



Observer-based boundary control for distributed parameter system under denial-of-service attacks

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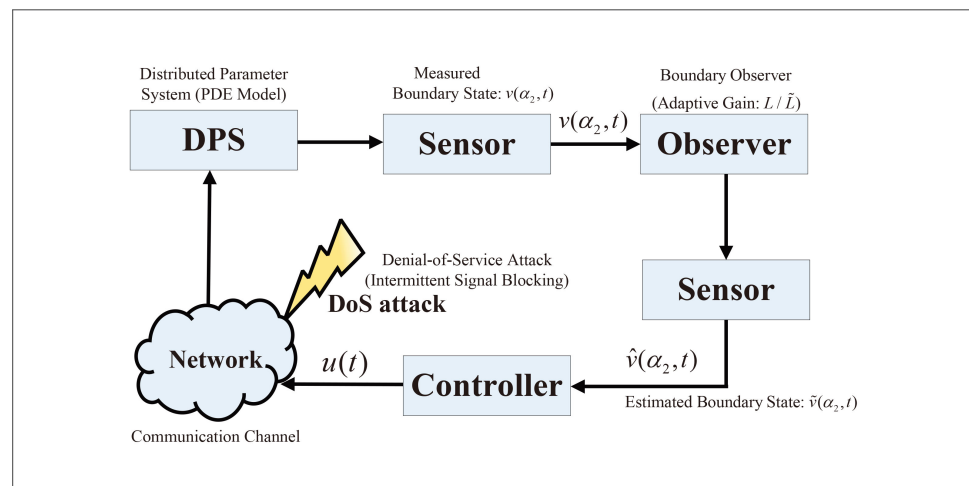
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Abstract

This paper focuses on the problem of boundary control for a distributed parameter system (DPS) under denial-of-service (DoS) attacks. Initially, a DPS model is employed. Considering the incomplete measurement of the DPS's state, a novel boundary observer is then proposed, which only relies on the right boundary state instead of full-domain information to achieve accurate state estimation, significantly reducing the measurement cost. Subsequently, an anti-DoS observer-based boundary controller is designed, which is applied only to the spatial boundary to lower actuator deployment costs while improving robustness to intermittent DoS attacks. In addition, a Lyapunov-Krasovskii functional is introduced, and the design methods for the controller and observer are derived by solving linear matrix inequalities. Finally, the feasibility of the control strategy is verified through an example.



1. INTRODUCTION

Unlike ordinary differential equations (ODEs), partial differential equations (PDEs) can simultaneously capture a system's temporal and spatial characteristics^[1,2,3]. This unique feature allows scholars to delve deeper into nonlinear dynamic phenomena within complex processes, including fluid dynamics^[4] and gene regulatory

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networks^[5,6]. As distributed parameter systems (DPSs) are generally modeled by PDEs, their control-related problems have attracted extensive attention from researchers. For complex DPSs, Wang *et al.*^[7] proposed an adaptive spatial model-based predictive controller with real-time linearization. To ensure the exponential stability of the closed-loop parabolic DPS, Wang *et al.*^[8] proposed a fuzzy boundary-based sampled-data control method.

As a widely studied method, the distributed control method needs to be applied to the entire spatial domain, which has excellent control performance but increases the control cost significantly. On the other hand, the boundary control method^[9,10] implies that the controller is only imposed on the boundary of the spatial domain. From a cost perspective, the number of actuators is greatly reduced^[11], which saves control costs and also decreases the probability of accidents due to machine failures. Moreover, since it is not always feasible to place controllers throughout the entire domain, boundary control not only offers economic benefits but also presents practical advantages when considering spatial constraints on controller placement. Man *et al.*^[10] proposed a boundary control scheme based on the T-S fuzzy model for a nonlinear parabolic PDE system, which effectively satisfies the needs in practical applications. To address the exponential consensus problem of multi-agent systems described by impulsive PDEs, an observer-based event-triggered boundary control strategy was designed in^[12], and the Zeno phenomenon was effectively excluded.

Networked control systems often face complex communication constraints, for instance, the authors in^[13], in-depth performance analysis of Multiple-Input Multiple-Output (MIMO) time-delay systems under multiple communication parameters and message queue effects. While network transmission offers great efficiency and convenience, cyber-attacks remain a hidden risk. Common types include replay attacks^[14,15], deception attacks^[16,17], and denial-of-service (DoS) attacks^[18,19,20]. DoS attacks, which need no complex techniques, can interrupt data transmission, disable normal network services, and even cause economic losses. Most existing boundary control schemes for DPSs^[8,21] ignore cyber-attacks (especially DoS attacks), lacking targeted solutions for DPSs with spatiotemporal dynamics. Furthermore, existing anti-DoS control methods^[19,22] rarely combine boundary control with observer design, failing to address the dual challenges of incomplete state measurement and intermittent communication disruptions in DPSs simultaneously. To our knowledge, few studies have focused on the control problem of DPS under DoS attacks.

To fill the gap of lacking anti-DoS boundary control schemes for DPSs with incomplete state measurement, this paper aims to design a cost-effective and robust control protocol. The proposed scheme can handle both practical measurement limitations and intermittent DoS attacks, thereby providing a feasible solution for the safe operation of complex engineering systems modeled by DPSs. Inspired by the above, this paper addresses the problem of anti-DoS observer-based control for DPSs. This paper's main contributions and novelties are concentrated in the following three points:

- (1) Although numerous advancements have been made in DPS control^[21,23], there is a noticeable lack of studies exploring control problems under boundary observers. To address the incomplete measurability of the system, this paper introduces a boundary observer based on the right boundary state of DPS.
- (2) Traditional observers^[24,25] rely on full-domain state information or multiple boundary measurements, leading to high deployment costs. Taking into account the cost issues of distributed control, a boundary controller is proposed that enables it to be applied to the boundary of the spatial domain.
- (3) Most existing anti-DoS control studies^[19,22] focus on ODE systems, and few target DPSs with spatiotemporal dynamics. Regarding the control problem of DPS, this paper fully considers the impact of DoS attacks, thereby endowing the designed control method with stronger anti-DoS attack performance.

The rest of the paper is structured as follows: Section 2 presents the modeling of DPS, boundary observer, and boundary controller, along with the error system. In Section 3, we give the Lyapunov-Krasovskii functional (LKF), conduct stability analysis, and outline the controller and observer design approach. In Section 4, an example is used to demonstrate the feasibility of the results. Section 5 concludes the article.

