaction, structurally closer to the BCS and SCS cases. Under such a simplified representation, the wetland system would be expected to inherit the multistationarity pattern that BCS and SCS exhibit, in particular multistationarity in the positive Anderies class. The reversal observed here, with monostationarity in the positive class once the two-way exchange between the atmosphere and the wetland is explicitly

represented, indicates that this inherited behavior may not be the right baseline expectation for WR. Whether the same reversal appears in higher-dimensional climate models is a question this framework cannot settle on its own, since the kinetic orders q_4, p_4, q_5, and p_5 would need to be estimated from observational or modeling data before the ratio $(q_4 - q_5)/(p_5 - p_4)$ could be evaluated for a specific deployment context. What the framework does offer is a structural reason to expect that the level of detail at which wetland–atmosphere exchange is represented can change the qualitative steady-state behavior of the coupled system, not just the quantitative size of the carbon flux.

COMPARATIVE ANALYSIS AND IMPLICATIONS

Comparative analysis

Table 4 summarizes the steady-state capacity of the four CDR systems by Anderies class. The results established in Section "CONDITIONS FOR MULTISTATIONARITY IN CDR REACTION NETWORKS" are sufficient conditions. They identify specific parameter combinations under which a system is guaranteed to contain multistationary or monostationary subsets. They do not preclude other behaviors under different parameter combinations. The Anderies class, defined by the signs of p_1, p_2, q_1, q_2, is a necessary first input for every entry in the table but not a complete one: each proposition additionally requires sign conditions on kinetic orders specific to the CDR technology itself, so no cell in Table 4 is determined by the Anderies class alone.

Table 4. Visual summary of steady-state capacity of the four CDR systems by Anderies class

System	Positive	Negative	P-null	Q-null
BCS	MS[a]	mono[b]	MS[a]	MS[a]
OF	MS[a]	mono[b]	mono[b]	mono[b]
SCS	MS[a]	mono[b]	MS[a]	MS[a]
WR	mono[b]	MS[a]	—	both[†c]

[†]Depending on the sign of $(q_4 - q_5)/(p_5 - p_4)$. [a]indicates multistationary subsets identified; [b]indicates monostationary subsets identified; [c]indicates both identified depending on parameter values; a dash indicates no conclusion established.

It bears emphasis that a system belonging to the positive class is not guaranteed to be multistationary for all parameter combinations; rather, there exist specific parameter regions within that class where multiple positive steady states can occur. Establishing whether a particular real-world deployment falls within such a region requires empirical estimation of the kinetic orders, a potential research direction discussed in Section "SUMMARY, CONCLUSION, AND FUTURE RESEARCH".

With this caveat in mind, the following observations can be made. The OF system stands alone in having multistationary subsets identified exclusively in the positive Anderies class. This pattern does not mean the Anderies class alone determines the outcome for OF: as for the other three systems, every case in Proposition 2 additionally requires sign conditions specific to the OF reactions themselves. The BCS and SCS systems share the same multistationarity profile: multistationary subsets have been identified in the positive class and also in both the P-null and Q-null classes, with monostationarity guaranteed only in the negative class. This shared profile follows from shared structural properties of the two networks. Both are deficiency-one networks in which the CDR-specific reactions extend the natural carbon cycle without directly modulating its rates. For both, the DOA identifies parameter conditions under which multistationarity arises in the positive, P-null, and Q-null classes. The WR system departs most significantly from the others: monostationary subsets have been identified in the positive class and multistationary subsets in the negative class, with the Q-null class containing both monostationary and multistationary subsets depending on the sign of $(q_4 - q_5)/(p_5 - p_4)$.

These differences can be understood in terms of the structural features of each CDR system. Among the four, the OF system is the deficiency-two network in which the CDR-specific reactions do not directly modulate the rates of the natural carbon cycle reactions R_1–R_4, while adding a parallel atmosphere-to-ocean flux. This does not make the technology's own parameters irrelevant, however: whether the multistationarity flagged by the positive Anderies class is actually realized still depends on sign conditions specific to the OF reactions (Proposition 2). The BCS and SCS systems are both deficiency-one networks whose CDR-specific reactions also leave the natural carbon cycle reactions R_1–R_4 structurally unchanged. Applied to these networks, the DOA identifies parameter conditions under which multistationarity arises in the P-null and Q-null classes. Here the additional pathways to multistationarity come from network-level structural properties rather than from direct coupling between the CDR storage and the natural cycle. The WR system goes furthest in structural complexity. Bidirectional transfers between the atmosphere and WR storage introduce additional kinetic orders q_4, p_4, q_5 and p_5. These give rise to a new ratio $(q_4 - q_5)/(p_5 - p_4)$, which governs the Q-null class behavior independently of the Anderies subnetwork class. This structural richness produces multistationarity behavior that is the most parameter-dependent among the four systems, as captured in Proposition 4.

