



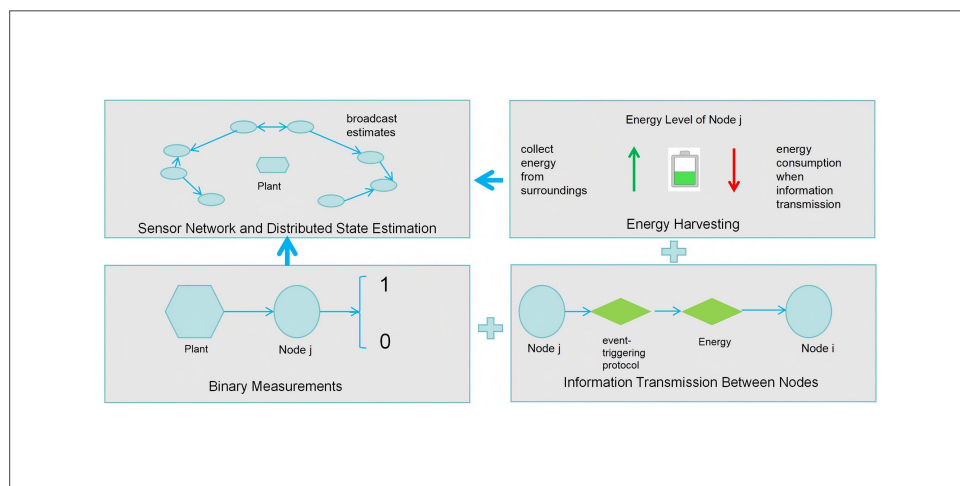
Distributed state estimation over binary sensor networks with energy harvester and dynamic event-triggering protocol: a scalable design

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Abstract

This paper addresses the distributed H_∞ -consensus state estimation issue for a class of discrete time-varying systems operating within binary sensor networks. An integral measurement output model is developed for each node to formulate the time intervals associated with sampling. Every binary sensor is equipped with an energy harvester to improve power efficiency. Information transmission between sensor nodes and their neighboring nodes is carefully orchestrated through a dynamic event-triggering protocol. Valuable information for estimation purposes is obtained by analyzing the discrepancies between the real and predicted inputs of binary sensors. Information from neighboring nodes is only transmitted when the node's energy level is positive and the event-triggering condition is met. Two random variables are introduced to represent the energy level and the information from neighboring nodes to be received or not, respectively. Based on the available information, a distributed estimator is constructed for every binary sensor, and the expected performance

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constraints are given for the dynamic characteristics of estimation errors within a finite horizon. Sufficient conditions are constructed to obtain the desired distributed estimation performance constraint, and associated estimator gains are achieved by resolving the recursive linear matrix inequalities at each node, indicating the excellent scalability of the proposed approach. Ultimately, the effectiveness of the distributed estimation algorithm proposed in this paper is validated through an extensive simulation analysis.

1. INTRODUCTION

In the past decades, wireless sensor networks have gained rapidly prominence in the tech world and have been widely adopted in a multitude of critical sectors^[1], including target tracking^[2], high-source localization^[3], parameter identification^[4], hazardous environment exploration^[5], parabolic systems^[6], and earthquake detection^[7]. Sensor networks are generally made up of numerous sensor nodes that collaborate to observe specific regions and collect environmental information. As a new kind of estimation scheme, distributed estimation has been developed on the hardware basis of sensor networks. Scalable design has become a hot topic when studying the distributed estimation problem due to the large scale of sensor networks. In accordance with vector dissipation theory, the local performance analysis (LPA) method has been shown to be an effective tool to achieve scalable design of distributed estimation^[8,9].

Due to their cost-effectiveness, binary sensor networks (BSNs) have garnered significant attention^[10,11,12,13], resulting in greater focus on the estimation and control challenges associated with binary measurements (BMs)^[14,15,16,17]. It is important to recognize that a single binary sensor can supply only a bit of data, which is inadequate for estimation purposes. To obtain meaningful insights from BM, an effective information extraction method has been proposed based on deterministic thresholds in^[14,17]. The fundamental principle of this approach involves considering the binary sensor's threshold as a weighted sum of two basis models. Then it leverages the discrepancy between the actual and predicted values to extract valuable insights. This method offers an effective means of acquiring valuable information from BM, thereby enhancing the estimation and control of the state of the system.

Most researchers implicitly assume that measurement outputs are functions of the system's condition at the present time interval. Nevertheless, this presumption often falls short in real-world scenarios. Due to delayed data collection, system measurements are obtained within a given time interval, leading to the phenomenon of integral measurements. This phenomenon frequently occurs in processes such as chemical processes^[18] and nuclear reactions^[19]. Recently, the challenge of estimating integral measurements has started to attract research attention^[20,21]. For example, state estimation and fault reconstruction schemes have been proposed in^[20] in a specific category of discrete-time systems subject to partially decoupled disturbances. The state estimation problem has been studied by Shen *et al.*^[21] for multirate artificial neural networks that incorporate integral measurements under the H_∞ performance constraint. However, the distributed state estimation problems within BSNs using integral measurements have not been adequately investigated, which drives the present study.

In network control systems, limited bandwidth would result in signal transmission bottlenecks, which could give rise to various undesirable network-induced phenomena such as time delays, data collision, packet losses, *etc.*^[22,23,24]. To mitigate the frequency of transmissions, an effective strategy is to implement the so-called event-triggering protocol (ETP), which operates on the principle of transmitting essential information solely when a predefined event-triggering condition is satisfied^[27,26,25,28,29,30,31]. It should be noted that most existing ETPs are static, which implies that the threshold in event-triggering conditions cannot be adjusted. To optimize the utilization of communication resources, the concept of dynamic ETP has been introduced in previous works^[32,34,33], which leverages non-negative internal variables to adaptively adjust the threshold parameters.

Specifically, when it comes to resource-constrained wireless sensor networks, dynamic ETP has been applied by Ge et al. [33] to schedule local measurement transmissions between neighboring nodes to address distributed set-membership estimation problems.

In the realm of estimation problems, the core of consensus-based estimation schemes lies in fusing each node's own estimation information with that received from its neighboring nodes and then transmitting this integrated estimation result to the entire estimation system. However, it should be noted that a crucial practical factor is often overlooked in most of the existing literature: the energy consumption required for information transmission between neighboring nodes [35,36,37]. In real-world applications, when sensor nodes directly transmit information, to sustain prolonged communication abilities, it is essential for sensors to discover efficient methods for energy restoration and effective energy replenishment. In recent years, energy harvesting (EH) technology has garnered significant research interest due to its sustainability and environmental benefits. This innovative approach has been successfully implemented in various industrial sectors [38,39].

For the filtering issues of EH sensors, the academic community has also achieved a series of promising research results [40,41,42,43]. However, the EH process is formulated by significant randomness, which directly leads to randomness in the amount of energy stored by sensors. In some cases, sensors may be unable to maintain communication with other sensors due to insufficient energy, thereby restricting communication and resulting in intermittent data transmission. Faced with this discontinuous communication mode, the analysis of transmission probability becomes particularly critical in the design of distributed estimators. Especially in distributed estimation, where multiple sensors are involved, the issue of energy consumption is particularly prominent. Therefore, introducing EH mechanisms into distributed estimation systems and proposing a novel estimation method that considers energy constraints for sensors with EH capabilities represent the main research motivation of this paper.

By summarizing the arguments so far, the key focus of our research lies in integrating the issues discussed above, particularly in scheduling dynamic ETPs to optimize the effectiveness of distributed state estimation utilizing EH. The key obstacles encountered in this procedure are detailed below: (1) How can we model dynamic ETPs, EH, and BMs within a unified framework? (2) Given the probability of energy availability, how can we leverage dynamic ETPs to derive an optimal transmission strategy? (3) To alleviate the computational load, what methods can we employ to define adequate conditions for H_∞ -consensus by leveraging stochastic analysis and vector dissipation principles? These anticipated challenges will be tackled in our present study.

In response to the above challenging problems, the main innovations are summarized as three folds: (1) an LPA technique is proposed to address the distributed H_∞ -consensus state estimation problem under integral measurements and BMs for EH and dynamic ETP; (2) a novel model is introduced, which reflects the combined effects of dynamic ETP, EH, and integral measurements on the transmitted signals; (3) local sufficient criteria are established for every node to ensure the targeted filtering efficiency using the LPA technique.

The structure of the remaining part of this paper is outlined as follows. Section 2 elaborates on the problem to be studied, including the system, integral measurements, dynamic ETP, methods for BM information extraction, energy level parameter settings for EH sensors, the distributed estimator and the desired performance constraints. In Section 3, the required distributed estimation scheme is developed using the LPA method. Moreover, the estimator gains are derived by resolving a series of recursive linear matrix inequalities (RLMIs). Section 4 provides a case study to illustrate the effectiveness of the suggested distributed estimation method. In Section 5, the final conclusions are presented.

2. PROBLEM FORMULATION

2.1. Topology of sensor network

As for a BSN, its topology is formulated as a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, \dots, N\}$, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and $\mathcal{A} = [a_{ij}]$ with $a_{ij} \geq 0$ denote the node set, the edge set, and the adjacency matrix, respectively. For different sensors $i \in \mathcal{V}$ and $j \in \mathcal{V}$, an edge is expressed as an ordered pair $(i, j) \in \mathcal{E}$. Moreover, if $(i, j) \in \mathcal{E}$, which indicates that j can send message to i , then $a_{ij} = 1$; otherwise $a_{ij} = 0$. $\mathcal{N}_i = \{j | (i, j) \in \mathcal{E}, j \neq i, j \in \mathcal{V}\}$ denotes the neighboring set of node i . If the in-degree and out-degree of node i are denoted as $p_i = \sum_j a_{ij}$ and $q_i = \sum_j a_{ji}$, respectively, then one further has $\mathcal{N}_i \triangleq \{j_{i1}, j_{i2}, \dots, j_{ip_i}\}$, where j_{id} denotes the d -th neighboring node.

2.2. System formulations

Consider a state-space mode of discrete time-varying systems as follows:

$$\bar{x}_{u+1} = \bar{A}_u \bar{x}_u + \bar{B}_u \omega_u \quad (1)$$

where $\bar{x}_u \in \mathbb{R}^{n_{\bar{x}}}$ and $\omega_u \in \mathbb{R}^{n_{\omega}}$ are the system state and the external disturbance belonging to $l_2[0, n-1]$, respectively; $\bar{A}_u \in \mathbb{R}^{n_{\bar{x}} \times n_{\bar{x}}}$ and $\bar{B}_u \in \mathbb{R}^{n_{\bar{x}} \times n_{\omega}}$ are known time-varying matrices; u is the sample time belonging to $\mathcal{H} \triangleq \{0, 1, \dots, n-1\}$. $\bar{x}_0$ is the initial state of the system.