Notation: $(N + *)$ symbolizes $(N + N^T)$ and $\text{diag}\{\dots\}$ represents a block-diagonal matrix. For a symmetric matrix x , $x > 0$ represents that x is positive definite. Let $\mathcal{H}^n \triangleq \mathcal{L}^2([\alpha_1, \alpha_2]; \mathbb{R}^n)$ represent the Hilbert space with inner product

$$\langle \eta_1(t, s), \eta_2(t, s) \rangle \triangleq \int_{\Omega} \langle \eta_1(t, s), \eta_2(t, s) \rangle_{\mathbb{R}^n} ds$$

and $\|\eta_i(t, s)\|_2 \triangleq \langle \eta_i(t, s), \eta_i(t, s) \rangle^{\frac{1}{2}}$ with $\eta_1(t, s), \eta_2(t, s) \in \mathcal{H}^n$. For any integer l , set $\langle l \rangle = \{1, 2, \dots, l\}$.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. System description and problem formulation

Consider the following DPS

$$\begin{cases} \frac{\partial v(x, t)}{\partial t} = \Theta \frac{\partial^2 v(x, t)}{\partial x^2} + A_1 v(x, t), \\ v(x, 0) = v_0(x), \quad v_{out}(t) = v(\alpha_2, t), \\ \frac{\partial v(x, t)}{\partial x} \big|_{x=\alpha_1} = 0, \quad \frac{\partial v(x, t)}{\partial x} \big|_{x=\alpha_2} = u(t), \end{cases} \quad (1)$$

where $t \in [0, \infty)$ and $x \in \Omega \triangleq [\alpha_1, \alpha_2] \subset \mathbb{R}$ denote the time and space, respectively. $v(x, t) \in \mathbb{R}^{n_s}$ is the state of the system, $u(t) \in \mathbb{R}^{n_u}$ is control input. $\Theta > 0 \in \mathbb{R}^{n_s \times n_s}$, $A_1 \in \mathbb{R}^{n_s \times n_s}$ and $B \in \mathbb{R}^{n_s \times n_u}$ are known constant matrices with appropriate dimensions. The initial condition is represented as $v_0(x)$. Considering factors including cost saving and ease of application in complex systems, the observer and controller in this paper are mainly designed through the state information at $x = \alpha_2$ (the right boundary of the system and the observer), based on which we set $v_{out}(t) = v(\alpha_2, t)$.

2.2. DoS attacks

DoS attacks aim to intermittently disrupt communication channels as indicated in [22,26]. Such attacks have particularly notable impacts on the controller, rendering it challenging to control the DPS Equation (1).

The model of DoS attacks is expressed as follows:

$$D_{DoS} = \begin{cases} 0, & t \in [l_n, s_n), \\ 1, & t \in [s_n, l_{n+1}), \end{cases} \quad (2)$$

with time series $\{l_n\}_{n \in \mathbb{N}}$ and $\{s_n\}_{n \in \mathbb{N}}$ satisfy $0 = l_0 < s_0 \leq l_1 < \dots < l_n < s_n \leq \dots$ for $n \in \mathbb{N}$.

Take the n -th interval as an example. The interval $[l_n, s_n)$ indicates that the channel is not blocked and data can be transmitted, and $[s_n, l_{n+1})$ represents that the channel is blocked.

For $t \geq t_r \geq 0$, $n(t_r, t)$ means the cumulative number of DoS attacks start and stop status changes during the period from t_r to t . Additionally, $\Phi_{DoS}(t_r, t)$ indicates the collection of durations for which the attacks were active over the interval $[t_r, t)$. Based on the above discussion, during $[t_r, t)$, the attack sleep interval is

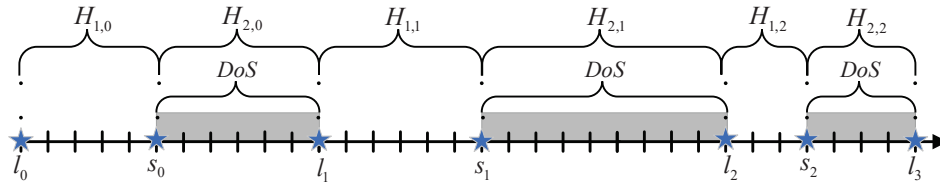


Figure 1. The framework of DoS attacks.

$$\bar{\Phi}_{sleep}(t_r, t) = (t_r, t) / \bar{\Phi}_{DoS}(t_r, t).$$

The framework of DoS attacks is shown in Figure 1. To facilitate the analysis of DoS attacks, the following assumptions are formulated:

Assumption 1. (DoS Duration) Suppose an interval $0 \leq t_z \leq t$ and set $T_{D1} > 0$. Then we can get

$$\bar{\Phi}_{DoS}(t_r, t) \leq \frac{t}{T_{D1}}.$$

2.3. Boundary observer description and problem formulation

In real-world applications, acquiring complete domain state information for Equation (1) is usually unavailable due to limitations such as high measurement cost. Therefore, a boundary state observer is constructed to estimate the state of Equation (1), and subsequently an anti-DoS observer-based controller is designed to achieve effective control for Equation (1).

Taking into account practical factors, this paper assumes that only the state information at $x = \alpha_2$ (right boundary of the system) is available, then we set

$$\begin{cases} \frac{\partial \hat{v}(x, t)}{\partial t} = \Theta \frac{\partial^2 \hat{v}(x, t)}{\partial x^2} + A_2 \hat{v}(x, t) + L(v_{out}(t) - \hat{v}_{out}(t)), \\ \hat{v}(x, 0) = \hat{v}_0(x), \quad \hat{v}_{out}(t) = \hat{v}(\alpha_2, t), \\ \frac{\partial \hat{v}(x, t)}{\partial x} \big|_{x=\alpha_1} = 0, \quad \frac{\partial \hat{v}(x, t)}{\partial x} \big|_{x=\alpha_2} = u(t) \end{cases} \quad (3)$$

where $\hat{v}(x, t) \in \mathbb{R}^{n_s}$ denotes the observer state. The observer initial condition is given as $\hat{v}_0(x)$. The output of the observer is $\hat{v}_{out}(t) \triangleq \hat{v}(\alpha_2, t)$. $A_2 \in \mathbb{R}^{n_s \times n_s}$ is known matrix. $L \in \mathbb{R}^{n_s \times n_s}$ is the boundary observer gain.

Considering the DoS attacks, the observation gain L is expressed in:

$$L = \begin{cases} L, & t \in H_{1,n}, \\ \tilde{L}, & t \in H_{2,n}, \end{cases}$$

where $\tilde{L} \in \mathbb{R}^{n_s \times n_s}$ represents the controller gain under DoS attacks.

The aim of this paper is to design an anti-DoS observer-based controller. First, we set a piecewise function $p(t)$

$$p(t) = \begin{cases} 1, & t \in H_{1,n}, \\ 2, & t \in H_{2,n}, \end{cases}$$

then we can get the feasible set of $p(t)$: $S_p(s_{11}, s_{12}; s_{21}, s_{22}) = \{p(t) : s_{11} \leq s_n - l_n \leq s_{12}; s_{21} \leq l_{n+1} - s_n \leq s_{22}\}$, in which $s_{ij} > 0, i, j \in \langle 2 \rangle$ are constants.

Drawing upon the above discussions, the controller is proposed:

$$u(t) = \begin{cases} BK\hat{v}_{out}(t), & t \in H_{1,n}, \\ 0, & t \in H_{2,n}. \end{cases} \quad (4)$$

with $B \in \mathbb{R}^{n_s \times n_s}$ is known matrix, $K \in \mathbb{R}^{n_s \times n_s}$ is the controller gain.

