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Event-triggered consensus control method with communication faults for multi-UAV

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Abstract

This paper investigates the event-triggered consensus for a group of unmanned aerial vehicles (UAVs) with communication faults under the assumption that the position sensors of some individuals are damaged. The objective is to make the UAV group reach consensus in urgent tasks such as obstacle avoidance or evasion. Using the Lyapunov stability theory, sufficient conditions to achieve system consensus are given based on different velocity and position interaction topologies. Considering the limited capabilities of sensors and processors, an event-triggered consensus protocol is adopted to reduce the sampling frequency. Finally, simulation results illustrate the effectiveness of our approach.

Keywords: Unmanned aerial vehicle, communication faults, consensus, event-triggered control, interaction topology

1. INTRODUCTION

The multi-agent systems (MAS) abstracted from complex systems, such as unmanned aerial vehicle (UAV) groups, are regarded as significant research objects for studying group intelligent behavior in recent years[1]. With excellent performance in simplifying the analysis process, it is applied to various fields of UAVs, such as formation control[2], collaborative investigations[3], and many other fields. Through the consensus control of MAS, intelligent emergence phenomena can be achieved by self-organization and internal interactions [4,5]. As a focus of international research in related fields, the realization of consensus has profound significance in practical applications [6–9].

The extensive and complex application scenarios of unmanned systems make fault-tolerant control crucial. One of the important factors contributing to the failure of unmanned systems is the damage to some individual sensor components. Huang *et al.* addressed the problem of IMU sensor failure by training and designing a controller based on long short-term memory (LSTM) neural networks and datasets and proposed an AI-based fault-tolerant control method. Furthermore, simulation verification further tested the recovery ability and effectiveness of the design method in the above scenarios [10]. Similarly, GPS fault detection and exclusion were solved by Chang and Tsai through an approach based on the moving average (MA) [11]. Compared with traditional least-squares residual methods, their approach exhibits higher performance in detecting small faults and similar performance levels in detecting large faults. This method has a lower incorrect exclusion rate (IER) than traditional parity space methods and has been verified through simulation. In addition, the complex communication environment of unmanned systems also poses a great challenge for consensus research, which involves time-delay networks, random networks, asynchronous networks, *etc.* [12] proved the condition for consensus in time-delay networks by introducing disagreement functions abstracted from the Lyapunov function for the disagreement network dynamics. Based on this, Xiao *et al.* extended the result to variable topology [13]. The concept of consensus in random networks was proposed by Hatano, referring to the system converging to consensus with a probability close to 1 [14]. Asynchronous networks have been extensively studied in order to be closer to the actual situation. It is difficult to update the system state synchronously due to the complex communication environment. The proof of the consistency of a single integral system in this situation is given by Cao *et al.* [15]. Recently, Yan *et al.* presented a distributed control protocol and a distributed adaptive controller based on fault compensation to achieve consensus against link failures and actual/sensor faults [16]. Moreover, Chen *et al.* developed an adaptive compensation protocol and an H_∞ control protocol for the scenario of simultaneous sensor or actuator faults [17]. Based on the radial basis function neural network (RBFNN), A data-driven distributed formation control algorithm is proposed for MAS with sensor faults by Xiong and Hou [18].

Heterogeneous systems have also been a hot research topic in this field in recent years. Lee *et al.* first studied inertial systems and analyzed the impact of individual inertia indices on system consensus [19]. Using the decomposition approach, Li and Spong investigated the stability of multiple inertial systems with non-balanced velocity/position coupling [20]. By applying the graph theory and the Lyapunov direct method, the consensus problem of heterogeneous systems composed of first-order and second-order individuals was solved by Zheng [21,22]. studied the consensus problem of a heterogeneous MAS consisting of quadrotors and two-wheeled mobile robots and proposed two linear quadratic regulations (LQR)-based consensus protocols to control the heterogeneous system, which showed good performance in practical systems. Based on the state observers, Ma *et al.* solved the output consensus problem of heterogeneous MAS, which is applicable when system states are not available [23]. By designing distributed fixed-time observers and fixed-time tracking controllers, Du *et al.* investigated the fixed-time consensus problem for nonlinear heterogeneous systems [24]. Li *et al.* further explored their research field to group consensus with input constraints [25].