The measurement of binary sensor $i \in \mathcal{V}$ is given as follows:

$$\begin{cases} y_{i,u} = h_i(z_{i,u}) = \begin{cases} 1, & \text{if } z_{i,u} \geq \tau_i; \\ 0, & \text{if } z_{i,u} < \tau_i, \end{cases} \\ z_{i,u} = \sum_{q=0}^s \bar{C}_{i,u-q} \bar{x}_{u-q} + D_{i,u} v_{i,u} \end{cases} \quad (2)$$

where $y_{i,u} \in \mathbb{R}$ and $z_{i,u} \in \mathbb{R}$ are the measurement output (that is directly accessible) and input (that is inaccessible) of the binary sensor i ; $\tau_i \in \mathbb{R}$ is a fixed threshold, $v_{i,u} \in \mathbb{R}^{n_v}$ is the measurement noise belonging to $l_2[0, n-1]$; $s \in \mathbb{R}$ is the duration designated for data collection, and $\bar{C}_{i,u-q} \in \mathbb{R}^{1 \times n_{\bar{x}}}$ and $D_{i,u} \in \mathbb{R}^{1 \times n_v}$ are two known time-varying matrices.

Regarding the integral measurements, the sample-state augmentation method (i.e., the lifting method) has been utilized to address integral measurements^[18]. Subsequently, the lifting method has become a widely adopted approach for studying estimation problems involving integral measurements^[44,45]. For integral measurement (2), denote $x_u \triangleq [\bar{x}_u^T \quad \bar{x}_{u-1}^T \quad \dots \quad \bar{x}_{u-s}^T]^T$. As such, (1) and (2) are rewritten as:

$$\begin{cases} x_{u+1} = A_u x_u + B_u \omega_u, \\ z_{i,u} = C_{i,u} x_u + D_{i,u} v_{i,u}, \\ y_{i,u} = h_i(z_{i,u}) \end{cases} \quad (3)$$

where

$$A_u \triangleq \begin{bmatrix} \bar{A}_u & 0 & 0 & \cdots & 0 \\ I & 0 & 0 & \cdots & 0 \\ 0 & I & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix},$$

$$B_u \triangleq \begin{bmatrix} \bar{B}_u^T & 0 & 0 & \cdots & 0 \end{bmatrix}^T,$$

$$C_{i,u} \triangleq \begin{bmatrix} \bar{C}_{i,u} & \bar{C}_{i,u-1} & \cdots & \bar{C}_{i,u-s} \end{bmatrix}.$$

2.3. Distributed estimator for binary sensor

Construct a distributed estimator for binary sensor i using all the available information:

$$\hat{x}_{i,u+1} = A_u \hat{x}_{i,u} + \sum_{j \in \mathcal{N}_i} G_{ij,u} (\hat{x}_{j,u} - \hat{x}_{i,u}) + \theta_{i,u} K_{i,u} (\tau_i - C_{i,u} \hat{x}_{i,u}), \quad (4)$$

$$\theta_{i,u} = \begin{cases} 1, & \text{if } y_{i,u} \neq \bar{y}_{i,u}; \\ 0, & \text{otherwise;} \end{cases} \quad (5)$$

$$\bar{z}_{i,u} \triangleq C_{i,u} \hat{x}_{i,u}, \quad (6)$$

$$\bar{y}_{i,u} = h_i(\bar{z}_{i,u}) = \begin{cases} 1, & \text{if } \bar{z}_{i,u} \geq \tau_i; \\ 0, & \text{otherwise;} \end{cases} \quad (7)$$

where $\hat{x}_{i,u}$, $\bar{z}_{i,u}$ and $\bar{y}_{i,u}$ are the estimated state, measurement output, and measurement input, respectively; $K_{i,u}$ and $G_{ij,u}$ are the estimator gains to be calculated later; $\theta_{i,u}$ expresses whether the information from the BMs is used or not in the estimator.

Now, introduce the principle of obtaining useful information for estimation purposes. From (2), (5) and (7), under the case of $\theta_{i,u} = 1$, i.e., $y_{i,u} \neq \bar{y}_{i,u}$, which means that $\bar{z}_{i,u} > \tau_i$ and $z_{i,u} < \tau_i$. As a consequence, τ_i can be expressed as linear combination of $\bar{z}_{i,u}$ and $z_{i,u}$, i.e.,

$$\tau_i = 0.5(1 - \delta_{i,u})\bar{z}_{i,u} + 0.5(1 + \delta_{i,u})z_{i,u} \quad (8)$$

where the uncertain scalar $\delta_{i,u}$ belongs to $[-1, 1]$.

By resorting to (8), one further has

$$\tau_i - C_{i,u} \hat{x}_{i,u} = 0.5(1 + \delta_{i,u})C_{i,u}(x_{i,u} - \hat{x}_{i,u}) + 0.5(1 - \delta_{i,u})D_{i,u}v_{i,u}. \quad (9)$$

According to (4), $\hat{x}_{j,u} (j \in \mathcal{N}_i)$ will be transferred to node i . To conserve sensor power and reduce network congestion, the following dynamic ETP is used to schedule the information transmission between neighboring nodes.

2.4. Dynamic event-triggering protocol

This subsection uses a dynamic ETP to determine whether the neighbor's information should be sent to node i at the current time. The event-triggering time sequence $0 \leq t_{j,0} < t_{j,1} < \dots < t_{j,u} < \dots$ is determined by the following dynamic ETP:

$$t_{j,u+1} = \min_{u \in \mathbb{Z}^+} \left\{ u > t_{j,u} \mid \frac{1}{\mu_j} r_{j,u} + \varsigma_j - \varepsilon_{j,u}^T \varepsilon_{j,u} \leq 0 \right\} \quad (10)$$

where

$$\varepsilon_{j,u} \triangleq \hat{x}_{j,u} - \hat{x}_{j,t_{j,u}}, \quad (11)$$

and $r_{j,u}$ is an internal dynamic variable whose evolution is given as

$$r_{j,u+1} = \lambda_j r_{j,u} + \varsigma_j - \varepsilon_{j,u}^T \varepsilon_{j,u}, \quad r_{j,0} \geq 0 \quad (12)$$

where μ_j , λ_j , and ς_j are given positive scalars. Moreover, the constraint $\lambda_j \mu_j \geq 1$ is satisfied. As such, for all $u \in [0, +\infty)$, one has $r_{j,u} \geq 0$. Here, $r_{j,u} \geq 0$ is a dynamic variable to adjust the fixed threshold ς_j in the traditional event-triggering condition for reducing the frequency of information transmission of node j .

In addition, to describe the effect of the dynamic ETP, define the following indicative variable as follows:

$$\zeta_{j,u} \triangleq \begin{cases} 1, & \frac{1}{\mu_j} r_{j,u} + \varsigma_j - \varepsilon_{j,u}^T \varepsilon_{j,u} \leq 0; \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

2.5. Energy harvesting

The inherent constraints in energy storage units within the sensors often prove inadequate for sustained information transmission between neighboring nodes. Consequently, our focus shifts to EH techniques, where a sensor, armed with an energy harvester, has the ability to harvest energy from the surrounding environment and convert it into electrical energy for the purposes of information transmission. Following a similar idea as in [47,48,46], the EH model is formulated for a neighboring node j of node i .

To begin with, the energy collected by the sensor node j is denoted by a random variable $h_{j,u}$ at the time instant u . Moreover, assume that $h_{j,u}$ is finite and satisfies the well-known binomial distribution, the distribution law of which is given as

$$\mathbb{P}(h_{j,u} = m) = \frac{b!}{m!(b-m)!} p_j^m (1-p_j)^{b-m}, \quad m = 0, 1, 2, \dots, b \quad (14)$$

where the integer b is the upper bound of the collected energy, and the scalar p_j satisfies the constraint $0 < p_j < 1$. For convenience in expression, $\mathbb{P}(h_{j,u} = m)$ is abbreviated as $\bar{p}_{j,u}^m$ satisfying $0 \leq \bar{p}_{j,u}^m \leq 1$. In addition, $h_{j,u}$ and other stochastic variables in this paper are mutually independent for different nodes j and time instant u .

Remark 1 Under certain assumptions, the energy harvested by sensor node j , which follows a binomial distribution, can be interpreted based on the principles of photovoltaic power generation. For simplicity, let b denote the total number of photons incident on a specific area of the material surface during the time interval from $n-1$ to n .

Each photon has two possible outcomes upon incidence: it is either absorbed (defined as "success") or not absorbed (defined as "failure," i.e., reflected or transmitted). Assume that the probability of absorption is a constant p , and correspondingly, the probability of non-absorption is $1 - p$. When a photon is absorbed, it contributes one unit of stored energy. It is generally accepted that photons do not interact with each other under typical light intensities (thus, the bosonic bunching effect can be neglected) and the absorption of each photon can therefore be considered an independent event. Consequently, during the time interval from $u - 1$ to u , given that b photons illuminate the surface, the probability that exactly m photons are absorbed follows a binomial distribution.

Next, the energy level of the sensor node j ($j \in \mathcal{N}_i$) at time instant u is denoted by $\kappa_{j,u} \in \{0, 1, 2, \dots, s_j\}$, where $s_j > 0$ represents the maximum capacity that can be stored. At instant u , the sensor node j ($j \in \mathcal{N}_i$) will consume c_j units of energy to broadcast its information simultaneously to all its out-neighboring nodes i if $\kappa_{j,u} > 0$. Moreover, any surplus energy generated beyond the battery's storage limit will be automatically jettisoned. In light of the above formulations, the dynamics of the energy level $\kappa_{j,u}$ is modeled as

$$\kappa_{j,u+1} = \min\{\kappa_{j,u} + h_{j,u} - \sigma_{j,u}\zeta_{j,u}c_j, s_j\} \quad (15)$$

where

$$\sigma_{j,u} \triangleq \begin{cases} 1, & \text{if } \kappa_{j,u} + h_{j,u} - \zeta_{j,u}c_j \geq 0; \\ 0, & \text{otherwise} \end{cases}$$

Here, $\sigma_{j,u} = 0$ means that the neighboring node j cannot transmit $\hat{x}_{j,u}$ to node i due to a lack of enough energy.

Remark 2 As introduced in the Introduction, EH and ETPs have been investigated for distinct underlying motivations. Specifically, EH is primarily motivated by the need to model the impact of energy consumption during information transmission. In contrast, the ETP aims to conserve energy by reducing the frequency of information transmissions through the imposition of a predefined triggering condition. In this paper, building upon existing results, we aim to investigate the precise energy-saving effect achieved within a framework that integrates both ETPs and EH techniques.