The estimation error is specified as

$$e(x, t) \triangleq v(x, t) - \hat{v}(x, t). \quad (5)$$

Based on (5), we have

$$\begin{aligned} \frac{\partial e(x, t)}{\partial t} &= \frac{\partial v(x, t)}{\partial t} - \frac{\partial \hat{v}(x, t)}{\partial t} \\ &= \Theta \left(\frac{\partial^2 v(x, t)}{\partial x^2} - \frac{\partial^2 \hat{v}(x, t)}{\partial x^2} \right) + A_1 v(x, t) - A_2 \hat{v}(x, t) - L(v_{out}(t) - \hat{v}_{out}(t)) \\ &= \Theta \frac{\partial^2 e(x, t)}{\partial x^2} + A_1 v(x, t) + A_2 v(x, t) - A_2 \hat{v}(x, t) - L(v_{out}(t) - \hat{v}_{out}(t)) \\ &= \Theta \frac{\partial^2 e(x, t)}{\partial x^2} + (A_1 - A_2)v(x, t) + A_2 e(x, t) - L(v_{out}(t) - \hat{v}_{out}(t)). \end{aligned}$$

Then, we derive the subsequent error system

$$\begin{cases} \frac{\partial e(x, t)}{\partial t} = \Theta \frac{\partial^2 e(x, t)}{\partial x^2} + A_2 e(x, t) + (A_1 - A_2)v(x, t) - L(v_{out}(t) - \hat{v}_{out}(t)), \\ \frac{\partial e(x, t)}{\partial x} \Big|_{x=\alpha_1} = \frac{\partial e(x, t)}{\partial x} \Big|_{x=\alpha_2} = 0, \\ e(x, 0) = e_0(x), \end{cases} \quad (6)$$

The overall control framework is illustrated in Figure 2. This paper aims to develop an anti-DoS observer-based controller Equation (4), such that the DPS Equation (1) is exponentially stable. For the convenience of stability analysis, the following lemma is presented.

Lemma 1. (Wirtinger's Inequality^[21]) Set $z \in \mathbb{R}^{n_s}$ with $z(\alpha_2) = 0$ or $z(\alpha_1) = 0$. If there exists any matrix $U \geq 0$, the following inequality is satisfied

$$\int_{\Omega} z^T(x) U z(x) dx \leq \frac{4(\alpha_2 - \alpha_1)^2}{\pi^2} \int_{\Omega} \frac{dz^T}{dx} U \frac{dz}{dx} dx.$$

In addition, if $z(\alpha_2) = 0$ and $z(\alpha_1) = 0$, we can deduce that

$$\int_{\Omega} z^T(x) U z(x) dx \leq \frac{(\alpha_2 - \alpha_1)^2}{\pi^2} \int_{\Omega} \frac{dz^T}{dx} U \frac{dz}{dx} dx.$$

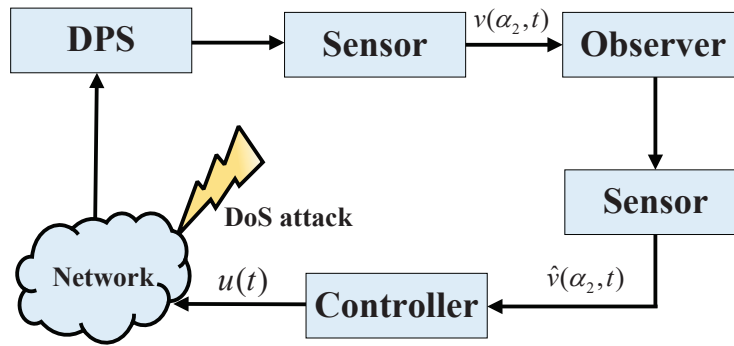


Figure 2. Observer-based boundary control scheme for DPS under DoS attacks.

3. MAIN RESULTS

In this part, the design of observers and controllers for DPSs Equations (1) is analyzed and derived based on sufficient conditions obtained from LKF.

Inspired from [27], as for $n \in \mathbb{N}$, the following piecewise linear functions are defined to support the subsequent stability analysis.

$$\begin{aligned} \theta_{11}(t) &= \frac{t-l_n}{s_n-l_n}, \quad \theta_{12}(t) = \frac{s_n-t}{s_n-l_n}, \quad \theta_{10}(t) = \frac{1}{s_n-l_n}, \quad t \in H_{1,n}, \\ \theta_{21}(t) &= \frac{t-s_n}{l_{n+1}-s_n}, \quad \theta_{22}(t) = \frac{l_{n+1}-t}{l_{n+1}-s_n}, \quad \theta_{20}(t) = \frac{1}{l_{n+1}-s_n}, \quad t \in H_{2,n}. \end{aligned}$$

Moreover, we can obtain

$$\begin{aligned} \theta_{11}(l_n) &= \theta_{22}(l_n^-) = 0, \quad \theta_{11}(s_n^-) = \theta_{22}(s_n) = 1, \\ \theta_{12}(l_n) &= \theta_{21}(l_n^-) = 1, \quad \theta_{12}(s_n^-) = \theta_{21}(s_n) = 0. \end{aligned} \quad (7)$$

Obviously, there are constants $\varsigma_{ij} > 0$, $i, j \in \langle 2 \rangle$ such that $s_n - l_n \in [\varsigma_{11}, \varsigma_{12}]$, $l_{n+1} - s_n \in [\varsigma_{21}, \varsigma_{22}]$ and we can deduce that $\phi_{h1}(t) : \mathbb{R}_+ \rightarrow [0, 1]$, satisfy

$$\begin{aligned} \theta_{h0}(t) &= \frac{\phi_{h1}(t)}{\varsigma_{h1}} + \frac{\phi_{h2}(t)}{\varsigma_{h2}}, \\ \phi_{h2}(t) &= 1 - \phi_{h1}(t), \end{aligned} \quad (8)$$

with $h \in \langle 2 \rangle$.

Subsequently, the design of controller Equations (4) and observer Equations (3) will be analyzed.

Theorem 1. For given constants $\alpha_1 > 0$, $\mu_1 > 1$ and $\mu_2 > 1$, the control gain κ , the observer gain L , if there exist matrices $P_{lm} \in \mathbb{R}^{n_s \times n_s} > 0$, $l, m \in \langle 2 \rangle$, such that

$$P_{22} \leq \mu_1 P_{11}, \quad P_{12} \leq \mu_2 P_{21}, \quad (9)$$

$$\Xi_1^m < 0, \quad (10)$$

$$\Xi_2^m < 0, \quad (11)$$

$$\frac{\ln \mu_1}{\varsigma_{12}} - \alpha_1 > 0, \quad \frac{\ln \mu_2}{\varsigma_{22}} - \alpha_1 < 0, \quad (12)$$

$$-(\frac{\ln \mu_1}{s_{1l}} - \alpha_1)(1 - \frac{1}{T_{D1}}) - (\frac{\ln \mu_2}{s_{2l}} - \alpha_1) \frac{1}{T_{D1}} < 0, \quad l \in \langle 2 \rangle \quad (13)$$

where

$$\Xi_1^{ml} = \begin{bmatrix} \Delta_{11}^{ml} & \Delta_{12}^m & \Delta_{13}^m & \Delta_{14}^m \\ * & \Delta_{22}^m & \Delta_{23}^m & \Delta_{24}^m \\ * & * & \Delta_{33}^{ml} & \Delta_{34}^m \\ * & * & * & \Delta_{44}^m \end{bmatrix},$$

$$\Xi_2^{ml} = \begin{bmatrix} \Lambda_{11}^{ml} & \Lambda_{12}^m & \Lambda_{13}^m & \Lambda_{14}^m \\ * & \Lambda_{22}^m & \Lambda_{23}^m & \Lambda_{24}^m \\ * & * & \Lambda_{33}^{ml} & \Lambda_{34}^m \\ * & * & * & \Lambda_{44}^m \end{bmatrix},$$