Implications for CDR risk assessment

The findings of this study have three important implications for CDR risk assessment and carbon neutrality planning.

First, the Anderies class of the natural carbon cycle should be treated as a prerequisite screening criterion in CDR risk assessment. Before evaluating any specific technology, policymakers and modelers could check whether the background carbon cycle is in a positive, negative, or null class. For the OF system, multistationary subsets were identified in the positive Anderies class alone. That determination is not by itself sufficient, however: Proposition 2 additionally requires sign conditions on the OF-specific kinetic orders p_3, q_3, p_4, p_5, q_5 and the condition $p_3 \mu_{A_2} > M_3$, so the Anderies class is a necessary first input rather than a complete characterization. For the BCS, SCS, and WR systems, the Anderies class remains an important first input, but it does not by itself characterize tipping-point risk. BCS and SCS exhibit multistationary subsets across the positive, P-null, and Q-null classes, while WR exhibits a more nuanced pattern that includes a reversal between the positive and negative classes. Empirical estimates of p_1, p_2, q_1, q_2 from observed carbon flux data (even rough ones) therefore provide substantial risk-screening value at low computational cost, but should be paired with attention to CDR-specific structural features for the latter three systems.

Second, the four systems demonstrate that the network-level structure of a CDR system is itself risk-relevant, and that this structure can take qualitatively different forms. The OF system illustrates a deficiency-two case in which the CDR-specific reactions do not directly modulate the natural carbon cycle reactions; even so, the system's steady-state behavior is not organized by the Anderies class alone, since realizing the multistationarity it flags additionally requires sign conditions specific to the OF reactions (Proposition 2). The BCS and SCS systems illustrate the deficiency-one case in which the CDR-specific reactions extend the natural carbon cycle without modulating its rates, and the DOA identifies parameter conditions under which multistationarity arises in classes that would be monostationary at the Anderies level. The WR system illustrates the case in which bidirectional coupling between the storage pool and the atmosphere, encoded in the kinetic orders q_4, p_4, q_5, p_5, can actually suppress multistationarity in the positive class, a potentially counterintuitive protective effect. Taken together, these contrasts suggest that the structural design of a CDR technology, in particular its network deficiency and whether and how its storage pool feeds back into natural carbon cycle reactions, warrants attention as a risk-relevant design parameter in CDR evaluation frameworks.

Third, the parameter-minimal nature of the RNCDR framework means that the conclusions above hold without requiring precise knowledge of rate constants. The framework relies primarily on the graphical and kinetic structure of the network, rather than on specific parameter values, making it particularly well-suited to systems

in which parameter uncertainty is high and exhaustive numerical simulation is impractical. As a practical example, the sign conditions on p_1, p_2, q_1, q_2 identified in Propositions 1–4 provide a concrete set of criteria that could guide the empirical estimation of kinetic orders from observed carbon flux data in future work. This approach complements existing numerical modeling studies by providing a systematic and tractable basis for assessing tipping-point risk across CDR strategies.

Relation to other CDR assessment methods

Table 5 places the RNCDR framework alongside the other families of methods used to assess CDR strategies. The framework is not an alternative to numerical simulation. It is a screen applied before simulation, and the asymmetry noted in Section "Multistationarity and tipping points: what this framework does and does not establish" defines its practical scope: a guaranteed monostationarity result excludes a configuration from further dynamical investigation, whereas a multistationarity result identifies where such investigation is warranted. Its computational cost is negligible by comparison, because the propositions follow from the structure of the network rather than from integrating the governing equations.