According to the previous analysis, the following distributed estimator is constructed:

$$\hat{x}_{i,u+1} = A_u \hat{x}_{i,u} + \sum_{j \in \mathcal{N}_i} \sigma_{j,u} \zeta_{j,u} G_{ij,u} (\hat{x}_{j,u} - \hat{x}_{i,u}) + \theta_{i,u} K_{i,u} (\tau_i - C_{i,u} \hat{x}_{i,u}). \quad (16)$$

2.6. Problem statement

Denoting $e_{i,u} \triangleq x_u - \hat{x}_{i,u}$, one has the estimation-error dynamic equation:

$$\begin{aligned} e_{i,u+1} = & (A_u - 0.5\theta_{i,u}K_{i,u}C_{i,u} - \sum_{j \in \mathcal{N}_i} \sigma_{j,u}\zeta_{j,u}G_{ij,u})e_{i,u} + \sum_{j \in \mathcal{N}_i} \sigma_{j,u}\zeta_{j,u}G_{ij,u}(e_{j,u} + \varepsilon_{j,u}) \\ & + B_u\omega_u - 0.5\theta_{i,u}K_{i,u}D_{i,u}v_{i,u} - 0.5\delta_{i,u}\theta_{i,u}(K_{i,u}C_{i,u}e_{i,u} + K_{i,u}D_{i,u}v_{i,u}). \end{aligned} \quad (17)$$

Letting $\xi_{i,u} \triangleq \begin{bmatrix} w_u^T & v_{i,u}^T \end{bmatrix}^T$, one further has

$$e_{i,u+1} = (A_{i,u} - 0.5\delta_{i,u}\theta_{i,u}K_{i,u}C_{i,u} - \sum_{j \in \mathcal{N}_i} \sigma_{j,u}\zeta_{j,u}G_{ij,u})e_{i,u} + (B_{i,u} + \delta_{i,u}E_{i,u})\xi_{i,u} + \sum_{j \in \mathcal{N}_i} \sigma_{j,u}\zeta_{j,u}G_{ij,u}(e_{j,u} + \varepsilon_{j,u}) \quad (18)$$

where

$$A_{i,u} \triangleq A_u - 0.5\theta_{i,u}K_{i,u}C_{i,u}, \quad B_{i,u} \triangleq \begin{bmatrix} B_u & -0.5\theta_{i,u}K_{i,u}D_{i,u} \end{bmatrix}, \quad E_{i,u} \triangleq \begin{bmatrix} 0 & -0.5\theta_{i,u}K_{i,u}D_{i,u} \end{bmatrix}.$$

In the following, the system dynamics Equation 18 is provided with the specified performance metric.

Definition 1 For any node $i \in \mathcal{V}$, let positive constants γ and ι_i , weighted matrices $U_i > 0$, $R_{i,u} > 0$, and $T_{i,u} > 0$ be given. Under energy constraints Equation 15 and dynamic ETPs Equation 10, system dynamics Equation 18 satisfies the H_∞ -consensus performance constraint over the finite horizon $\mathcal{X}$ in the mean-square sense if the subsequent inequality holds

$$\sum_{i=1}^N \sum_{u=0}^{n-1} \mathbb{E} \left\{ \|e_{i,u}\|_{T_{i,u}}^2 + \sum_{j \in \mathcal{N}_i} \|e_{j,u} - e_{i,u}\|_{R_{i,u}}^2 \right\} \leq \gamma^2 \sum_{i=1}^N (e_{i,0}^T U_i e_{i,0} + \iota_i) + \gamma^2 \sum_{i=1}^N \sum_{u=0}^{n-1} \|\xi_{i,u}\|^2. \quad (19)$$

The aim of this paper is to find the appropriate estimator gains, denoted as $K_{i,u}$ and $G_{ij,u}$ ($i \in \mathcal{V}$ and $j \in \mathcal{N}_i$), so that the inequality constraint Equation 19 holds.

3. MAIN RESULTS

3.1. Probabilistic distribution law of energy level

As discussed earlier, the transmission of information between neighboring nodes can only be achieved if the energy level is positive (that is, $\kappa_{j,u} > 0$) due to the introduction of the EH technique. Next, we present the probabilistic distribution law of the random variable $\sigma_{j,u}$.

Denote $p_{j,u}^m \triangleq \mathbb{P}(\kappa_{j,u} = m)$ for $m = 0, 1, \dots, s_j$, $\phi_{j,u} \triangleq [p_{j,u}^0 \quad p_{j,u}^1 \quad \dots \quad p_{j,u}^{s_j}]^T$ and $\gamma_{j,u}^2 \triangleq \mathbb{E}\{(\sigma_{j,u} - \bar{\sigma}_{j,u})^2\}$.

Lemma 1 [43] The following conclusions hold

$$\gamma_{j,u}^2 = \bar{\sigma}_{j,u}(1 - \bar{\sigma}_{j,u}) \quad (20)$$

where

$$\bar{\sigma}_{j,u} \triangleq 1 - \mathbf{a}_j \phi_{j,u}, \quad \mathbf{a}_j \triangleq [1 \quad \underbrace{0 \quad \dots \quad 0}_{s_j}],$$

and $\phi_{j,u}$ satisfies the following dynamic equation

$$\phi_{j,u+1} = \zeta_j + \mathbf{M}_{j,u} \phi_{j,u} \quad (21)$$

where

$$\zeta_j \triangleq \underbrace{[0 \cdots 0]_{s_j}}_{s_j} 1]^T,$$

$$\mathbf{M}_{j,u} \triangleq \begin{bmatrix} \bar{p}_{j,u}^0 & \bar{p}_{j,u}^0 & 0 & \cdots & 0 \\ \bar{p}_{j,u}^1 & \bar{p}_{j,u}^1 & \bar{p}_{j,u}^0 & \cdots & 0 \\ \bar{p}_{j,u}^2 & \bar{p}_{j,u}^2 & \bar{p}_{j,u}^1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{p}_{j,u}^{s_j-1} & \bar{p}_{j,u}^{s_j-1} & \bar{p}_{j,u}^{s_j-2} & \cdots & \bar{p}_{j,u}^0 \\ -\sum_{m=0}^{s_j-1} \bar{p}_{j,u}^m & -\sum_{m=0}^{s_j-1} \bar{p}_{j,u}^m & -\sum_{m=0}^{s_j-2} \bar{p}_{j,u}^m & \cdots & -\bar{p}_{j,u}^0 \end{bmatrix}.$$

3.2. Vector dissipation

This subsection provides the LPA method to derive sufficient local conditions for any node $i \in \mathcal{V}$ to ensure that Equation 18 satisfies the desired performance criterion Equation 19. Vector dissipation is the basis of the LPA method. As such, let us recall the definition of vector dissipation.

Let $\mathbf{V}_u \triangleq [v_{1,u} \cdots v_{N,u}]^T$ and $\mathbf{S}_u \triangleq [s_{1,u} \cdots s_{N,u}]^T$.

Definition 2 It is said that the system dynamics Equation 18 is stochastic dissipative regarding the supply rate function $s_{i,u}$, if there exist a storage function $v_{i,u} \geq 0$ (with $v_0 = 0$) and a dissipation matrix $W \in \mathbb{R}^{N \times N}$ such that the following inequality holds

$$\mathbb{E}\{V_{i,u+1}\} < [W\mathbf{V}_u]_i + S_{i,u}. \quad (22)$$

Furthermore, the system dynamics Equation 18 is vector dissipation regarding the vector supply rate function $\mathbf{S}_u$, if Equation 22 holds for any node $i \in \mathcal{V}$ and $u \in \mathcal{H}$, i.e.,

$$\mathbb{E}\{\mathbf{V}_{u+1}\} \ll W\mathbf{V}_u + \mathbf{S}_u. \quad (23)$$

Here, for any two vectors with the same dimensions, $\mathbf{e} \triangleq [e_1 \ e_2 \ \cdots \ e_N]^T$ and $\mathbf{f} \triangleq [f_1 \ f_2 \ \cdots \ f_N]^T \in \mathbb{R}^N$, $\mathbf{e} \ll \mathbf{f}$ implies that $e_i < f_i$ ($\forall i \in \mathcal{V}$).

Here, as the focus of the LPA method, Equation 23 refers to the vector dissipation inequality, which is constructed by seeking suitable local conditions of node $i \in \mathcal{V}$ with its neighboring nodes. In addition, the dissipation matrix W needs to be nonsingular and the column sub-stochastic, which will be constructed later.

3.3. Scalable distributed state estimation scheme

For proceeding further, for any node $i \in \mathcal{V}$, introduce an interval-valued function regarding q_i :

$$\mathcal{J}_{q_i} \triangleq \begin{cases} (0, \frac{1+q_i}{2q_i}), & \text{if } q_i \neq 0; \\ (0, 1], & \text{if } q_i = 0, \end{cases} \quad (24)$$

and choose the supply rate function:

$$S_{i,u} \triangleq \gamma^2 \|\xi_{i,u}\|^2 - \|e_{i,u}\|_{T_{i,u}}^2 - \sum_{j \in \mathcal{N}_i} \|e_{j,u} - e_{i,u}\|_{R_{i,u}}^2 + \frac{S_i}{n\mu_i} + \frac{\vartheta_i}{n} \sum_{j \in \mathcal{N}_i} s_j,$$

where $\vartheta_i \in \mathbb{R}$ is the positive parameter to be given by the subsequent theorem.

With the help of the LPA method, a sufficient criterion regarding the performance constraint Equation 19 is established in the subsequent theorem for the system dynamics Equation 18 by means of the vector dissipation of system Equation 18 regarding the supply rate function $s_{i,u}$.