$\Delta_{11}^m = \alpha_1 P_{1m} + \frac{1}{s_{1l}}(P_{11} - P_{12}) + (P_{1m}A_2 + *) - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{1m}\Theta + *)$, $\Delta_{12}^m = \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{1m}\Theta + *)$, $\Delta_{13}^m = (P_{1m}(A_1 - A_2))^T$, $\Delta_{14}^m = P_{1m}L$, $\Delta_{22}^m = \frac{2}{\alpha_2 - \alpha_1}P_{1m}BK - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{1m}\Theta + *)$, $\Delta_{33}^m = \alpha_1 P_{1m} + \frac{1}{s_{1l}}(P_{11} - P_{12}) + (P_{1m}A_2 + *) + (P_{1m}(A_1 - A_2) + *) - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{1m}\Theta + *)$, $\Delta_{34}^m = -P_{1m}L + \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{1m}\Theta + *)$, $\Delta_{44}^m = -\frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{1m}\Theta + *)$, $\Lambda_{11}^m = \alpha_1 P_{2m} + \frac{1}{s_{2l}}(P_{21} - P_{22}) + (P_{2m}A_2 + *) - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{2m}\Theta + *)$, $\Lambda_{12}^m = \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{2m}\Theta + *)$, $\Lambda_{13}^m = (P_{2m}(A_1 - A_2))^T$, $\Lambda_{14}^m = P_{2m}\tilde{L}$, $\Lambda_{22}^m = -\frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{2m}\Theta + *)$, $\Lambda_{33}^m = \alpha_1 P_{2m} + \frac{1}{s_{2l}}(P_{21} - P_{22}) + (P_{2m}A_2 + *) + (P_{2m}(A_1 - A_2) + *) - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{2m}\Theta + *)$, $\Lambda_{34}^m = -P_{2m}\tilde{L} + \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{2m}\Theta + *)$, $\Lambda_{44}^m = -\frac{\pi^2}{4(\alpha_2 - \alpha_1)^2}(P_{2m}\Theta + *)$, the part of Δ and Λ that are not specified are zero. Then, the DPS Equations (1) and the error system Equations (6) are exponentially stable.

Proof. Replacing P_{1m} , P_{2m} , $\frac{1}{s_{1l}}$ and $\frac{1}{s_{2l}}$ in Equations (10) and (11) by $P_1(t)$, $P_2(t)$, $\theta_{10}(t)$ and $\theta_{20}(t)$ we have

$$\Xi_1(t) = \begin{bmatrix} \Delta_{11}(t) & \Delta_{12}(t) & \Delta_{13}(t) & \Delta_{14}(t) \\ * & \Delta_{22}(t) & \Delta_{23}(t) & \Delta_{24}(t) \\ * & * & \Delta_{33}(t) & \Delta_{34}(t) \\ * & * & * & \Delta_{44}(t) \end{bmatrix}, \quad (14)$$

$$\Xi_2(t) = \begin{bmatrix} \Lambda_{11}(t) & \Lambda_{12}(t) & \Lambda_{13}(t) & \Lambda_{14}(t) \\ * & \Lambda_{22}(t) & \Lambda_{23}(t) & \Lambda_{24}(t) \\ * & * & \Lambda_{33}(t) & \Lambda_{34}(t) \\ * & * & * & \Lambda_{44}(t) \end{bmatrix}, \quad (15)$$

with $P_{p(t)}(t) = \sum_{d=1}^2 \theta_{p(t),d}(t)P_{p(t),d}$.

Construct the Lyapunov functional as

$$V_{p(t)}(t) = \sum_{s=1}^2 V_{p(t),s}(t), \quad (16)$$

where

$$V_{p(t),1}(t) = \psi_{p(t)}(t) \int_{\Omega} \hat{v}^T(s, t) P_{p(t)}(t) \hat{v}(s, t) ds, \quad (17)$$

$$V_{p(t),2}(t) = \psi_{p(t)}(t) \int_{\Omega} e^T(s,t) P_{p(t)}(t) e(s,t) ds, \quad (18)$$

and $\psi_{p(t)}(t) = \mu_{p(t)}^{\theta_{p(t),1}(t)}$.

For $t \in [l_n, s_n)$, $p(t) = 1$, we can deduce that

$$\begin{aligned} V_{11}(t) &= \psi_1(t) \int_{\Omega} \hat{v}^T(x,t) P_1(t) \hat{v}(x,t) dx, \\ V_{12}(t) &= \psi_1(t) \int_{\Omega} e^T(x,t) P_1(t) e(x,t) dx, \end{aligned}$$

then we obtain the derivative of $V_{11}(t)$ as:

$$\begin{aligned} \frac{\partial V_{11}(t)}{\partial t} &= \frac{\ln \mu_1}{s_n - l_n} V_{11}(t) - \alpha_1 V_{11}(t) + \psi_1(t) \int_{\Omega} \hat{v}^T(x,t) \alpha_1 P_1(t) \hat{v}(x,t) dx \\ &\quad + \psi_1(t) \int_{\Omega} \hat{v}^T(x,t) \theta_{10}(t) (P_{11} - P_{12}) \hat{v}(x,t) dx + 2\psi_1(t) \int_{\Omega} \hat{v}^T(x,t) P_1(t) \frac{\partial \hat{v}(x,t)}{\partial t} dx. \end{aligned} \quad (19)$$

Based on Equations (3) and (19), we have

$$\begin{aligned} &2\psi_1(t) \int_{\Omega} \hat{v}^T(x,t) P_1(t) \frac{\partial \hat{v}(x,t)}{\partial t} dx \\ &= 2\psi_1(t) \int_{\Omega} \hat{v}^T(x,t) P_1(t) \left(\Theta \frac{\partial^2 \hat{v}(x,t)}{\partial x^2} + A_2 \hat{v}(x,t) + Le(\alpha_2, t) \right) dx \\ &= 2\psi_1(t) \int_{\Omega} \hat{v}^T(x,t) P_1(t) \Theta \frac{\partial^2 \hat{v}(x,t)}{\partial x^2} dx + 2\psi_1(t) \int_{\Omega} \hat{v}^T(x,t) P_1(t) A_2 \hat{v}(x,t) dx \\ &\quad + 2\psi_1(t) \int_{\Omega} \hat{v}^T(x,t) P_1(t) Le_0(\alpha_2, t) dx, \end{aligned} \quad (20)$$

Furthermore, we can get

$$\begin{aligned} &2\psi_1(t) \int_{\Omega} \hat{v}^T(x,t) P_1(t) \Theta \frac{\partial^2 \hat{v}(x,t)}{\partial x^2} dx \\ &= 2\psi_1(t) \hat{v}^T(x,t) P_1(t) \Theta \frac{\partial \hat{v}(x,t)}{\partial x} \Big|_{\alpha_1}^{\alpha_2} - 2\psi_1(t) \int_{\Omega} \frac{\partial \hat{v}^T(x,t)}{\partial x} P_1(t) \Theta \frac{\partial \hat{v}(x,t)}{\partial x} dx \\ &= 2\psi_1(t) \hat{v}^T(\alpha_2, t) P_1(t) \Theta \frac{\partial \hat{v}(x,t)}{\partial x} \Big|_{\alpha_2} - 2\psi_1(t) \int_{\Omega} \frac{\partial \hat{v}^T(x,t)}{\partial x} P_1(t) \Theta \frac{\partial \hat{v}(x,t)}{\partial x} dx \\ &= 2\psi_1(t) \hat{v}^T(\alpha_2, t) P_1(t) \Theta B K \hat{v}(\alpha_2, t) - 2\psi_1(t) \int_{\Omega} \frac{\partial \hat{v}^T(x,t)}{\partial x} P_1(t) \Theta \frac{\partial \hat{v}(x,t)}{\partial x} dx. \end{aligned} \quad (21)$$