Table 5. Families of methods used in CDR assessment, and the question each answers

Approach	Question answered	Requirements	Cannot establish
Earth system models [15,16]	Transient response to prescribed CDR forcing	Full parameterization; high-performance computing	Equilibrium multiplicity
Integrated assessment models [9,17]	Portfolio composition under economic constraints	Cost and land-use data	Carbon-cycle dynamics
Reduced-complexity box models [3,7,8]	Trajectories and equilibria of aggregated pools	Rate constants	Generality across parameter values
Permanence and verification frameworks [18,19]	Durability and auditability of a deployment	Site-level monitoring	System-level feedbacks
RNCDR (this work)	Whether more than one positive steady state can exist	Network structure and kinetic-order signs	Stability, basins, timescales

Limitations

Several limits bear on how these results should be used. The conditions established in Section "CONDITIONS FOR MULTISTATIONARITY IN CDR REACTION NETWORKS" are sufficient rather than necessary. They identify parameter regions in which multistationarity or monostationarity is guaranteed, but they do not partition the parameter space, and behavior outside those regions is not determined by the present analysis. Nor does the existence of multiple positive steady states imply that more than one of them is an attractor. Establishing bistability would require a linearized stability analysis that we do not perform, which is the distinction drawn in Section "Multistationarity and tipping points: what this framework does and does not establish": multistationarity is a structural precondition for one tipping mechanism rather than a demonstration of tipping. The scope is also narrower than the word "tipping" suggests. We take a tipping point to be an abrupt transition from one positive steady state to another, and the results speak only to that. Abrupt behavior that ends somewhere else (e.g., in a limit cycle) lies outside the definition; so does rate-induced tipping in a system with a unique steady state. This is not a remote concern for the systems studied here. The compost-bomb instability, in which soil carbon is released explosively above a critical rate of warming, is a rate-induced phenomenon. It arises in a soil-carbon model whose steady state is unique and globally attracting [48,56]. A monostationarity result for the SCS system would not exclude behavior of that kind.

The representation is also highly aggregated. It has five carbon pools, no spatial resolution and no explicit temperature state variable, so climate feedbacks enter only through the kinetic orders rather than as dynamical variables in their own right.

Two sets of parameters remain unconstrained by observation. The Anderies classification depends on p_1, p_2, q_1 and q_2, which have not been estimated from carbon flux data; the sign conditions in Propositions 1–4 therefore identify which qualitative behaviors are possible for this class of model rather than which occur in any particular deployment. The storage parameters in Table 3 are likewise nominal, chosen to span the parameter space rather than measured. Supplementary Section 2 sets out which propositions are sensitive to them.

Finally, the analysis covers single-technology deployments only; portfolios of simultaneously deployed strategies are not treated here, although the framework accommodates them. And a steady-state analysis is silent on time. It says nothing about how long a transition between states would take, which is precisely the quantity that determines whether monitoring, reporting and verification systems could detect one.

SUMMARY, CONCLUSION, AND FUTURE RESEARCH

This study applied the RNCDR framework to analyze the steady-state multiplicity of four CDR strategies, namely biochar sequestration, ocean fertilization, soil carbon sequestration, and wetland restoration, by translating each system into a chemical reaction network with power-law kinetics and examining the graphical and kinetic structure of the resulting networks. The analysis identified sufficient conditions under which each system may exhibit multistationarity or is guaranteed to be monostationary, without requiring precise knowledge of rate constants. The key findings are summarized below.

- Among the four systems, OF is the one whose multistationary subsets are confined to a single Anderies class, the positive class ($R > 0$), with monostationary subsets identified in the negative, P-null, and Q-null classes. This confinement to one class does not mean the Anderies class alone determines the outcome, however: every case in Proposition 2, including the positive class, additionally requires sign conditions specific to the OF reactions, so the technology's own parameters remain part of the determination for OF exactly as they do for BCS, SCS, and WR.

- The BCS and SCS systems share the broadest reach of multistationarity across Anderies classes among the four strategies analyzed. Both exhibit multistationary subsets in the positive, P-null, and Q-null classes, with monostationarity guaranteed only in the negative class under appropriate sign conditions. This shared profile is a consequence of shared structural properties: both BCS and SCS are deficiency-one networks in which the CDR-specific reactions extend the natural carbon cycle without directly modulating its rates, and the DOA identifies parameter conditions under which multistationarity arises in classes that would be monostationary at the level of the Anderies subnetwork in isolation.

- The WR system exhibits the most parameter-dependent multistationarity behavior among the four systems analyzed. In contrast to the BCS, OF, and SCS systems, the WR system is monostationary in the positive Anderies class and multistationary in the negative class. The Q-null class contains both monostationary and multistationary subsets, depending on the sign of $(q_4 - q_5)/(p_5 - p_4)$. This behavior is a consequence of the bidirectional coupling between the atmosphere and WR storage introduced by the kinetic orders q_4, p_4, q_5, and p_5, which gives rise to a new ratio governing the Q-null class behavior independently of the Anderies subnetwork class.