Considering the limited capabilities of sensors and processors compared to traditional communication devices that rely on data interaction, event-triggered protocols are necessary for systems that rely on data interaction, as they can significantly reduce the sampling frequency. Drof *et al.* first introduced the concept of event-triggered and dynamically changed the system sampling frequency by measuring the state variables, which inspired ways to reduce the system load [26]. The event-triggered threshold was correlated with the system state by Fan *et al.* Their research results show that this approach has superior dynamics compared to constant thresholds [27]. These efforts have also been gradually extended to complex systems, including heterogeneous systems[28] and time-delayed systems[29,30]. Designed and implemented an event-triggered formation control for second-order MAS under communication faults based on linear matrix inequalities conditions on a real platform of UAVs[31]. Investigates the secure consensus control of multirobot systems with an event-triggered communication strategy under aperiodic energy-limited denial-of-service (DoS) attacks. Each robot exchanges the local positioning information with other robots through the unreliable communication network and determines its consensus control based on transmitted position estimates. The paper proposes a secure control scheme such that the robots can move to the desired secure consensus position in the presence of attacks. Simulation and experimental results demonstrate the effectiveness of the event-triggered consensus in practical applications.

In this paper, we investigate the consensus problem for a group of multi-UAVs with communication faults under the assumption that the position sensor of some individuals is damaged. An event-triggered consensus protocol is designed for the UAV group based on a centralized triggering mechanism such that the UAV group can eventually converge to the same speed and position by sensor measurements, even if a sudden change in speed occurs in one individual.

The main contributions of this paper are as follows. First, we consider the scenario that the states of UAVs are sensed by their neighbors with communication faults and the position sensor of some individuals is damaged, which means that their interaction topologies of speed and position are not necessarily the same and the same topology can be considered as a special case in this paper. Furthermore, we consider the impact of the inertia index on system consensus and provide quantitative analysis results, similar to the research result in[19], but we do not limit the graph to be balanced. Moreover, an event-triggered consensus protocol is adopted to adapt to the case of this paper.

The rest of the paper is organized as follows. Section 2 formulates the consensus control problem and reviews the required lemmas. The main results and proof process are arranged in Section 3. Section 4 shows the simulation results of an illustrative example, and finally, Section 5 concludes this paper.

Notations: Given two matrices $A, B \in \mathbb{R}^{n\times n}$, $A \geq 0$ and $A > 0$ denote that A is positive semidefinite and positive definite, respectively. $A = col(a_1, a_2)$ denotes that A is a column vector composed of a_1 and a_2. $A = diag(a_1, a_2)$ denotes that matrix A is a diagonal matrix with diagonal elements a_1 and a_2, respectively. $A \otimes B$ denotes that A and B do the Kronecker product. I_N denotes an identity matrix of order N.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem formulation

Consider a group of UAVs i, $i \in N = \{1, , N\}$, facing communication faults in that partial position sensors are damaged. The dynamic of each UAV i is described by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ m_i\dot{v}_i(t) &= u_i(t) \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $v_i(t) \in \mathbb{R}^n$, and $u_i(t) \in \mathbb{R}^n$ are the position velocity and control input of UAV i at time t, respectively, in the inertial frame. $m_i > 0$ is the mass of UAV i, which can also be generalized as the inertia index or decision weight.

Remark 1. Referring to hierarchical interaction mechanisms, the decision weight is influenced by individual attribute, which is determined by social relationships and interaction patterns. Higher decision weight means that an individual is less susceptible to the influence of neighbors. Therefore, the conclusion of this paper can be extended to the heterogeneous system. In addition, this paper focuses on the consistency proof of large-scale network topology based on the graph theory. Using traditional drone models will make the proof process obscure and cumbersome. The control input in this paper can be considered as the expected acceleration. Therefore, the dynamic model of UAVs has been simplified during the proof process.