Theorem 1 Consider any binary sensor $i \in \mathcal{V}$ with its neighboring nodes $j (j \in \mathcal{N}_i)$. Suppose that the parameters $\alpha_i \in \mathcal{J}_{q_i}$, $\alpha_j \in \mathcal{J}_{q_j}$, $\gamma > 0$, $s_i > 0$, $s_j > 0$, $\varrho_i > 0$, $\vartheta_i > 0$, and $\bar{r}_i > 0$ satisfying the following conditions:

$$\frac{S_i}{\mu_i} + \vartheta_i \sum_{j \in \mathcal{N}_i} s_j \leq \gamma^2 \varrho_i, \lambda_i + \alpha_i < 1, (1 + q_j)\vartheta_i < \alpha_j, r_{i,0} \leq n\gamma^2 \bar{r}_i.$$

Let U_i and $P_{i,0}$ be given positive definite matrices satisfying $0 < P_{i,0} \leq \gamma^2 U_i$, and let $K_{i,u}$ and $G_{ij,u}$ be the given estimator gains. Then, the system dynamics Equation 18 achieves the performance constraint Equation 19, if there exists a set of positive definite matrices $P_{i,u+1} > 0$ such that the following $\theta_{i,u}$ -dependent inequality holds for any $u \in \mathcal{H}$:

$$\Pi_{i0,u} + \Pi_{i1,u}^T P_{i,u+1} \Pi_{i1,u} + \sum_{d=1}^{p_i} \Pi_{ijd,u}^T P_{i,u+1} \Pi_{ijd,u} \leq 0 \quad (\theta_{i,u} = 0, 1) \quad (25)$$

where

$$\begin{aligned} \Pi_{i0,u} &\triangleq \begin{bmatrix} \Pi_{i0,u}^{11} & * & * & * & * & * & * \\ \Pi_{i0,u}^{21} & \Pi_{i0,u}^{22} & * & * & * & * & * \\ 0 & 0 & \Pi_{i0,u}^{33} & * & * & * & * \\ 0 & 0 & 0 & \Pi_{i0,u}^{44} & * & * & * \\ 0 & 0 & 0 & 0 & \Pi_{i0,u}^{55} & * & * \\ 0 & 0 & 0 & 0 & 0 & \Pi_{i0,u}^{66} & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \Pi_{i0,u}^{77} \end{bmatrix}, \\ \Pi_{i0,u}^{11} &\triangleq -(1 - \alpha_i)P_{i,u} + T_{i,u} + (p_i + 1)R_{i,u}, \quad \Pi_{i0,u}^{21} \triangleq -\text{col}_{p_i}\{R_{i,u}\}, \quad \Pi_{i0,u}^{22} \triangleq \text{diag}_{p_i} \left\{ -\frac{\alpha_{j_d}}{1 + q_{j_d}} P_{j_d,u} + R_{i,u} \right\}, \\ \Pi_{i0,u}^{33} &\triangleq -\gamma^2 I, \quad \Pi_{i0,u}^{44} \triangleq -\frac{1}{n\mu_i} I, \quad \Pi_{i0,u}^{55} \triangleq \text{diag}_{p_i} \left\{ -\frac{\vartheta_j}{n\mu_j} I \right\}, \quad \Pi_{i0,u}^{66} \triangleq \frac{\lambda_i + \alpha_i - 1}{n\mu_i}, \quad \Pi_{i0,u}^{77} \triangleq \text{diag}_{p_i} \left\{ \frac{(1 + q_j)\vartheta_i - \alpha_j}{(1 + q_j)n\mu_j} \right\}, \\ \Pi_{i1,u} &\triangleq \begin{bmatrix} \Pi_{i1,u}^{(11)} & \Pi_{i1,u}^{(22)} & \Pi_{i1,u}^{(33)} & \Pi_{i1,u}^{(44)} & 0 & 0 & 0 \end{bmatrix}, \\ \Pi_{i1,u}^{(11)} &\triangleq A_{i,u} - 0.5\delta_{i,u}\theta_{i,u}K_{i,u}C_{i,u} - \sum_{j \in \mathcal{N}_i} \bar{\sigma}_{j,u}\zeta_{j,u}G_{ij,u}, \quad \Pi_{i1,u}^{(22)} \triangleq \bar{\sigma}_{\mathcal{N}_i,u} \circ \zeta_{\mathcal{N}_i,u} \circ G_{\mathcal{N}_i,u} \\ \Pi_{i1,u}^{(33)} &\triangleq B_{i,u} + \delta_{i,u}E_{i,u}, \quad \Pi_{i1,u}^{(44)} \triangleq \bar{\sigma}_{\mathcal{N}_i,u} \circ \zeta_{\mathcal{N}_i,u} \circ G_{\mathcal{N}_i,u}, \\ \Pi_{ijd,u} &\triangleq \begin{bmatrix} \Pi_{ijd,u}^{(11)} & -\Pi_{ijd,u}^{(22)} & 0 & -\Pi_{ijd,u}^{(44)} & 0 & 0 & 0 \end{bmatrix}, \quad \Pi_{ijd,u}^{(11)} \triangleq \gamma_{j_d,u}\zeta_{j_d,u}G_{ij_d,u}, \quad \bar{\sigma}_{\mathcal{N}_i,u} \triangleq [\bar{\sigma}_{j_{i_1},u} \dots \bar{\sigma}_{j_{i_{p_i}},u}], \\ \Pi_{ijd,u}^{(22)} &\triangleq \begin{bmatrix} \underbrace{0 \dots 0}_{d-1} & \gamma_{j_d,u}\zeta_{j_d,u}G_{ij_d,u} & \underbrace{0 \dots 0}_{p_i-d} \end{bmatrix}, \quad \Pi_{ijd,u}^{(44)} \triangleq \begin{bmatrix} \underbrace{0 \dots 0}_{d-1} & \gamma_{j_d,u}\zeta_{j_d,u}G_{ij_d,u} & \underbrace{0 \dots 0}_{p_i-d} \end{bmatrix}, \\ G_{\mathcal{N}_i,u} &\triangleq [G_{ij_1,u} \dots G_{ij_{p_i},u}], \quad \gamma_{\mathcal{N}_i,u} \triangleq [\gamma_{j_{i_1},u} \dots \gamma_{j_{i_{p_i}},u}], \quad \zeta_{\mathcal{N}_i,u} \triangleq [\zeta_{j_{i_1},u} \dots \zeta_{j_{i_{p_i}},u}]. \end{aligned}$$

Proof 1 For node i , choose the scalar storage function with the following form:

$$V_{i,u} \triangleq V_{i1,u} + V_{i2,u} \quad (26)$$

where

$$V_{i1,u} \triangleq e_{i,u}^T P_{i,u} e_{i,u}, \quad V_{i2,u} \triangleq \frac{1}{n\mu_i} r_{i,u}.$$

Calculate the mathematical expectation of the storage function along the dynamics of Equation 12 and Equation 19

$$\begin{aligned} & \mathbb{E}\{V_{i1,u+1}\} \\ &= \mathbb{E}\{e_{i,u+1}^T P_{i,u+1} e_{i,u+1}\} \\ &= \mathbb{E}\left\{\left((A_{i,u} - 0.5\delta_{i,u}\theta_{i,u}K_{i,u}C_{i,u} - \sum_{j \in \mathcal{N}_i} \sigma_{j,u}\zeta_{j,u}G_{ij,u})e_{i,u} + (B_{i,u} + \delta_{i,u}E_{i,u})\xi_{i,u} + \sum_{j \in \mathcal{N}_i} \sigma_{j,u}\zeta_{j,u}G_{ij,u}(e_{j,u} + \varepsilon_{j,u})\right)^T P_{i,u+1}\right. \\ & \quad \times \left.\left((A_{i,u} - 0.5\delta_{i,u}\theta_{i,u}K_{i,u}C_{i,u} - \sum_{j \in \mathcal{N}_i} \sigma_{j,u}\zeta_{j,u}G_{ij,u})e_{i,u} + (B_{i,u} + \delta_{i,u}E_{i,u})\xi_{i,u} + \sum_{j \in \mathcal{N}_i} \sigma_{j,u}\zeta_{j,u}G_{ij,u}(e_{j,u} + \varepsilon_{j,u})\right)\right\} \\ &= \mathbb{E}\left\{\left((A_{i,u} - 0.5\delta_{i,u}\theta_{i,u}K_{i,u}C_{i,u} - \sum_{j \in \mathcal{N}_i} \bar{\sigma}_{j,u}\zeta_{j,u}G_{ij,u})e_{i,u} + (B_{i,u} + \delta_{i,u}E_{i,u})\xi_{i,u} + \sum_{j \in \mathcal{N}_i} \bar{\sigma}_{j,u}\zeta_{j,u}G_{ij,u}(e_{j,u} + \varepsilon_{j,u})\right)^T P_{i,u+1}\right. \\ & \quad \times \left.\left((A_{i,u} - 0.5\delta_{i,u}\theta_{i,u}K_{i,u}C_{i,u} - \sum_{j \in \mathcal{N}_i} \bar{\sigma}_{j,u}\zeta_{j,u}G_{ij,u})e_{i,u} + (B_{i,u} + \delta_{i,u}E_{i,u})\xi_{i,u} + \sum_{j \in \mathcal{N}_i} \bar{\sigma}_{j,u}\zeta_{j,u}G_{ij,u}(e_{j,u} + \varepsilon_{j,u})\right)\right\} \\ & \quad + \mathbb{E}\left\{\left(\sum_{j \in \mathcal{N}_i} (\sigma_{j,u} - \bar{\sigma}_{j,u})\zeta_{j,u}G_{ij,u}(e_{i,u} - e_{j,u} - \varepsilon_{j,u})\right)^T P_{i,u+1} \left(\sum_{j \in \mathcal{N}_i} (\sigma_{j,u} - \bar{\sigma}_{j,u})\zeta_{j,u}G_{ij,u}(e_{i,u} - e_{j,u} - \varepsilon_{j,u})\right)\right\} \\ &= \chi_{i,u}^T \Pi_{i1,u}^T P_{i,u+1} \Pi_{i1,u} \chi_{i,u} + \sum_{j \in \mathcal{N}_i} \gamma_{j,u}^2 \zeta_{j,u}^2 (e_{i,u} - e_{j,u} - \varepsilon_{j,u})^T G_{ij,u}^T P_{i,u+1} G_{ij,u} (e_{i,u} - e_{j,u} - \varepsilon_{j,u}) \\ &\triangleq \chi_{i,u}^T \Pi_{i1,u}^T P_{i,u+1} \Pi_{i1,u} \chi_{i,u} + \sum_{d=1}^{p_i} \chi_{i,u}^T \Pi_{ijd,u}^T P_{i,u+1} \Pi_{ijd,u} \chi_{i,u} \end{aligned}$$

where

$$\begin{aligned} \chi_{i,u} &\triangleq \begin{bmatrix} e_{i,u}^T & e_{\mathcal{N}_i,u}^T & \xi_{i,u}^T & \varepsilon_{i,u}^T & \varepsilon_{\mathcal{N}_i,u}^T & \sqrt{r_{i,u}} & \sqrt{r_{\mathcal{N}_i,u}}^T \end{bmatrix}^T, \quad e_{\mathcal{N}_i,u} \triangleq \text{col}_{p_i}\{e_{ji_d,u}\}, \\ \sqrt{r_{\mathcal{N}_i,u}} &\triangleq \begin{bmatrix} \sqrt{r_{ji_1,u}} & \sqrt{r_{ji_2,u}} & \cdots & \sqrt{r_{ji_{p_i},u}} \end{bmatrix}^T, \quad \varepsilon_{\mathcal{N}_i,u} \triangleq \begin{bmatrix} \varepsilon_{ji_1,u}^T & \varepsilon_{ji_2,u}^T & \cdots & \varepsilon_{ji_{p_i},u}^T \end{bmatrix}^T. \end{aligned}$$