From Lemma 1, we can conclude that

$$\begin{aligned} &2\psi_1(t) \int_{\Omega} \frac{\partial \hat{v}^T(x,t)}{\partial x} P_1(t) \Theta \frac{\partial \hat{v}(x,t)}{\partial x} dx \\ &\leq -\psi_1(t) \frac{\pi^2}{4(\alpha_2 - \alpha_1)} \int_{\Omega} (\hat{v}(x,t) - \hat{v}(\alpha_2, t))^T (P_1(t) \Theta + (P_1(t) \Theta)^T) (\hat{v}(x,t) - \hat{v}(\alpha_2, t)) dx. \end{aligned} \quad (22)$$

Then, from the derivative of $V_{12}(t)$, we can deduce that

$$\begin{aligned} \frac{\partial V_{12}(t)}{\partial t} &= \frac{\ln \mu_1}{s_n - l_n} V_{12}(t) - \alpha_1 V_{12}(t) + \psi_1(t) \int_{\Omega} e^T(x, t) \alpha_1 P_1(t) e(x, t) dx \\ &\quad + \psi_1(t) \int_{\Omega} e^T(x, t) \theta_{10}(t) (P_{11} - P_{12}) e(x, t) dx + 2\psi_1(t) \int_{\Omega} e^T(x, t) P_1(t) \frac{\partial e(x, t)}{\partial t} dx, \end{aligned} \quad (23)$$

in which

$$\begin{aligned} &2\psi_1(t) \int_{\Omega} e^T(x, t) P_1(t) \frac{\partial e(x, t)}{\partial t} dx \\ &= 2\psi_1(t) \int_{\Omega} e^T(x, t) P_1(t) \left\{ \Theta \frac{\partial^2 e(x, t)}{\partial x^2} + A_2 e(x, t) + (A_1 - A_2) v(x, t) - Le(\alpha_2, t) \right\} dx \\ &= 2\psi_1(t) \int_{\Omega} e^T(x, t) P_1(t) \Theta \frac{\partial^2 e(x, t)}{\partial x^2} dx + 2\psi_1(t) \int_{\Omega} e^T(x, t) P_1(t) A_2 e(x, t) dx \\ &\quad + 2\psi_1(t) \int_{\Omega} e^T(x, t) P_1(t) (A_1 - A_2) (e(x, t) + \hat{v}(x, t)) dx - 2\psi_1(t) \int_{\Omega} e^T(x, t) P_1(t) Le(\alpha_2, t) dx. \end{aligned} \quad (24)$$

Similar to Equations (21) and (22), one has

$$\begin{aligned} &\psi_1(t) \int_{\Omega} e^T(x, t) P_1(t) \Theta \frac{\partial^2 e(x, t)}{\partial x^2} dx \\ &\leq -\psi_1(t) \frac{\pi^2}{4(\alpha_2 - \alpha_1)} \int_{\Omega} (e(x, t) - e(\alpha_2, t))^T (P_1(t) \Theta + (P_1(t) \Theta)^T) (e(x, t) - e(\alpha_2, t)) dx. \end{aligned} \quad (25)$$

Combining Equations (19)-(25), we obtain the following inequality

$$\frac{\partial V_1(t)}{\partial t} = \psi_1(t) \int_{\Omega} \varepsilon_1^T \Xi_1(t) \varepsilon_1 dx + \left(\frac{\ln \mu_1}{s_n - l_n} - \alpha_1 \right) V_1(t), \quad (26)$$

with $\varepsilon_1 = \text{col}(\hat{v}(x, t), \hat{v}(\alpha_2, t), e(x, t), e(\alpha_2, t))$.

From Equations (10) and (14), one can derive that

$$\frac{\partial V_1(t)}{\partial t} \leq \left(\frac{\ln \mu_1}{s_n - l_n} - \alpha_1 \right) V_1(t), \quad (27)$$

where $\beta_1 = \frac{\ln \mu_1}{s_n - l_n} - \alpha_1 > 0$, due to Equation (7), we can easily deduce that

$$V_1(t) \leq e^{-\beta_1(t-l_n)} V_1(l_n), t \in H_{1,n}. \quad (28)$$

For $t \in [s_n, l_{n+1})$, $p(t) = 2$, we can deduce that

$$\begin{aligned} V_{21}(t) &= \psi_2(t) \int_{\Omega} \hat{v}^T(x, t) P_2(t) \hat{v}(x, t) dx, \\ V_{22}(t) &= \psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) e(x, t) dx. \end{aligned} \quad (29)$$

Taking the derivative of $V_{21}(t)$, we can deduce that:

$$\begin{aligned} \frac{\partial V_{21}(t)}{\partial t} &= \frac{\ln \mu_2}{l_{n+1} - s_n} V_{21}(t) - \alpha_1 V_{21}(t) + \psi_2(t) \int_{\Omega} \hat{v}^T(x, t) \alpha_1 P_2(t) \hat{v}(x, t) dx \\ &\quad + \psi_2(t) \int_{\Omega} \hat{v}^T(x, t) \theta_{20}(t) (P_{21} - P_{22}) \hat{v}(x, t) dx + 2\psi_2(t) \int_{\Omega} \hat{v}^T(x, t) P_2(t) \frac{\partial \hat{v}(x, t)}{\partial t} dx. \end{aligned} \quad (30)$$

Furthermore, it is obvious that

$$\begin{aligned} &2\psi_2(t) \int_{\Omega} \hat{v}^T(x, t) P_2(t) \frac{\partial \hat{v}(x, t)}{\partial t} dx \\ &= 2\psi_2(t) \int_{\Omega} \hat{v}^T(x, t) P_2(t) \Theta \frac{\partial^2 \hat{v}(x, t)}{\partial x^2} dx + 2\psi_2(t) \int_{\Omega} \hat{v}^T(x, t) P_2(t) A_2 \hat{v}(x, t) dx \\ &\quad + 2\psi_2(t) \int_{\Omega} \hat{v}^T(x, t) P_2(t) L e(\alpha_2, t) dx, \end{aligned} \quad (31)$$

Similar to Equation (21), it is evident that

$$\begin{aligned} &2\psi_2(t) \int_{\Omega} \hat{v}^T(x, t) P_2(t) \Theta \frac{\partial^2 \hat{v}(x, t)}{\partial x^2} dx \\ &= 2\psi_2(t) \hat{v}^T(x, t) P_2(t) \Theta \frac{\partial \hat{v}(x, t)}{\partial x} \Big|_{\alpha_1}^{\alpha_2} - 2\psi_2(t) \int_{\Omega} \frac{\partial \hat{v}^T(x, t)}{\partial x} P_2(t) \Theta \frac{\partial \hat{v}(x, t)}{\partial x} dx \\ &= -2\psi_2(t) \int_{\Omega} \frac{\partial \hat{v}^T(x, t)}{\partial x} P_2(t) \Theta \frac{\partial \hat{v}(x, t)}{\partial x} dx \\ &\leq -\psi_2(t) \frac{\pi^2}{4(\alpha_2 - \alpha_1)} \int_{\Omega} (\hat{v}(x, t) - \hat{v}(\alpha_2, t))^T (P_2(t) \Theta + (P_2(t) \Theta)^T) (\hat{v}(x, t) - \hat{v}(\alpha_2, t)) dx. \end{aligned} \quad (32)$$