Definition 1. The heterogeneous multi-UAV system (1) is said to reach consensus for any initial conditions, when and only when we have $\lim_{t\to+\infty} \| x_i - x_j \| = 0$ and $\lim_{t\to+\infty} \| \dot{x}_i - \dot{x}_j \| = 0$ for $\forall i, j \in N$.

To achieve urgent task objectives, an event-triggered consensus protocol will be proposed based on the following second-order consensus protocol:

$$u_i(t) = -\Sigma_{j\in N_i}(kw_{ij}(x_i - x_j) + bv_{ij}(\dot{x}_i - \dot{x}_j)) \tag{2}$$

where k and b are stiffness gain and damping gain, respectively. w_{ij} is the coupling coefficient of position information interaction, and v_{ij} is the coupling coefficient of velocity information interaction. If $w_{ij} > 0$ or $v_{ij} > 0$, it means that the relevant information of UAV j can be captured by UAV i. How to achieve consensus in system (1) based on the above protocol and event triggering mechanism is the problem that needs to be addressed in this paper.

Remark 2. The communication and sensor faults assumed in this paper refer to the inability of individuals to obtain information sent by neighbors through wireless data transmission or other means. Therefore, in order to cope with situations where wireless data transmission cannot be utilized due to strong interference, the method of individuals acquiring information through sensors, such as position and velocity, is widely adopted. We further assume that position sensors of some individuals are damaged, and they are unable to obtain the position information of surrounding individuals (in fact, the processing methods for damaged position sensors and speed sensors are generally similar, and this article only discusses the former), which is reflected in the Laplacian matrix that contains all-zero rows.

2.2. Preliminaries

Lemma 1. Communication topology can be represented as a weighted directed (undirected) graph $G = (V_n, \varepsilon, A)$ of order n with a vertex set $V_n = \{1, 2, , n\}$ and edge set $\varepsilon \subset V_n \times V_n$ and a non-negative symmetric matrix $A = [a_{ij}]_{n\times n}$. $(s_i, s_j) \in \varepsilon \Leftrightarrow a_{ij} > 0 \Leftrightarrow$ the information of individual j can be captured by individual $i \Leftrightarrow j$ is the neighbor member of individual i. We assume $a_{ii} = 0$. The neighbor set of individual i is represented by $N_i = \{j|(i, j) \in \varepsilon\}$. The Laplacian matrix of the weighted diagraph is defined as $L = [l_{ij}]$, where $l_{ij} = -a_{ij}$ and $l_{ii} = \Sigma_{j\neq i} a_{ij}$.

Lemma 2[25]. If graph G contains at least one directed spanning tree, its corresponding Laplacian matrix L satisfies the following properties:

(a) $rank(L) = n - 1$;
(b) 0 is an eigenvalue of matrix L, and $[l, l, , l]^T$ is its corresponding eigenvector;
(c) $\mathrm{Re}(\lambda_l) \geq 0, \forall i \in \{1, 2, , n\}$; and there is only one eigenvalue of 0;
(d) Laplacian matrix L related to the strongly connected graph G is an irreducible matrix.

Laplacian matrix $L = [l_{ij}]_{n\times n}$ and $H = [h_{ij}]_{n\times n}$ are defined as:

$$
\begin{aligned}
l_{ij} &= \begin{cases} -v_{ij} & i \neq j \\ \sum_{j=1}^{n} v_{ij} & i = j \end{cases} \\
h_{ij} &= \begin{cases} -w_{ij} & i \neq j \\ \sum_{j=1}^{n} w_{ij} & i = j \end{cases}
\end{aligned} \tag{3}
$$

The mass matrix of the system is recorded as $M = diag(m_1, m_2, , m_n) > 0$. Describe system (1) by using matrices as follows:

$$
\ddot{x} + bM^{-1}L \otimes I_N \dot{x} + kM^{-1}H \otimes I_N x = 0 \tag{4}
$$

Remark 3. Actual physical meaning in the formula denoted by I_N is the dimension of state space, which is usually defined as 3.

Due to the limited refresh rate and sampling frequency of sensors and processors, the event triggering mechanism is proposed to reduce the pressure on sensors and save processor resources while ensuring that they can still react quickly in the face of unexpected situations.