- The structural design of a CDR system is itself a risk-relevant parameter, and it can take qualitatively different forms across CDR strategies. The four systems analyzed here span a range from a deficiency-two network with no direct modulation of the natural cycle (OF), to deficiency-one networks whose CDR-specific reactions extend the natural cycle without modulating its rates while admitting multistationarity in additional classes through the conditions identified by the DOA (BCS and SCS), to a deficiency-two network with bidirectional

coupling between the storage pool and the atmosphere (WR). Each of these forms produces a distinct multistationarity profile across Anderies classes, demonstrating that the network-level structure of a CDR system can either introduce or suppress tipping-point risk in ways that are not predictable from the Anderies subnetwork class alone.

- The RNCDR framework provides a structure-based route to screening for multistationarity, which is a necessary condition for an abrupt transition between steady states. By relying on the graphical and kinetic structure of the network rather than precise parameter values, the framework can characterize multistationarity conditions across all four CDR systems without exhaustive numerical simulation. The sign conditions on p_1, p_2, q_1, q_2 identified in Propositions 1–4 provide a concrete set of criteria that could guide the empirical estimation of kinetic orders from observed carbon flux data in future work.

For Earth system science, this study demonstrates that CRNT offers a tractable and systematic complement to conceptual and numerical carbon-cycle modeling for the analysis of steady-state multiplicity, and hence of one structural precondition for tipping-point behavior. In this framework, carbon pools are represented as interacting components and carbon transfers as reactions, and the question of whether the system can settle into more than one equilibrium is answered from the structure of these interactions rather than from numerical simulation. The RNCDR framework identifies structural drivers of multistationarity from the graphical and kinetic structure of simple heuristic box models, complementing the insights available from high-dimensional computational models. The framework is designed to accommodate other CDR technologies and portfolios of simultaneously deployed strategies, as noted in Section "Limitations", though demonstrating this extensibility remains a direction for future work.

For climate policy, the findings underscore that the reliability of a CDR strategy as a climate mitigation tool cannot be assessed independently of the dynamic regime of the natural carbon cycle into which it is deployed. The natural carbon cycle can operate in qualitatively different regimes, characterized here by the Anderies class, which determines whether the system is capable of settling into more than one equilibrium carbon distribution. A CDR system deployed into a multistationary regime may settle into a steady state characterized by low net CO_2 removal performance, undermining the carbon accounting assumptions underlying its deployment. The results suggest that a basic characterization of this background regime, through estimation of how photosynthesis and respiration respond to changes in carbon pool sizes, should be treated as a prerequisite screening step in CDR risk assessment, prior to the evaluation of any specific technology. Furthermore, the structural design of CDR technologies, specifically the nature of the feedback between the CDR storage pool and the natural carbon cycle, should be recognized as a risk-relevant design parameter in the development and evaluation of CDR deployment strategies. Aligning these findings with IPCC-style scenario analyses could provide further insights into the role of each CDR strategy in achieving long-term mitigation targets and inform the prioritization of technologies in national and global climate action plans[12].

Several directions for future research emerge from this study. Empirical validation of the kinetic order conditions identified here will be essential to strengthen the credibility and applicability of the results. In particular, estimating the kinetic orders p_1, p_2, q_1, q_2 for the natural carbon cycle and the CDR-specific kinetic orders from real-world data would allow the Anderies class and the multistationarity conditions to be evaluated for specific geographical regions or deployment scenarios.

Extending the framework to portfolios of multiple CDR technologies deployed simultaneously is another important direction. The RNCDR framework is designed to accommodate such combinations, and analyzing the multistationarity behavior of CDR portfolios could reveal synergies or trade-offs that are not apparent from single-technology analyses. One case is of particular interest. BCS and SCS have multistationarity broadly

distributed across Anderies classes, whereas that of OF is anchored more narrowly to a single class. Whether combining the two kinds reduces the overall tipping-point risk of a portfolio is an open question.

Finally, the model outcomes could be aligned with IPCC-style scenario analyses[12]. Doing so would give insight into the role of each CDR strategy in meeting long-term mitigation targets, and would inform how technologies are prioritized in national and global climate action plans.