In addition, we also obtain

$$\mathbb{E}\{V_{i2,u+1}\} = \mathbb{E}\left\{\frac{1}{n\mu_i} r_{i,u+1}\right\} = \frac{1}{n\mu_i} (\lambda_i r_{i,u} + \varsigma_i - \varepsilon_{i,u}^T \varepsilon_{i,u}). \quad (27)$$

According to Equation 15, we introduce a positive scalar ϑ_i , such that the following inequality holds:

$$\frac{\vartheta_i}{n} \sum_{j \in \mathcal{N}_i} \left(\frac{1}{\mu_j} r_{j,u} + \varsigma_j - \varepsilon_{j,u}^T \varepsilon_{j,u} \right) \geq 0. \quad (28)$$

As such, one further has

$$\begin{aligned}
 \mathbb{E}\{V_{i,u+1}\} &= \mathbb{E}\{V_{i1,u+1}\} + \mathbb{E}\{V_{i2,u+1}\} \\
 &= \chi_{i,u}^T \Pi_{i1,u}^T P_{i,u+1} \Pi_{i1,u} \chi_{i,u} + \sum_{d=1}^{p_i} \chi_{i,u}^T \Pi_{ij_d,u}^T P_{i,u+1} \Pi_{ij_d,u} \chi_{i,u} + \frac{1}{n\mu_i} (\lambda_i r_{i,u} + \varsigma_i - \varepsilon_{i,u}^T \varepsilon_{i,u}) \\
 &\leq \chi_{i,u}^T \Pi_{i1,u}^T P_{i,u+1} \Pi_{i1,u} \chi_{i,u} + \sum_{d=1}^{p_i} \chi_{i,u}^T \Pi_{ij_d,u}^T P_{i,u+1} \Pi_{ij_d,u} \chi_{i,u} + \frac{1}{n\mu_i} (\lambda_i r_{i,u} + \varsigma_i - \varepsilon_{i,u}^T \varepsilon_{i,u}) \\
 &\quad + \frac{1}{n\mu_i} (\lambda_i r_{i,u} + \varsigma_i - \varepsilon_{i,u}^T \varepsilon_{i,u}) + \frac{\vartheta_i}{n} \sum_{j \in \mathcal{N}_i} \left(\frac{1}{\mu_j} r_{j,u} + \varsigma_j - \varepsilon_{j,u}^T \varepsilon_{j,u} \right),
 \end{aligned}$$

which implies that

$$\begin{aligned}
 \mathbb{E}\{V_{i,u+1}\} &- S_{i,u} - (1 - \alpha_i) V_{i,u} - \sum_{j \in \mathcal{N}_i} \frac{\alpha_j}{1 + q_j} V_{j,u} \\
 &\leq \chi_{i,u}^T \Pi_{i1,u}^T P_{i,u+1} \Pi_{i1,u} \chi_{i,u} + \sum_{d=1}^{p_i} \chi_{i,u}^T \Pi_{ij_d,u}^T P_{i,u+1} \Pi_{ij_d,u} \chi_{i,u} + \frac{1}{n\mu_i} (\lambda_i r_{i,u} - \varepsilon_{i,u}^T \varepsilon_{i,u}) + \frac{\vartheta_i}{n} \sum_{j \in \mathcal{N}_i} \left(\frac{1}{\mu_j} r_{j,u} - \varepsilon_{j,u}^T \varepsilon_{j,u} \right) \\
 &\quad - \gamma^2 \|\xi_{i,u}\|^2 + \|e_{i,u}\|_{T_{i,u}}^2 + \sum_{j \in \mathcal{N}_i} \|e_{j,u} - e_{i,u}\|_{R_{i,u}}^2 - (1 - \alpha_i) V_{i,u} - \sum_{j \in \mathcal{N}_i} \frac{\alpha_j}{1 + q_j} V_{j,u} \\
 &\triangleq \chi_{i,u}^T \Pi_{i0,u} \chi_{i,u} + \chi_{i,u}^T \Pi_{i1,u}^T P_{i,u+1} \Pi_{i1,u} \chi_{i,u} + \sum_{d=1}^{p_i} \chi_{i,u}^T \Pi_{ij_d,u}^T P_{i,u+1} \Pi_{ij_d,u} \chi_{i,u}.
 \end{aligned} \tag{29}$$

Then, in light of Equation 29, one immediately has

$$\begin{aligned}
 \mathbb{E}\{V_{i,u+1}\} &\leq S_{i,u} + \sum_{j \in \mathcal{N}_i} \frac{\alpha_j}{1 + q_j} V_{j,u} + (1 - \alpha_i) V_{i,u} \\
 &\triangleq S_{i,u} + [W \mathbf{V}_u]_i
 \end{aligned} \tag{30}$$

where

$$W \triangleq \begin{bmatrix} 1 - \alpha_1 & \cdots & a_{1N} \frac{\alpha_N}{1 + q_N} \\ \vdots & \ddots & \vdots \\ a_{N1} \frac{\alpha_1}{1 + q_1} & \cdots & 1 - \alpha_N \end{bmatrix}.$$

Moreover, since $\alpha_i \in \mathcal{I}_{q_i}$ for $i \in \mathcal{V}$, one immediately has w is the desired dissipation matrix.

By means of the double expectation theorem, one obtains that

$$\mathbb{E}\{V_{i,u+1}\} < \mathbb{E}\{[W \mathbf{V}_u]_i\} + \mathbb{E}\{S_{i,u}\}, \tag{31}$$

which can be rewritten in a compact form:

$$\mathbb{E}\{\mathbf{V}_{u+1}\} \ll \mathbb{E}\{W \mathbf{V}_u\} + \mathbb{E}\{\mathbf{S}_u\}. \tag{32}$$

Next, a scalar form is derived by making the linear transformation to Equation 32:

$$\mathbf{1}_{1 \times N} \mathbb{E}\{\mathbf{V}_{u+1}\} < \mathbf{1}_{1 \times N} \mathbb{E}\{W \mathbf{V}_s\} + \mathbf{1}_{1 \times N} \mathbb{E}\{\mathbf{S}_u\}, \quad (33)$$

which is further calculated in light of the property of dissipation matrix w as follows:

$$\sum_{i=1}^N \mathbb{E}\{V_{i,u+1}\} < \sum_{i=1}^N \mathbb{E}\{S_{i,u}\} + \sum_{i=1}^N \sum_{j=1}^N W_{ij} V_{i,u} = \sum_{i=1}^N \mathbb{E}\{S_{i,u}\} + \sum_{i=1}^N V_{i,u}.$$

Furthermore, the above inequality can be rewritten as

$$\sum_{i=1}^N \mathbb{E}\{V_{i,u+1} - V_{i,u}\} \leq \sum_{i=1}^N \mathbb{E}\{S_{i,u}\}. \quad (34)$$

Next, Equation 34 is summed regarding u from 0 to $n-1$ and one has

$$\mathbb{E}\{V_{i,n+1} - V_{i,0}\} \leq \sum_{u=0}^{n-1} \sum_{i=1}^N \mathbb{E}\{S_{i,u}\}, \quad (35)$$

which indicates that

$$\mathbb{E}\{V_{i,n+1}\} \leq \sum_{u=0}^{n-1} \sum_{i=1}^N \mathbb{E}\{S_{i,u}\} + \mathbb{E}\{V_{i,0}\}. \quad (36)$$

Noticing that $\mathbb{E}\{V_{i,n+1}\} \geq 0$, one derives from Equation 36 and $S_{i,u}$ that

$$\sum_{i=1}^N \sum_{u=0}^{n-1} \mathbb{E} \left\{ \|e_{i,u}\|_{T_{i,u}}^2 + \sum_{j \in \mathcal{N}_i} \|e_{j,u} - e_{i,u}\|_{R_{i,u}}^2 \right\} \leq \sum_{i=1}^N \left(\frac{S_i}{\mu_i} + \vartheta_i \sum_{j \in \mathcal{N}_i} s_j \right) + \sum_{i=1}^N \left(e_{i,0}^T P_{i,0} e_{i,0} + \frac{r_{i,0}}{n\mu_i} \right) + \gamma^2 \sum_{i=1}^N \sum_{u=0}^{n-1} \|\xi_{i,u}\|^2. \quad (37)$$

and, therefore, one has Equation 19 according to $P_{i,0} \leq \gamma^2 U_i$, $\frac{S_i}{\mu_i} + \vartheta_i \sum_{j \in \mathcal{N}_i} s_j \leq \gamma^2 \varrho_i$, and $r_{i,0} \leq n\gamma^2 \bar{r}_i$.

Furthermore, denoting $\iota_i \triangleq \varrho_i + \bar{r}_i$, the proof is complete according to Definition 1.

In the following, the desired estimator gains are calculated in terms of recursive linear matrix inequalities by resorting to the Schur Complement Lemma.

Theorem 2 Consider any binary sensor $i \in \mathcal{T}$ with its neighboring nodes $j (j \in \mathcal{N}_i)$. Suppose that the parameters $\alpha_i \in \mathcal{J}_{q_i}$, $\alpha_j \in \mathcal{J}_{q_j}$, $\gamma > 0$, $s_i > 0$, $s_j > 0$, $\varrho_i > 0$, $\vartheta_i > 0$, and $\bar{r}_i > 0$ satisfying the following conditions:

$$\frac{S_i}{\mu_i} + \vartheta_i \sum_{j \in \mathcal{N}_i} s_j \leq \gamma^2 \varrho_i, \lambda_i + \alpha_i < 1, (1 + q_j) \vartheta_i < \alpha_j, r_{i,0} \leq n\gamma^2 \bar{r}_i.$$