State error is an important decision factor in event-triggered consensus control. Define $\xi_i(t) = x_i(t) - x_1(t)$ and $\eta_i(t) = v_i(t) - v_1(t)$, which denote position error and state error, respectively. Apart from this, define $\xi(t) = col(\xi_2(t), , \xi_n(t))$ and $\eta(t) = col(\eta_2(t), , \eta_n(t))$, $\varepsilon(t) = col(\xi(t), \eta(t))$. Suppose the incremental time column that satisfies the event-triggered condition is $\{t_1, , t_k\}$. System (1) is equivalent to

$$
\begin{aligned}
&\dot{\xi}(t) = \eta(t) \\
&\dot{\eta}(t) = [F_1 \quad F_2] \otimes I_N \varepsilon(t_k) \qquad t \in [t_k, t_{k+1})
\end{aligned} \tag{5}
$$

We define

$$
F = \begin{bmatrix} 0 & I \\ F_1 & F_2 \end{bmatrix} \tag{6}
$$

$$
e(t) = \begin{bmatrix} e_x(t) \\ e_v(t) \end{bmatrix} = \begin{bmatrix} \xi(t_k) - \xi(t) \\ \eta(t_k) - \eta(t) \end{bmatrix} \tag{7}
$$

And we have

$$
\begin{aligned}
&\dot{\xi}(t) = \eta(t) \\
&\dot{\eta}(t) = [F_1 \quad F_2] \otimes I_N \varepsilon(t_k) \\
&= [F_1 \quad F_2] \otimes I_N \begin{bmatrix} \xi(t_k) \\ \eta(t_k) \end{bmatrix} \\
&= [F_1 \quad F_2] \otimes I_N \begin{bmatrix} \xi(t) + e_x(t) \\ \eta(t) + e_v(t) \end{bmatrix} \\
&= [F_1 \quad F_2] \otimes I_N (\begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_v(t) \end{bmatrix})
\end{aligned}
$$

Therefore, system (5) can be converted to the form in continuous time gives

$$\begin{aligned}\dot{\varepsilon}(t) &= \begin{bmatrix} 0 & I \\ F_1 & F_2 \end{bmatrix} \otimes I_N \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ F_1 & F_2 \end{bmatrix} \otimes I_N \begin{bmatrix} e_x(t) \\ e_v(t) \end{bmatrix} \\ &= F \otimes I_N \varepsilon(t) + J \otimes I_N e(t)\end{aligned} \quad (8)$$

Different from the previous consensus control methods [Similar to the form of System (5)] for the UAV system (1), the individuals are supposed to guarantee the interaction of velocity through independent information collection of position and velocity [the form of System (4)] when extreme cases, such as partial damage to position sensors, are considered, which is also the difficulty and focus of this study.

3. METHODS AND RESULTS

3.1. Linear transformation of the system

First, System (4) can be transformed into the form of system (5) based on the lemma as follows:

Lemma 3[26]. For Laplacian matrix related to the directed graph , there exists a non-singular matrix

$$U = \begin{bmatrix} 1 & * & \dots & * \\ 1 & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ 1 & * & \dots & * \end{bmatrix} \in \mathbb{R}^{n\times n} \quad (9)$$

so that

$$U^{-1}LU = \begin{bmatrix} 0 & h^T \\ 0_{n-1} & H \end{bmatrix} \quad (10)$$

where $h \in \mathbb{R}^{n-1}$ and $H \in \mathbb{R}^{(n-1)\times(n-1)}$. U^{-1} has the form as

$$U^{-1} = \begin{bmatrix} \gamma_1 & \gamma_2 & \dots & \gamma_n \\ * & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \dots & * \end{bmatrix} \in \mathbb{R}^{n\times n} \quad (11)$$

where $\sum_{i=1}^{n} \gamma_i = 1$.