DECLARATIONS

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Availability of data and materials

The original contributions presented in this study are included in the article/Supplementary Materials. Further inquiries can be directed to the corresponding author.

AI and AI-assisted tools statement

During the preparation of this manuscript, the AI tool Claude (Anthropic; versions Claude Opus 4.7, released 2026-04-16, and Claude Opus 5, released 2026-07-24) was used solely for language editing and improving the readability of portions of the manuscript. The tool did not influence the study design, data collection, analysis, interpretation, or the scientific content of the work. All authors take full responsibility for the accuracy, integrity, and final content of the manuscript.

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Supplementary Materials
Supplementary Materials

REFERENCES

1. Bonan, G. B.; Doney, S. C. Climate, ecosystems, and planetary futures: the challenge to predict life in earth system models. *Science* **2018**, *359*, eaam8328. DOI
2. Flato, G. M. Earth system models: an overview. *WIREs Clim. Chang.* **2011**, *2*, 783-800. DOI
3. Anderies, J. M.; Carpenter, S. R.; Steffen, W.; Rockström, J. The topology of non-linear global carbon dynamics: from tipping points to planetary boundaries. *Environ. Res. Lett.* **2013**, *8*, 044048. DOI
4. Lenton, T. M.; Held, H.; Kriegler, E.; et al. Tipping elements in the earth's climate system. *Proc. Natl. Acad. Sci. USA* **2008**, *105*, 1786-93. DOI
5. Rockström, J.; Steffen, W.; Noone, K.; et al. Planetary boundaries: exploring the safe operating space for humanity. *Ecol. Soc.* **2009**, *14*, art32. DOI
6. Armstrong McKay, D. I.; Staal, A.; Abrams, J. F.; et al. Exceeding 1.5°C global warming could trigger multiple climate tipping points. *Science* **2022**, *377*, eabn7950. DOI
7. Mills, B. J. W.; Tennenbaum, S.; Schwartzman, D. Exploring multiple steady states in earth's long-term carbon cycle. *Am. J. Sci.* **2021**, *321*, 1033-44. DOI
8. Zhu, F.; Rose, B. E. J. Multiple equilibria in a coupled climate–carbon model. *J. Clim.* **2023**, *36*, 547-64. DOI
9. Chiquier, S.; Gurgel, A.; Morris, J.; Chen, Y. H.; Paltsev, S. Integrated assessment of carbon dioxide removal portfolios: land, energy, and economic trade-offs for climate policy. *Environ. Res. Lett.* **2025**, *20*, 024002. DOI
10. Keller, D. P.; Lenton, A.; Littleton, E. W.; Oschlies, A.; Scott, V.; Vaughan, N. E. The effects of carbon dioxide removal on the carbon cycle. *Curr. Clim. Change Rep.* **2018**, *4*, 250-65. DOI
11. Morrow, D. R.; Thompson, M. S.; Anderson, A.; et al. Principles for thinking about carbon dioxide removal in just climate policy. *One Earth* **2020**, *3*, 150-53. DOI
12. Shukla, P. R.; Skea, J.; Reisinger, A.; et al. Climate change 2022 - mitigation of climate change. In Proceedings of the Contribution of Working Group III Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press; 2023. DOI
13. Bindl, M.; Edwards, M. R.; Cui, R. Y. Risks of relying on uncertain carbon dioxide removal in climate policy. *Nat. Commun.* **2025**, *16*, 5958. DOI
14. Buylova, A.; Fridahl, M.; Nasiritousi, N.; Reischl, G. Cancel (out) emissions? The envisaged role of carbon dioxide removal technologies in long-term national climate strategies. *Front. Clim.* **2021**, *3*, 675499. DOI
15. Keller, D. P.; Lenton, A.; Scott, V.; et al. The carbon dioxide removal model intercomparison project (CDRMIP): rationale and experimental protocol for CMIP6. *Geosci. Model Dev.* **2018**, *11*, 1133-60. DOI
16. Asaadi, A.; Schwinger, J.; Lee, H.; et al. Carbon cycle feedbacks in an idealized simulation and a scenario simulation of negative emissions in CMIP6 earth system models. *Biogeosciences* **2024**, *21*, 411-35. DOI
17. Mendez, Q. R.; Creutzig, F.; Fuss, S. Deep uncertainty in carbon dioxide removal portfolios. *Environ. Res. Lett.* **2025**, *20*, 054013. DOI
18. Yao, Y.; Zhang, B. Life cycle assessment in the monitoring, reporting, and verification of land-based carbon dioxide removal: gaps and opportunities. *Environ. Sci. Technol.* **2025**, *59*, 11950-63. DOI
19. Streck, C.; Minoli, S.; Roe, S.; et al. Considering durability in carbon dioxide removal strategies for climate change mitigation. *Clim. Policy* **2026**, *26*, 493-501. DOI
20. Ampah, J. D.; Jin, C.; Liu, H.; et al. Deployment expectations of multi-gigatonne scale carbon removal could have adverse impacts on Asia's energy-water-land nexus. *Nat. Commun.* **2024**, *15*, 6342. DOI
21. Smith, S. M.; Geden, O.; Gidden, M. J.; et al. The state of carbon dioxide removal, 2nd ed. University of Oxford; 2024. DOI
22. Reed, K. A.; Medeiros, B.; Jablonowski, C.; Simpson, I. R.; Voigt, A.; Wing, A. A. Why idealized models are more important than ever in earth system science. *AGU Adv.* **2025**, *6*, e2025AV001716. DOI
23. Conradi, C.; Flockerzi, D.; Raisch, J.; Stelling, J. Subnetwork analysis reveals dynamic features of complex (bio)chemical networks. *Proc. Natl. Acad. Sci. USA* **2007**, *104*, 19175-80. DOI
24. Hernandez, B. S.; De La Cruz, R. J. L. Independent decompositions of chemical reaction networks. *Bull. Math. Biol.* **2021**, *83*, 76. DOI
25. Delmas, E.; Besson, M.; Brice, M. H.; et al. Analysing ecological networks of species interactions. *Biol. Rev.* **2018**, *94*, 16-36. DOI
26. Veloz, T.; Flores, D. Reaction network modeling of complex ecological interactions: endosymbiosis and multilevel regulation. *Complexity* **2021**, *2021*, 8760937. DOI
27. Avram, F.; Adenane, R.; Neagu, M. Advancing mathematical epidemiology and chemical reaction network theory via synergies between them. *Entropy* **2024**, *26*, 936. DOI