Let U_i and $P_{i,0}$ be given positive definite matrices satisfying $0 < P_{i,0} \leq \gamma^2 U_i$. Then, the system dynamics Equation 18 satisfies the H_∞ -consensus constraint in the mean square sense if there exists a sequence of positive definition matrices $P_{i,u+1} > 0$ such that the following $\theta_{i,u}$ -dependent inequality holds

$$\begin{bmatrix} \Upsilon_{i0,u} & * & * & * & * & * \\ \Upsilon_{i1,u} & -P_{i,u+1} & * & * & * & * \\ \Upsilon_{ij1,u} & 0 & -P_{i,u+1} & * & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \Upsilon_{ij_{p_i},u} & 0 & 0 & \cdots & -P_{i,u+1} & * \\ 0 & \bar{M}_{i1,u}^T & 0 & \cdots & 0 & -\mu_{i1,u}I \end{bmatrix} < 0 \quad (38)$$

where

$$\begin{aligned} \Upsilon_{i0,u} &\triangleq \begin{bmatrix} \Upsilon_{i0,u}^{11} & * & * & * & * & * & * \\ \Upsilon_{i0,u}^{21} & \Upsilon_{i0,u}^{22} & * & * & * & * & * \\ 0 & 0 & \Upsilon_{i0,u}^{33} & * & * & * & * \\ 0 & 0 & 0 & \Upsilon_{i0,u}^{44} & * & * & * \\ 0 & 0 & 0 & 0 & \Upsilon_{i0,u}^{55} & * & * \\ 0 & 0 & 0 & 0 & 0 & \Upsilon_{i0,u}^{66} & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \Upsilon_{i0,u}^{77} \end{bmatrix}, \\ \Upsilon_{i0,u}^{11} &\triangleq -(1 - \alpha_i)P_{i,u} + T_{i,u} + (p_i + 1)R_{i,u} + \mu_{i1,u}C_{i,u}^T C_{i,u}, \quad \Upsilon_{i0,u}^{21} \triangleq -\text{col}_{p_i}\{R_{i,u}\}, \\ \Upsilon_{i0,u}^{22} &\triangleq \text{diag}_{p_i}\left\{-\frac{\alpha_{j_{i,d}}}{1 + q_{j_{i,d}}}P_{j_{i,d},u} + R_{i,u}\right\}, \quad \Upsilon_{i0,u}^{33} \triangleq -\gamma^2 I + \mu_{i1,u}D_{i,u}^T D_{i,u}, \quad \Upsilon_{i0,u}^{44} \triangleq -\frac{1}{n\mu_i}I, \\ \Upsilon_{i0,u}^{55} &\triangleq \text{diag}\left\{-\frac{\theta_j}{n\mu_j}I\right\}, \quad \Upsilon_{i0,u}^{66} \triangleq \frac{\lambda_i + \alpha_i - 1}{n\mu_i}, \quad \Upsilon_{i0,u}^{77} \triangleq \text{diag}_{p_i}\left\{\frac{(1 + q_j)\theta_i - \alpha_j}{(1 + q_j)n\mu_j}\right\}, \\ \Upsilon_{i1,u} &\triangleq \begin{bmatrix} \Upsilon_{i1,u}^{(11)} & \Upsilon_{i1,u}^{(22)} & \Upsilon_{i1,u}^{(33)} & \Upsilon_{i1,u}^{(44)} & 0 & 0 & 0 \end{bmatrix}, \\ \Upsilon_{i1,u}^{(11)} &\triangleq \bar{A}_{i,u} - \sum_{j \in \mathcal{N}_i} \bar{\sigma}_{j,u} \zeta_{j,u} \bar{G}_{ij,u}, \quad \Upsilon_{i1,u}^{(22)} \triangleq \bar{\sigma}_{\mathcal{N}_i,u} \circ \zeta_{\mathcal{N}_i,u} \circ \bar{G}_{\mathcal{N}_i,u}, \quad \Upsilon_{i1,u}^{(33)} \triangleq \bar{B}_{i,u}, \\ \Upsilon_{i1,u}^{(44)} &\triangleq \bar{\sigma}_{\mathcal{N}_i,u} \circ \zeta_{\mathcal{N}_i,u} \circ \bar{G}_{\mathcal{N}_i,u}, \quad \bar{A}_{i,u} \triangleq P_{i,u+1}A_u - 0.5\theta_{i,u}\bar{K}_{i,u}C_{i,u}, \quad \bar{B}_{i,u} \triangleq \begin{bmatrix} P_{i,u+1}B_u & -0.5\theta_{i,u}\bar{K}_{i,u}D_{i,u} \end{bmatrix}, \\ \Upsilon_{ij_d,u} &\triangleq \begin{bmatrix} \Pi_{ij_d,u}^{(11)} & \Upsilon_{ij_d,u}^{(22)} & 0 & -\Pi_{ij_d,u}^{(44)} & 0 & 0 & 0 \end{bmatrix}, \\ \Upsilon_{ij_d,u}^{(11)} &\triangleq \gamma_{j_{i,d},u} \zeta_{j_{i,d},u} \bar{G}_{ij_d,u}, \quad \bar{M}_{i1,u} \triangleq \text{vec}\{-0.5\theta_{i,u}\bar{K}_{i,u}, 0, \bar{E}_{i,u}, 0, 0, 0, 0\}, \quad \bar{E}_{i,u} = \begin{bmatrix} 0 & -0.5\theta_{i,u}\bar{K}_{i,u}D_{i,u} \end{bmatrix}, \\ \Upsilon_{ij_d,u}^{(22)} &\triangleq \begin{bmatrix} \underbrace{0 \cdots 0}_{d-1} & \gamma_{j_{i,d},u} \zeta_{j_{i,d},u} \bar{G}_{ij_d,u} & \underbrace{0 \cdots 0}_{p_i-d} \end{bmatrix}, \quad \Upsilon_{ij_d,u}^{(44)} \triangleq \begin{bmatrix} \underbrace{0 \cdots 0}_{d-1} & \gamma_{j_{i,d},u} \zeta_{j_{i,d},u} \bar{G}_{ij_d,u} & \underbrace{0 \cdots 0}_{p_i-d} \end{bmatrix}. \end{aligned}$$

Furthermore, if Equation 38 is feasible, then one calculates

$$K_{i,u} = P_{i,u+1}^{-1} \bar{K}_{i,u}, \quad G_{ij,u} = P_{i,u+1}^{-1} \bar{G}_{ij,u}.$$

Proof 2 By resorting to the Schur Complement Lemma, it follows that Equation 25 holds if and only if

$$\Pi_{i,u} \triangleq \begin{bmatrix} \Pi_{i0,u} & * & * & * & * \\ \Pi_{i1,u} & -P_{i,u+1}^{-1} & * & * & * \\ \Pi_{ij_1,u} & 0 & -P_{i,u+1}^{-1} & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Pi_{ij_{p_i},u} & 0 & 0 & \dots & -P_{i,u+1}^{-1} \end{bmatrix} < 0. \quad (39)$$

Next, the left-hand side of Equation 39 is partitioned as

$$\Pi_{i,u} = \bar{\Pi}_{i,u} + \tilde{\Pi}_{i,u} = \begin{bmatrix} \Pi_{i0,u} & * & * & * & * \\ \bar{\Pi}_{i1,u} & -P_{i,u+1}^{-1} & * & * & * \\ \Pi_{ij_1,u} & 0 & -P_{i,u+1}^{-1} & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Pi_{ij_{p_i},u} & 0 & 0 & \dots & -P_{i,u+1}^{-1} \end{bmatrix} + \begin{bmatrix} 0 & * & * & * & * \\ \tilde{\Pi}_{i1,u} & 0 & * & * & * \\ 0 & 0 & 0 & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (40)$$

where

$$\begin{aligned} \bar{\Pi}_{i1,u} &\triangleq \begin{bmatrix} \bar{\Pi}_{i1,u}^{(11)} & \Pi_{i1,u}^{(22)} & \bar{\Pi}_{i1,u}^{(33)} & \Pi_{i1,u}^{(44)} & 0 & 0 & 0 \end{bmatrix}, \\ \bar{\Pi}_{i1,u}^{(11)} &\triangleq A_{i,u} - \sum_{j \in \mathcal{A}_i} \bar{\sigma}_{j,u} \zeta_{j,u} G_{ij,u}, \quad \bar{\Pi}_{i1,u}^{(33)} \triangleq B_{i,u}, \\ \tilde{\Pi}_{i1,u} &\triangleq \begin{bmatrix} \tilde{\Pi}_{i1,u}^{(11)} & 0 & \tilde{\Pi}_{i1,u}^{(33)} & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \tilde{\Pi}_{i1,u}^{(11)} &\triangleq -0.5\delta_{i,u}\theta_{i,u}K_{i,u}C_{i,u}, \quad \tilde{\Pi}_{i1,u}^{(33)} \triangleq \delta_{i,u}E_{i,u}. \end{aligned}$$

Next, one acquires the following compact form

$$\tilde{\Pi}_{i,u} = \tilde{M}_{i,u} F_{i,u} \tilde{N}_{i,u} + \tilde{N}_{i,u}^T F_{i,u} \tilde{M}_{i,u}^T \quad (41)$$

where

$$\begin{aligned} \tilde{M}_{i,u} &\triangleq \begin{bmatrix} 0 & \tilde{M}_{i1,u}^T & 0 & \dots & 0 \end{bmatrix}^T, \quad \tilde{M}_{i1,u} \triangleq \text{vec}\{-0.5\theta_{i,u}K_{i,u}, 0, E_{i,u}, 0, 0, 0, 0\}, \\ \tilde{N}_{i,u} &\triangleq \begin{bmatrix} \tilde{N}_{i1,u} & 0 & 0 & \dots & 0 \end{bmatrix}, \quad \tilde{N}_{i1,u} \triangleq \text{diag}\{C_{i,u}, 0, I, 0, 0, 0, 0\}, \\ F_{i,u} &\triangleq \text{diag}\{F_{i1,u}, 0, 0, \dots, 0\}, \quad F_{i1,u} \triangleq \text{diag}\{\delta_{i,u}, 0, \delta_{i,u}, 0, 0, 0, 0\}. \end{aligned}$$

By means of S-Procedure, $\Pi_{i,u} < 0$ holds with $F_{i,u} F_{i,u}^T \leq I$ if and only if there exists a constant $\mu_{i1,u} > 0$ such that

$$\bar{\Pi}_{i,u} + \mu_{i1,u}^{-1} \tilde{M}_{i,u} \tilde{M}_{i,u}^T + \mu_{i1,u} \tilde{N}_{i,u}^T \tilde{N}_{i,u} < 0. \quad (42)$$

By reusing Schur Complement Lemma, Equation 42 holds if and only if

$$\begin{bmatrix} \Upsilon_{i0,u} & * & * & * & * & * \\ \bar{\Pi}_{i1,u} & -P_{i,u+1}^{-1} & * & * & * & * \\ \Pi_{ij1,u} & 0 & -P_{i,u+1}^{-1} & * & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \Pi_{ij_{p_i},u} & 0 & 0 & \cdots & -P_{i,u+1}^{-1} & * \\ 0 & \tilde{M}_{i1,u}^T & 0 & \cdots & 0 & -\mu_{i1,u}I \end{bmatrix} < 0 \quad (43)$$

where

$$\Upsilon_{i0,u} \triangleq \Pi_{i0,u} + \mu_{i1,u} \tilde{N}_{i1,u}^T \tilde{N}_{i1,u}.$$

After performing the congruent transformation $\text{diag}\{I, P_{i,u+1}, \underbrace{P_{i,u+1}, \dots, P_{i,u+1}}_{p_i}, I\}$ to Equation 43 and introducing some new notations, one easily has Equation 38 following some simple algebraic operations. From Equation 38, one immediately calculates the estimator gains $K_{i,u}$ and $G_{ij,u}$. As a result, the proof is complete with the aid of Theorem 1.