For $\frac{\partial V_{22}(t)}{\partial t}$, we get the derivative as

$$\begin{aligned} \frac{\partial V_{22}(t)}{\partial t} &= \frac{\ln \mu_2}{l_{n+1} - s_n} V_{22}(t) - \alpha_1 V_{22}(t) + \psi_2(t) \int_{\Omega} e^T(x, t) \alpha_1 P_2(t) e(x, t) dx \\ &\quad + \psi_2(t) \int_{\Omega} e^T(x, t) \theta_{20}(t) (P_{21} - P_{22}) e(x, t) dx + 2\psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) \frac{\partial e(x, t)}{\partial t} dx. \end{aligned} \quad (33)$$

Similar to Equation (24), we use the same method and get

$$\begin{aligned} &2\psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) \frac{\partial e(x, t)}{\partial t} dx \\ &= 2\psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) \left\{ \Theta \frac{\partial^2 e(x, t)}{\partial x^2} + A_2 e(x, t) + (A_1 - A_2)(e(x, t) + \hat{v}(x, t)) - L e(\alpha_2, t) \right\} dx \\ &= 2\psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) \Theta \frac{\partial^2 e(x, t)}{\partial x^2} dx + \psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) A_2 e(x, t) dx \\ &\quad + 2\psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) (A_1 - A_2)(e(x, t) + \hat{v}(x, t)) dx - 2\psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) \tilde{L} e(\alpha_2, t) dx, \end{aligned} \quad (34)$$

where

$$\begin{aligned} &2\psi_2(t) \int_{\Omega} e^T(x, t) P_2(t) \Theta \frac{\partial^2 e(x, t)}{\partial x^2} dx \\ &\leq -\psi_2(t) \frac{\pi^2}{4(\alpha_2 - \alpha_1)} \int_{\Omega} (e(x, t) - e(\alpha_2, t))^T (P_2(t) \Theta + (P_2(t) \Theta)^T) (e(x, t) - e(\alpha_2, t)) dx. \end{aligned} \quad (35)$$

From Equations (30)–(35), it follows that

$$\frac{\partial V_2(t)}{\partial t} = \psi_2(t) \int_{\Omega} \varepsilon_2^T \Xi_2(t) \varepsilon_2 dx + \left(\frac{\ln \mu_2}{l_{n+1} - s_n} - \alpha_1 \right) V_2(t), \quad (36)$$

where $\varepsilon_2 = \text{col}(\hat{v}(x, t), \hat{v}(\alpha_2, t), e(x, t), e(\alpha_2, t))$.

One can derive from Equations (11) and (15) that

$$\frac{\partial V_2(t)}{\partial t} \leq \beta_2 V_2(t), \quad (37)$$

with $\beta_2 = \frac{\ln \mu_2}{l_{n+1} - s_n} - \alpha_1 < 0$. From Equation (7), which means that

$$V_2(t) \leq e^{-\beta_2(t-s_n)} V_2(s_n), \quad t \in H_{2,n}. \quad (38)$$

By combining Equations (28) and (38), it can be deduced that the conditions in Equations (7) and (9)–(12) ensure the feasibility of the following inequalities:

$$\begin{cases} V_2(s_n) \leq V_1(s_n^-), \\ V_1(l_n) \leq V_2(l_n^-). \end{cases} \quad (39)$$

In summary, we can conclude that

$$\begin{aligned} V(t) &\leq e^{-\beta_1(t-\frac{t}{T_D}) - \beta_2 \frac{t}{T_D}} V_1(0) \\ &= e^{\theta_0 t} V_1(0), \end{aligned} \quad (40)$$

in which $\theta_0 = -\beta_1(1 - \frac{1}{T_{D1}}) - \beta_2 \frac{1}{T_{D1}} < 0$.

It can be concluded that system Equation (1) and the error system Equation (6) are exponentially stable, thereby finishing the proof. $\square$

Next, we would like to elaborate on the design methods of boundary controller Equation (4) and observer Equation (3) using Theorem 2. First, we set

$$\begin{aligned} X_{11} &\triangleq P_{11}^{-1}, \quad X_{21} \triangleq P_{21}^{-1}, \\ P_{12} &= \chi_1 P_{11}, \quad P_{22} = \chi_2 P_{21}, \end{aligned} \quad (41)$$

where $\chi_1, \chi_2 > 0$ are constants.

Theorem 2. Consider constants $\alpha_1 > 0$, $\chi_l > 0$ and $\mu_l > 1$, if there exist matrices $X_{l1} \in \mathbb{R}^{n_s \times n_s} > 0$, $l, m \in \langle 2 \rangle$, such that the following linear matrix inequalities (LMIs) hold:

$$X_{11} \leq \mu_1 \chi_2^{-1} X_{21}, \quad X_{21} \leq \mu_2 \chi_1^{-1} X_{11}, \quad (42)$$

$$\bar{\Xi}_1^{ml} < 0, \quad (43)$$

$$\bar{\Xi}_2^{ml} < 0, \quad (44)$$

where

$$\bar{\Xi}_1^{ml} = \begin{bmatrix} \bar{\Delta}_{11}^{ml} & \bar{\Delta}_{12}^m & \bar{\Delta}_{13}^m & \bar{\Delta}_{14}^m \\ * & \bar{\Delta}_{22}^m & \bar{\Delta}_{23}^m & \bar{\Delta}_{24}^m \\ * & * & \bar{\Delta}_{33}^{ml} & \bar{\Delta}_{34}^m \\ * & * & * & \bar{\Delta}_{44}^m \end{bmatrix},$$

$$\bar{\Xi}_2^{ml} = \begin{bmatrix} \bar{\Lambda}_{11}^{ml} & \bar{\Lambda}_{12}^m & \bar{\Lambda}_{13}^m & \bar{\Lambda}_{14}^m \\ * & \bar{\Lambda}_{22}^m & \bar{\Lambda}_{23}^m & \bar{\Lambda}_{24}^m \\ * & * & \bar{\Lambda}_{33}^{ml} & \bar{\Lambda}_{34}^m \\ * & * & * & \bar{\Lambda}_{44}^m \end{bmatrix},$$