28. Alamin, A. J. L. J.; Cruz, M. J. T.; Hernandez, B. S.; Mendoza, E. R. The long-term impact of direct capture approaches to carbon dioxide removal. *Match Commun. Math. Comput. Chem.* **2026**, *1*, 161. DOI
29. Fortun, N.; Gaspar, P.; Jose, E.; Lao, A.; Mendoza, E.; Razon, L. A reaction network approach to modeling carbon dioxide removal systems. *Process Integr. Optim. Sustain.* **2025**, *10*, 1433-57. DOI
30. Fortun, N. T.; Lao, A. R.; Mendoza, E. R.; Razon, L. F. Parameter-minimal analysis of carbon dioxide removal through direct air capture. *Match Commun. Math. Comput. Chem.* **2026**, *95*, 695-729. DOI
31. Fortun, N. T.; Mendoza, E. R. Comparative analysis of carbon cycle models via kinetic representations. *J. Math. Chem.* **2023**, *61*, 896-932. DOI
32. Fortun, N. T.; Mendoza, E. R.; Razon, L. F.; Lao, A. R. A deficiency-one algorithm for power-law kinetic systems with reactant-determined interactions. *J. Math. Chem.* **2018**, *56*, 2929-62. DOI
33. Steffen, W.; Richardson, K.; Rockstrom, J.; et al. Planetary boundaries: guiding human development on a changing planet. *Science* **2015**, *347*, 1259855. DOI
34. Savageau, M. Biochemical Systems analysis: I. Some mathematical properties of the rate law for the component enzymatic reactions. *Am. J. Sci.* **1969**, *25*, 365-69. DOI
35. Savageau, M. Development of fractal kinetic theory for enzyme-catalysed reactions and implications for the design of biochemical pathways. *BioSystems* **1998**, *47*, 9-36. DOI
36. Voit, E. Biochemical systems theory: a review. *ISRN Biomath.* **2013**, *2013*, 1-53. DOI
37. García, J. H.; Torvanger, A. Carbon leakage from geological storage sites: implications for carbon trading. *Energy Policy* **2019**, *127*, 320-29. DOI
38. Stone, E. J.; Lowe, J. A.; Shine, K. P. The impact of carbon capture and storage on climate. *Energy Environ. Sci.* **2009**, *2*, 81-91. DOI
39. Beerling, D. J.; Reinhard, C. T.; James, R. H.; Khan, A.; Pidgeon, N.; Planavsky, N. J. Challenges and opportunities in scaling enhanced weathering for carbon dioxide removal. *Nat. Rev. Earth Environ.* **2025**, *6*, 672-86. DOI
40. Bergero, C.; Wise, M.; Lamers, P.; Wang, Y.; Weber, M. Biochar as a carbon dioxide removal strategy in integrated long-run mitigation scenarios. *Environ. Res. Lett.* **2024**, *19*, 074076. DOI
41. Buesseler, K. O.; Bianchi, D.; Chai, F.; et al. Next steps for assessing ocean iron fertilization for marine carbon dioxide removal. *Front. Clim.* **2024**, *6*, 1430957. DOI
42. Paustian K, Larson E, Kent J, Marx E, Swan A. Soil C sequestration as a biological negative emission strategy. *Front. Clim.* **2019**, *1*, 8. DOI