Remark 3 It should be noted that Theorems 1 and 2 provide the parameter constraints that are helpful in achieving the desirable performance constraint and reproducing the results and understanding the engineering implications of the model.

Remark 4 The scalable distributed H_∞ -consensus state estimation problem is investigated for a class of discrete time-varying system over BSNs with energy harvesters under the schedule of the dynamic ETP. Compared with the existing results, this paper exhibits the following characteristics: (1) the combined impact of the dynamic ETP and the EH techniques is studied within the framework of the scalable distributed estimation; (2) useful information is extracted from the BM subject to integral measurement; (3) a binomial distribution is used to formulate the limited energy collected from the surrounding environment, which is more in line with engineering practice; and (4) the scalable design is achieved by means of the LPA method, which implies that the distributed estimation scheme is implemented on every node.

4. AN ILLUSTRATIVE EXAMPLE

In this section, a demonstrative example is provided to show the efficiency and viability of the proposed distributed H_∞ consensus estimation approach for BSNs operating in EH and dynamic ETPs. For evaluating the performance of the approach, four cases are considered: dynamic ETP with energy constraints; dynamic ETP without energy constraints; static ETP without energy constraints; and energy constraints only (without ETP).

The topology structure of BSN is depicted by the digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where the binary sensor nodes are denoted by the set $\mathcal{V} = \{1, 2, 3, 4, 5, 6\}$, the edge set is $\mathcal{E} = \{(1, 5), (2, 1), (3, 1), (4, 2), (5, 3), (6, 4)\}$, and the adjacency matrix is as follows

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

The system parameters in Equation 1 and Equation 2 is chosen as follows:

$$\begin{aligned} \bar{A}_u &= \begin{bmatrix} 0.87 + 0.3 \sin(0.9u) & -0.4 \\ 0.15 & 0.98 + 0.1 \sin(6u) \end{bmatrix}, \quad \bar{B}_u = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \\ \bar{C}_{1,u} &= \begin{bmatrix} 0.5 & -0.5 \end{bmatrix}, \quad D_{i,u} = 0.15, \quad \bar{C}_{2,u} = \begin{bmatrix} 0.4 & -0.7 \end{bmatrix}, \\ \bar{C}_{3,u} &= \begin{bmatrix} 0.5 & -0.4 \end{bmatrix}, \quad \bar{C}_{4,u} = \begin{bmatrix} 0.4 & -0.6 \end{bmatrix}, \\ \bar{C}_{5,u} &= \begin{bmatrix} 0.4 & -0.3 \end{bmatrix}, \quad \bar{C}_{6,u} = \begin{bmatrix} 0.5 & -0.4 \end{bmatrix}. \end{aligned}$$

The sensor parameters are set as $\tau_1 = 0.1$, $\tau_2 = 0.1$, $\tau_3 = 0.2$, $\tau_4 = -0.7$, $\tau_5 = 0.2$, and $\tau_6 = 0.4$, respectively; the critical parameters in the dissipation matrix are set as $\alpha_i = 0.3$; the noise attenuation level is chosen as $\gamma^2 = 0.1$; the time interval to collect the data is $s = 1$; the dynamic ETP parameters are set as $\mu_i = 50$, $\varsigma_i = 0.4$, $\lambda_i = 0.5$, $r_{i,0} = 5$. The other parameters are given as $R_{i,u} = I$, $T_{i,u} = I$, $U_{i,u} = I$, $P_{i,0} = 10I$, and $n = 31$. All estimator gains are derived by resorting to Theorem 2 using the YALMIP toolbox.

The initial values are set as $x_0 = [2.1 \quad 2.1]^T$, $\hat{x}_{1,0} = [2.2 \quad 2.2]^T$, $\hat{x}_{2,0} = [1.9 \quad 2.1]^T$, $\hat{x}_{3,0} = [1.8 \quad 2.1]^T$, $\hat{x}_{4,0} = [1.7 \quad 2.7]^T$, $\hat{x}_{5,0} = [1.8 \quad 2.1]^T$, and $\hat{x}_{6,0} = [2.0 \quad 2.0]^T$, respectively. The external disturbance sequences w_u and $v_{i,u}$ are set as

$$w_u = \exp(-u), \quad v_{i,u} = \frac{\sin(10u + 1)}{3u + 1}, \quad i = 1, 2, 3, 4, 5, 6.$$

The energy harvester parameters for every node are set as $p_i = 0.5$, $b = 4$, $s_i = 4$, $\kappa_{i,0} = 4$, $c_i = 3$ and $\phi_{i,0} = [0.0625 \quad 0.25 \quad 0.375 \quad 0.25 \quad 0.0625]$, respectively; one calculates the probability distribution of the random variable $\kappa_{i,u}$ is calculated using Lemma 1, and one derives

$$M_{i,u} = \begin{bmatrix} 0.0625 & 0.0625 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.0625 & 0 & 0 \\ 0.375 & 0.375 & 0.25 & 0.0625 & 0 \\ 0.25 & 0.25 & 0.375 & 0.25 & 0.0625 \\ -0.9375 & -0.9375 & -0.6875 & -0.3125 & -0.0625 \end{bmatrix}$$

By means of Lemma 1, one then has $\gamma_{i,u}^2 = 3.955540933510626e - 05$. Finally, we derive the desirable estimator gains by resorting to Theorem 2 and the MATLAB (Matrix Laboratory) software.

Figures 1-8 plot all simulation results under Case I. More specifically, Figure 1 and Figure 2 depict the true state and its estimations from each binary sensor under the dynamic ETP and EH; Figure 3 plots the time instants of $\theta_{i,u} = 1$; Figure 4 plots the triggering time instants of dynamic ETP; Figure 5 describe the curves of $z_{1,u} - \bar{z}_{1,u}$ and $\tau_1 - \bar{z}_{1,u}$; Figure 6 depicts the changes of energy level on Node 1 under the conditions of EH and communication consumption; and Figure 7 and Figure 8 depict $\|x_u - \hat{x}_{i,u}\|^2$ (i.e., the estimation errors) and $\sum_{j \in \mathcal{N}_i} \|\hat{x}_{i,u} - \hat{x}_{j,u}\|^2$ (i.e., the consensus errors).

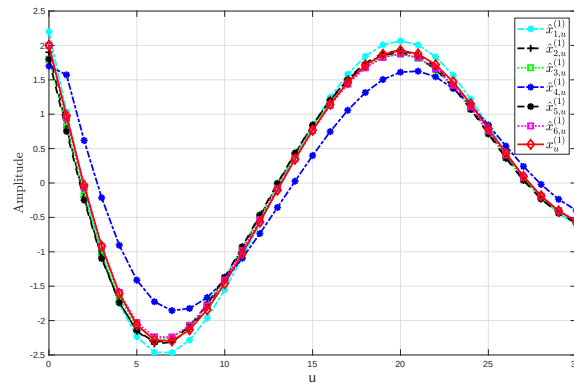


Figure 1. $x_u^{(1)}$ and its estimates under case I.

It follows from Figure 6 that the dynamic change in the amount of energy inside the sensor depends on both the communication consumption and the generation of EH. In light of Figure 3, at certain time instants, the useful information from every binary sensor is extracted for estimation purposes in light of Equation 5 and Equation 6. According to Figure 4, due to the introduction of dynamic ETP, the transmission frequency of the six nodes in the sensor network decreases significantly, and the triggering frequency of the transmission of information also decreases with time, further saving network resources. According to Figure 5, it is relatively small of the differences between $z_{1,u} - \bar{z}_{1,u}$ and $\tau_1 - \bar{z}_{1,u}$ at the time instants of information extraction. Therefore, selecting an appropriate threshold is helpful to increase the number of information extractions to improve the accuracy of the estimate.

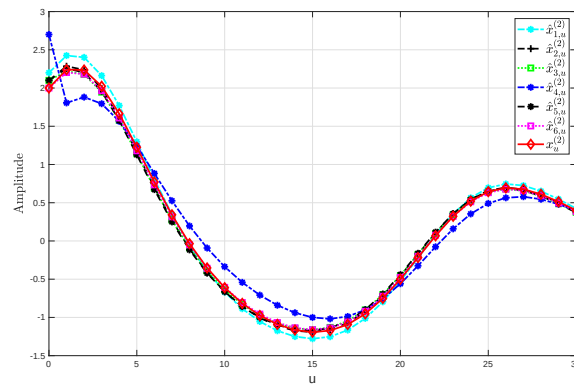


Figure 2. $x_u^{(2)}$ and its estimates under case I.

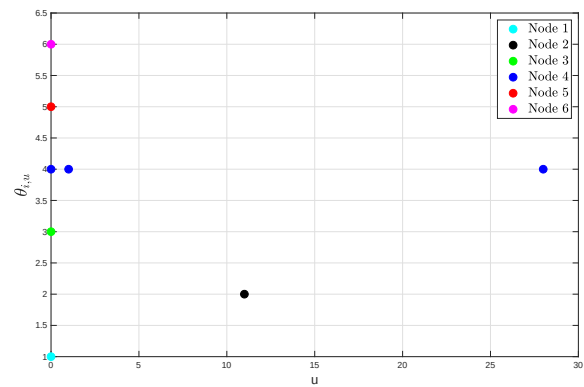


Figure 3. The time instants of $\theta_{i,u} = 1$ under case I.

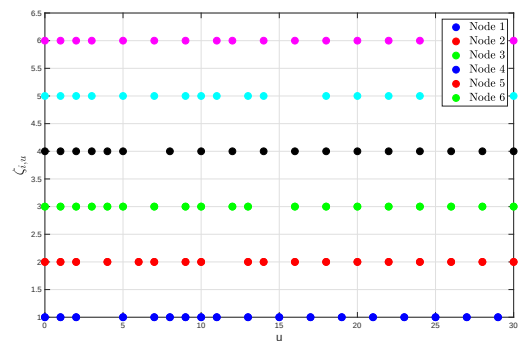


Figure 4. The information transmission instants under case I.