$$\begin{aligned} \bar{\Delta}_{11}^{ml} &= \alpha_1 \chi_1^{m-1} X_{11} + \frac{1}{s_{1l}} (X_{11} - \chi_1 X_{11}) + (\chi_1^{m-1} A_2 X_{11} + *) - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_1^{m-1} \Theta X_{11} + *), \quad \bar{\Delta}_{12}^m = \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_1^{m-1} \Theta X_{11} + *), \quad \bar{\Delta}_{13}^m = \\ & (\chi_1^{m-1} (A_1 - A_2) X_{11})^T, \quad \bar{\Delta}_{14}^m = \chi_1^{m-1} \bar{L}_{11}, \quad \bar{\Delta}_{22}^m = \frac{2}{\alpha_2 - \alpha_1} \chi_1^{m-1} B \bar{K}_{11} - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_1^{m-1} \Theta X_{11} + *), \quad \bar{\Delta}_{33}^{ml} = \alpha_1 \chi_1^{m-1} X_{11} + \frac{1}{s_{1l}} (X_{11} - \chi_1 X_{11}) + \\ & (\chi_1^{m-1} A_2 X_{11} + *) + (\chi_1^{m-1} (A_1 - A_2) X_{11} + *) - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_1^{m-1} \Theta X_{11} + *), \quad \bar{\Delta}_{34}^m = -\chi_1^{m-1} \bar{L}_{11} + \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_1^{m-1} \Theta X_{11} + *), \quad \bar{\Delta}_{44}^m = \\ & -\frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_1^{m-1} \Theta X_{11} + *), \quad \bar{\Lambda}_{11}^{ml} = \alpha_1 \chi_2^{m-1} X_{21} + \frac{1}{s_{2l}} (X_{21} - \chi_2 X_{21}) + (\chi_2^{m-1} A_2 X_{21} + *) - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_2^{m-1} \Theta X_{21} + *), \quad \bar{\Lambda}_{12}^m = \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_2^{m-1} \Theta X_{21} + *) \\ & *, \quad \bar{\Lambda}_{13}^m = (\chi_2^{m-1} (A_1 - A_2) X_{21})^T, \quad \bar{\Lambda}_{14}^m = \chi_2^{m-1} \bar{L}_{21}, \quad \bar{\Lambda}_{22}^m = -\frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_2^{m-1} \Theta X_{21} + *), \quad \bar{\Lambda}_{33}^{ml} = \alpha_1 \chi_2^{m-1} X_{21} + \frac{1}{s_{2l}} (X_{21} - \chi_2 X_{21}) + \\ & (\chi_2^{m-1} A_2 X_{21} + *) + (\chi_2^{m-1} (A_1 - A_2) X_{21} + *) - \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_2^{m-1} \Theta X_{21} + *), \quad \bar{\Lambda}_{34}^m = -\chi_2^{m-1} \bar{L}_{21} + \frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_2^{m-1} \Theta X_{21} + *), \quad \bar{\Lambda}_{44}^m = \\ & -\frac{\pi^2}{4(\alpha_2 - \alpha_1)^2} (\chi_2^{m-1} \Theta X_{21} + *), \end{aligned}$$

the part of $\bar{\Delta}$ and $\bar{\Lambda}$ that are not specified are zero. Moreover, the controller gains are designed as $K = \bar{K}_{11} X_{11}^{-1}$, and the observer gains $L = \bar{L}_{11} X_{11}^{-1}$, $\tilde{L} = \bar{L}_{21} X_{21}^{-1}$.

Proof. Pre- and post-multiplying Equations (14) and (15) by $\text{diag}\{X_{11}, X_{11}, X_{11}, X_{11}\}$ and $\text{diag}\{X_{21}, X_{21}, X_{21}, X_{21}\}$. Then we complete the proof. $\square$

4. NUMERICAL EXAMPLES

In this section, we provide an example to indicate the feasibility of the observer (3) and boundary controller (4). Consider the following DPS:

$$\begin{cases} \frac{\partial v(x,t)}{\partial t} = \Theta \frac{\partial^2 v(x,t)}{\partial x^2} + A_1 v(x,t), \\ v(x,0) = v_0(x), \quad v_{out}(t) = v(\alpha_2, t), \\ \frac{\partial v(x,t)}{\partial x} \big|_{x=\alpha_1} = 0, \quad \frac{\partial v(x,t)}{\partial x} \big|_{x=\alpha_2} = u(t), \end{cases}$$

and following observer:

$$\begin{cases} \frac{\partial \hat{v}(x,t)}{\partial t} = \Theta \frac{\partial^2 \hat{v}(x,t)}{\partial x^2} + A_2 \hat{v}(x,t) + L(v_{out}(t) - \hat{v}_{out}(t)), \\ \hat{v}(x,0) = \hat{v}_0(x), \quad \hat{v}_{out}(t) = \hat{v}(\alpha_2, t), \\ \frac{\partial \hat{v}(x,t)}{\partial x} \big|_{x=\alpha_1} = 0, \quad \frac{\partial \hat{v}(x,t)}{\partial x} \big|_{x=\alpha_2} = u(t). \end{cases}$$

where $v(x,t) = \text{col}\{v_1(x,t), v_2(x,t), v_3(x,t)\}$, $\hat{v}(x,t) = \text{col}\{\hat{v}_1(x,t), \hat{v}_2(x,t), \hat{v}_3(x,t)\}$, $\alpha_1 = 0$, $\alpha_2 = 5$, $A_1 = -4 \times I_3$, $A_2 = -I_3$, $\Theta = I_3$ and $B = I_3$.

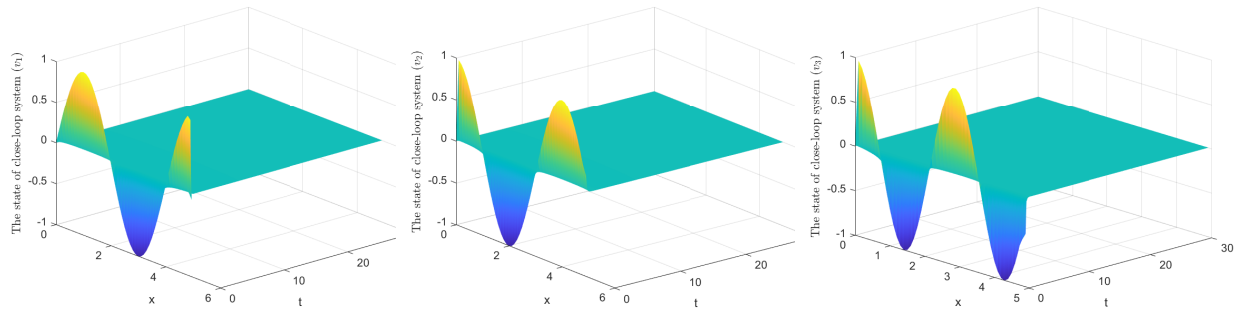


Figure 3. The state of close-loop system.

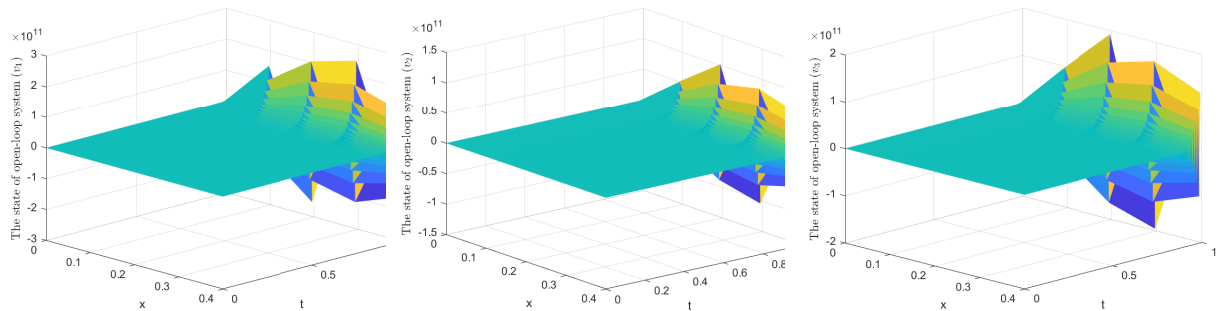


Figure 4. The state of open-loop system.