43. Taillardat, P.; Thompson, B. S.; Garneau, M.; Trottier, K.; Friess, D. A. Climate change mitigation potential of wetlands and the cost-effectiveness of their restoration. *Interface Focus* **2020**, *10*, 20190129. DOI
44. Ganti, G.; Gasser, T.; Bui, M.; et al. Evaluating the near- and long-term role of carbon dioxide removal in meeting global climate objectives. *Commun. Earth Environ.* **2024**, *5*, 377. DOI
45. Oschlies, A.; Bach, L. T.; Rickaby, R. E. M.; Satterfield, T.; Webb, R.; Gattuso, J. Climate targets, carbon dioxide removal, and the potential role of ocean alkalinity enhancement. In: Guide to Best Practices in Ocean Alkalinity Enhancement Research; *State Planet* **2023**, *2-oae2023*, 1-9. DOI
46. Ashwin, P.; Wieczorek, S.; Vitolo, R.; Cox, P. Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system. *Phil. Trans. R. Soc. A* **2012**, *370*, 1166-84. DOI
47. Farahbakhsh, I.; Bauch, C. T.; Anand, M. Tipping points in coupled human-environment system models: a review. *Earth Syst. Dynam.* **2024**, *15*, 947-67. DOI
48. Wieczorek, S.; Ashwin, P.; Luke, C. M.; Cox, P. M. Excitability in ramped systems: the compost-bomb instability. *Proc. R. Soc. A* **2011**, *467*, 1243-69. DOI
49. Schmidt, H. P.; Abiven, S.; Cowie, A.; et al. Biochar permanence: a policy commentary. *GCB Bioenergy* **2025**, *17*, e70092. DOI
50. Breure, T. S.; De Rosa, D.; Panagos, P.; Cotrufo, M. F.; Jones, A.; Lugato, E. Revisiting the soil carbon saturation concept to inform a risk index in European agricultural soils. *Nat. Commun.* **2025**, *16*, 2570. DOI
51. Emerson, D.; Sofen, L. E.; Michaud, A. B.; Archer, S. D.; Twining, B. S. A cost model for ocean iron fertilization as a means of carbon dioxide removal that compares ship- and aerial-based delivery, and estimates verification costs. *Earth's Future* **2024**, *12*, e2023EF003732. DOI
52. Shinar, G.; Feinberg, M. Design principles for robust biochemical reaction networks: what works, what cannot work, and what might almost work. *Math. Biosci.* **2011**, *231*, 39-48. DOI
53. Wiuf, C.; Feliu, E. Power-law kinetics and determinant criteria for the preclusion of multistationarity in networks of interacting species. *SIAM J. Appl. Dyn. Syst.* **2013**, *12*, 1685-721. DOI
54. Wiuf, C.; Feliu, E. A computational method to preclude multistationarity in networks of interacting species. *Bioinformatics* **2013**, *29*, 2327-34. DOI
55. Hernandez, B. S.; Mendoza, E. R.; Reyes V, A. A. D. L. A computational approach to multistationarity of power-law kinetic systems. *J. Math. Chem.* **2020**, *58*, 367-96. DOI
56. Luke, C. M.; Cox, P. M. Soil carbon and climate change: from the Jenkinson effect to the compost-bomb instability. *Eur. J. Soil Sci.* **2011**, *62*, 5-12. DOI