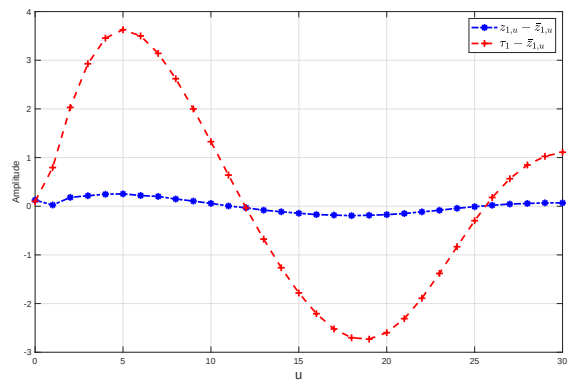


Figure 5. $z_{1,u} - \bar{z}_{1,u}$ and $\tau_1 - \bar{\tau}_{1,u}$ under case I.

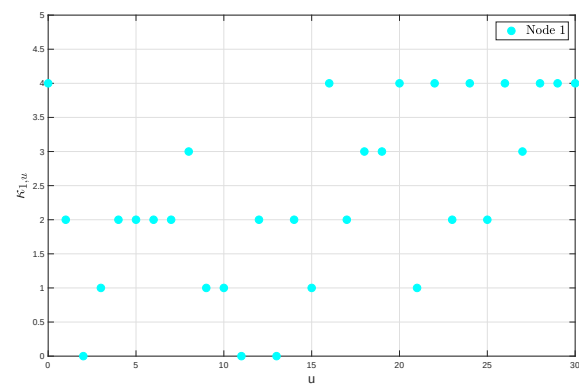


Figure 6. Energy level of node 1 under case I.

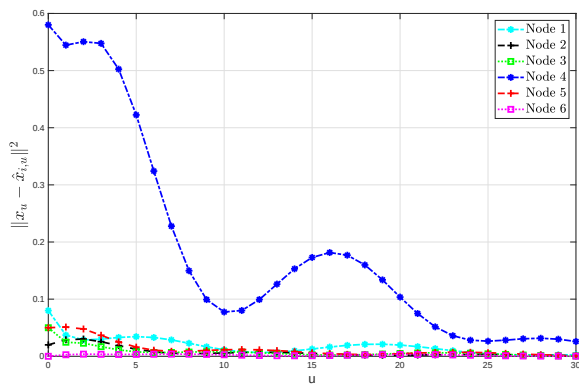


Figure 7. $\|x_u - \hat{x}_{i,u}\|^2$ under case I.

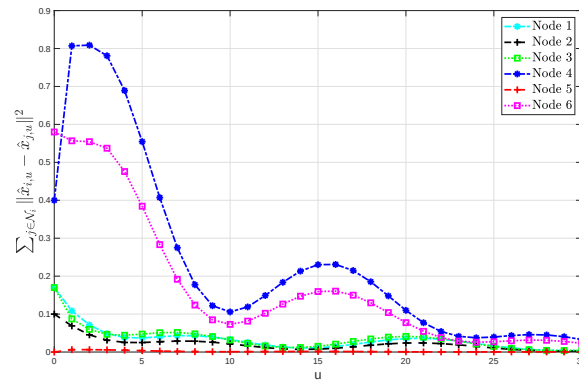


Figure 8. $\sum_{j \in \mathcal{N}_i} \|\hat{x}_{i,u} - \hat{x}_{j,u}\|^2$ under case I.

From Figure 1 and Figure 2, it follows that despite the limitations imposed by the energy harvester and dynamic ETP, the estimation of each binary sensor maintains a commendable level of accuracy in discerning the actual state. As depicted in Figure 7 and Figure 8, both the estimation errors and the consensus errors of every binary sensor meet the aspired performance criteria.

Next, consider Case II: the dynamic ETP without energy constraints. To this end, we set $c_i \equiv 0$, i.e., the energy consumption associated with information transmission is neglected. The corresponding simulation results are presented in Figure 9 and Figure 10, where Figure 10 confirms the absence of energy loss. Comparing these figures with those of Case I, it is observed that the number of information transmissions increases.

Next, consider Case III: the static ETP without energy constraints, i.e., $r_{i,u} \equiv 0$ and $c_i \equiv 0$. The corresponding simulation results are shown in Figure 11 and Figure 12, where Figure 12 again verifies zero energy loss. In comparison with Figure 4 and Figure 9, the number of information transmissions is found to increase further.

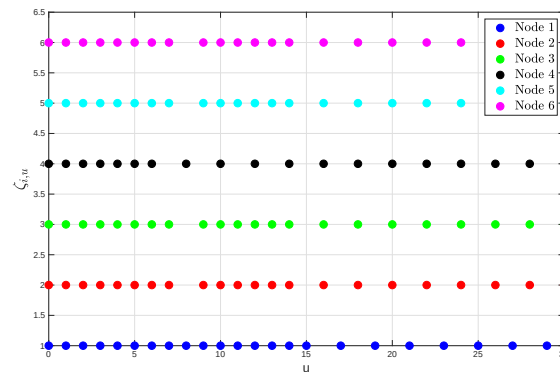


Figure 9. Information transmission time instants under case II.

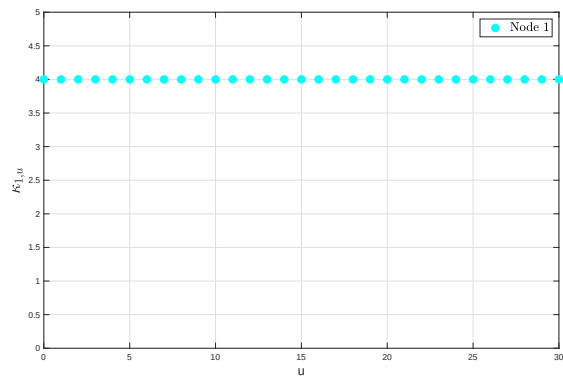


Figure 10. Energy level of node 1 under case II.

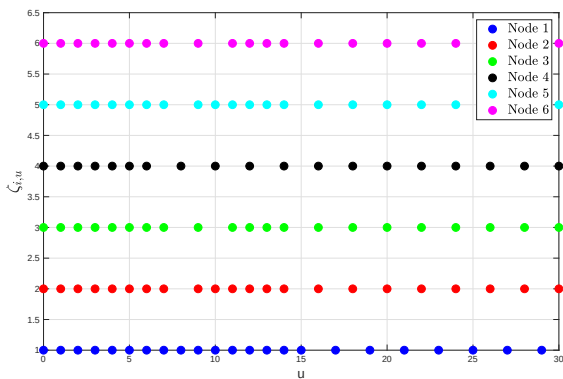


Figure 11. Information transmission time instants of node 1 under case III.

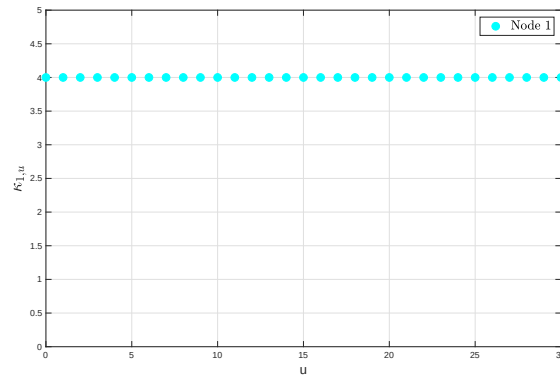


Figure 12. Energy level of node 1 under case III.

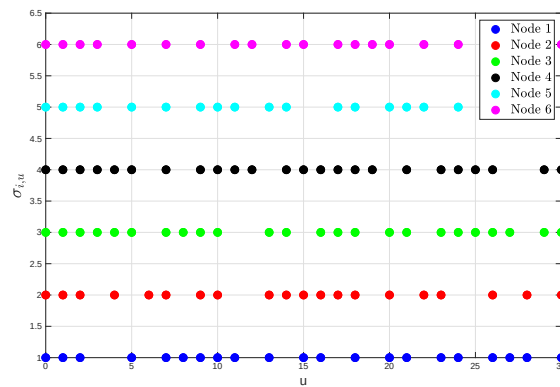


Figure 13. Information transmission time instants of node 1 under case IV.

Finally, consider Case IV: the scenario with only an energy constraint but no ETP, i.e., $r_{i,u} \equiv 0$ and $\varsigma_i \equiv 0$. The corresponding simulation results are displayed in Figure 13 and Figure 14. Compared with the results of the other three cases, there exist significant differences.

In conclusion, the simulations collectively demonstrate the desirable performance of the proposed distributed H_∞ consensus estimation scheme for binary sensors equipped with EH functions and a dynamic ETP.

5. CONCLUSIONS

This paper has proposed a distributed state estimation scheme using integral measurements in BSNs, considering the EH technique and dynamic ETP. A new BM model has been established for each node by choosing the integral measurements as inputs of the BMs. The random variable obeying the binomial distribution has been used to describe energy levels and an indicator variable has been introduced to formulate the effect of dynamic

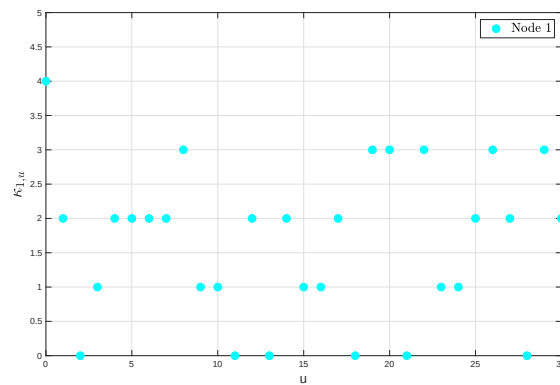


Figure 14. Energy level of node 1 under case IV.

ETP. The dynamic ETP and EH methods have been used cooperatively for each node to decide whether information should be transmitted from its neighboring nodes. Based on the available information, distributed consensus estimators have been constructed for each sensor. Through the LPA method, sufficient conditions have been established for each node to ensure that the estimation error of all nodes meets the desired H_∞ -consensus performance criteria. The estimator gains have been determined by solving the recursive LMIs on every node, which implies the scalability. The simulation results have validated the efficacy of the distributed estimation approach developed in this paper. In the future, the potential research directions would be to extend our main results to a 2-D system [6], study the distributed secure filtering [49], or adopt binary encoding schemes [50] to achieve information transmission in an efficient manner.

DECLARATIONS

Authors' contributions

Conceptualization and manuscript drafting: Han, F.; Ma, L.

Methodology and experiment: Song, Y.

Manuscript edition: Zhang, J.

Review and supervision: Shao, S.

Availability of data and materials

The authors generated all data and materials used in the research as an integral part of the study, with explicit details provided in the methodology section of the manuscript. These are available from the corresponding author upon reasonable request.

AI and AI-assisted tools statement

Not applicable.

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Conflicts of interest

All authors declared that there are no conflicts of interest.

Ethical approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

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