Subject to initial conditions

$$\begin{cases} v_1(x, 0) = \sin(0.5\pi x), \\ \hat{v}_1(x, 0) = \cos(0.5\pi x), \end{cases} \begin{cases} v_2(x, 0) = \cos(0.5\pi x), \\ \hat{v}_2(x, 0) = \sin(0.5\pi x), \end{cases} \begin{cases} v_3(x, 0) = \cos(0.7\pi x), \\ \hat{v}_3(x, 0) = \sin(0.8\pi x). \end{cases}$$

We use MATLAB (Matrix Laboratory) to verify that the LMIs presented in Theorem 2 have a feasible solution. Based on the results above, we can derive the corresponding controller gain matrix and observer gain matrices, and the results are as follows:

$$K = -1.5067 \times I_3, \quad L = 3.6189 \times I_3, \quad \tilde{L} = 4.8605 \times I_3.$$

As indicated in Figure 3, the controller drives the system to gradually achieve stability. Conversely, Figure 4 demonstrates that without the controller, the system exhibits divergence over time. The observed state of the system is depicted in Figure 5, and the observation error curve is shown in Figure 6. These results confirm that the observer provides accurate estimates of the system state and transmits this information to the controller, thereby ensuring the effectiveness of the control.

It is worth noting that most existing studies fail to consider the impact of DoS attacks, and conventional controllers typically rely directly on full system outputs or multi-point boundary measurements for feedback control [28].

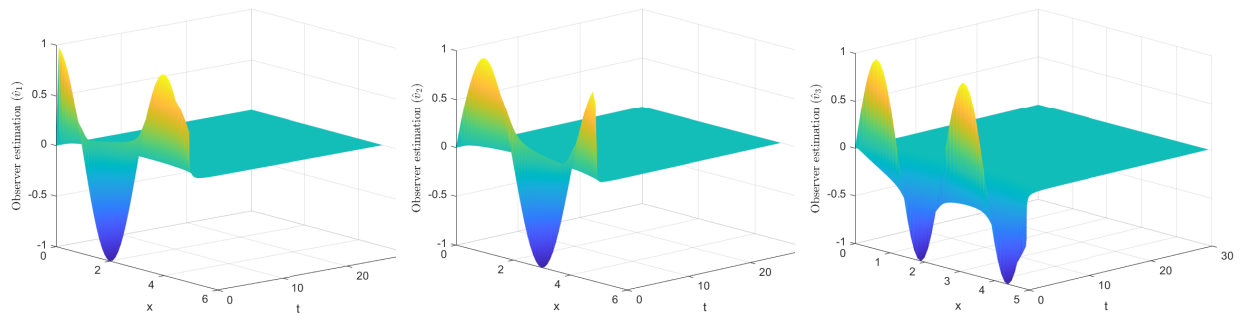


Figure 5. Observer state.

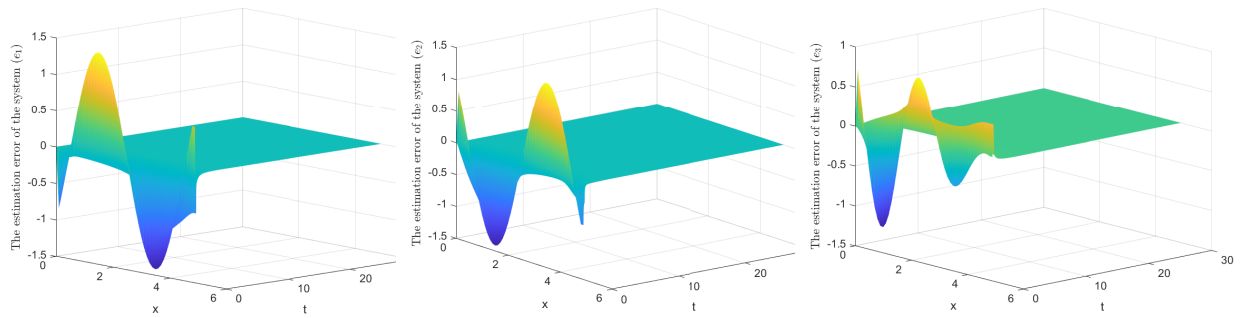


Figure 6. The estimation error of the system.

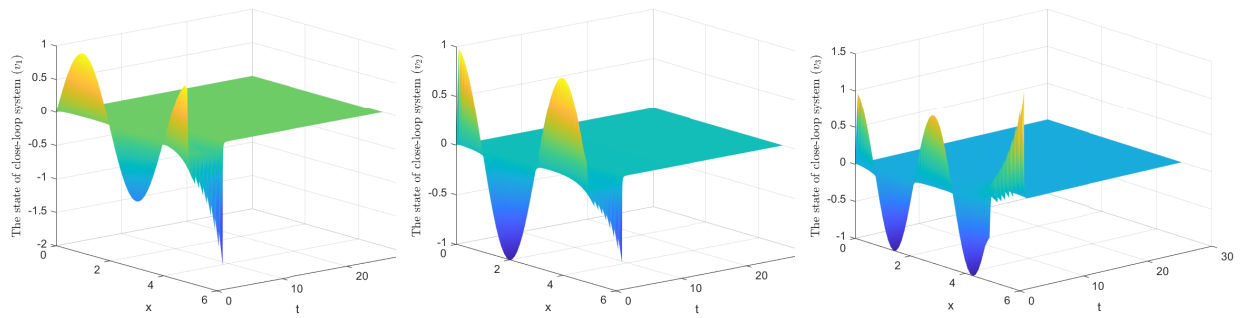


Figure 7. System states of the traditional strategy (Without DoS Attacks and Observer).

In traditional methods, the controller Equation (4) is usually designed as

$$u(t) = K_0 v_{out}(t).$$

When this controller is used to control the system Equation (1), the matrices and parameters of the system are consistent with those in the above. We ultimately obtain the controller gain $K_0 = -0.7983 \times I_3$.

The state of the closed-loop system obtained thereby is shown in Figure 7. The results demonstrate that although such traditional controllers can maintain the stability of system Equation (1) without considering DoS attacks or incorporating observers, the proposed method in this paper simultaneously integrates the suppression of DoS attack interference and state estimation via observers. It is more aligned with practical engineering scenarios involving communication attacks and constrained state measurement, thus exhibiting more prominent application value.

5. CONCLUSIONS

In this paper, the problem of boundary control for DPSs under DoS attacks has been addressed. First, a PDE-based DPS model has been established to accurately characterize the system's spatiotemporal dynamics, and a boundary observer dependent only on the right boundary state has been designed to tackle the practical issue of incomplete state measurement in complex DPSs. Then, an anti-DoS observer-based boundary controller has been proposed, which is applied solely to the spatial boundary to reduce actuator deployment costs while ensuring robustness against intermittent DoS attacks. Additionally, a systematic design strategy for the observer and controller gains has been presented by constructing a LKF and solving LMIs, which theoretically guarantees the exponential stability of the closed-loop system. Finally, the feasibility and effectiveness of the proposed method have been fully demonstrated through a numerical example. This strategy not only theoretically guarantees the exponential stability of the closed-loop system but also quantifies the trade-off between attack parameters and system stability performance, thereby filling the gap in quantitative design criteria for anti-DoS control of DPSs.

DECLARATIONS

Authors' contributions

Methodology, writing - original draft, investigation, conceptualization: Wang, Y.
Conceptualization, investigation, visualization: Zhao, N.

Availability of data and materials

The raw data and experimental datasets supporting this research were generated through numerical simulations based on the proposed theoretical model.

AI and AI-assisted tools statement

Not applicable.

Financial support and sponsorship

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Conflicts of interest

Both authors declared that there are no conflicts of interest.

Ethical approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

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