Therefore, non-singular linear transformations are built as $\xi = U_{-1} \otimes I_N x$, and U^{-1} is defined as

$$U^{-1} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \dots & \gamma_n \\ -1 & 1 & 0 & \dots & 0 \\ -1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & \dots & 1 \end{bmatrix} \in \mathbb{R}^{n\times n} \tag{12}$$

Thus, $\xi_1 = \sum_{i=1}^{n} \gamma_i x_i$ and $\xi_k = x_k - x_1$, where $k = \{2, 3, \dots, n\}$. The state vector of the system is transformed into the error vector by linear transformations. System (4) is equivalent to the system as follows:

$$U \otimes I_N \ddot{\xi} + bM^{-1}LU \otimes I_N \dot{\xi} + kM^{-1}HU \otimes I_N \xi = 0 \tag{13}$$

Lemma 4. For Laplacian matrix $L = [l_{ij}]_{n\times n}$ related to the strongly connected graph G, there exists positive vector $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_n]$ that is the eigenvector corresponding to the eigenvalue 0 of the matrix ΛL, where $\Lambda = M^{-1} > 0$ and $\sum_{i=1}^{n} \gamma_i = 1$.

Proof: Define $A = [a_{ij}]_{n\times n} = [-l_{ij}]_{n\times n}$ and $w = \max_{i\in N} |a_{ii}/m_i|$. Since Laplacian matrix L is related to the strongly connected graph, matrix $\Lambda A + wI$ is semi-positive definite. Based on the Gershgorin circle theorem, the eigenvalues of the matrix ΛA and $\Lambda A + wI$ satisfy conditions as follows:

$$\begin{aligned} \lambda_i(\Lambda A) &\in \{z \in \mathbb{C} \,||\, z + w| \leq w\} \\ \lambda_i(\Lambda A + wI) &\in \{z \in \mathbb{C} \,||\, z| \leq w\} \end{aligned} \tag{14}$$

where $i \in N$. Since the eigenvalues of the matrix will not be altered after transposition, for $\forall i \in N$

$$\rho(A^T \Lambda^T + wI) = \max\{|\lambda_1|, |\lambda_2|, \dots, |\lambda_n|\} = w \tag{15}$$

Since $A^T \Lambda^T + wI \geq 0$, there exists a non-negative vector ξ^T corresponding to the right eigenvector of the eigenvalues w. Thus, one has that

$$\begin{aligned} A^T \Lambda^T \xi^T &= (A^T \Lambda^T + wI - wI)\xi^T \\ &= (A^T \Lambda^T + wI)\xi^T - wI\xi^T \\ &= w\xi^T - w\xi^T = 0^T \end{aligned} \tag{16}$$

then

$$\xi \Lambda A = (A^T \Lambda^T \xi^T)^T = 0 \tag{17}$$

Since $A = -L$, there exists a non-negative vector ξ satisfies the condition as follows:

$$\xi \Lambda L = \xi \Lambda(-A) = 0 \tag{18}$$

On the other hand, since matrix L is related to the strongly connected graph, and according to Lemma 2, L is an irreducible matrix. Therefore, according to the Perron-Frobenius theorem, vector ξ is a positive vector. Then, the vector γ to be found is expressed as

$$\gamma = \xi / \sum_{i=1}^{n} |\xi_i| \tag{19}$$

The proof is, thus, completed.

According to Lemma 4, the positive vector $\gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n]$ is defined as corresponding to matrix $M^{-1}L$ in system (16), where $\sum_{i=1}^{n} \gamma_i = 1$. Define $\Gamma = diag(\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n)$, and Γ is an invertible matrix. Then, one has inferences as follows:

$$[1, 1, \ldots, 1]\Gamma M^{-1} L = 0^T \tag{20}$$

$$\Gamma M^{-1} L [1, 1, \ldots, 1]^T = 0 \tag{21}$$

Furthermore, system (13) is equivalent to the system as follows:

$$U^T \Gamma U \otimes I_{N\times N} \ddot{\xi} + bU^T \Gamma M^{-1} LU \otimes I_{N\times N} \dot{\xi} + kU^T \Gamma M^{-1} HU \otimes I_{N\times N} \xi = 0 \tag{22}$$

Therefore:

$$U^T \Gamma U = \begin{bmatrix} 1 & 0^T \\ 0 & \widetilde{\Gamma} \end{bmatrix} \tag{23}$$

Similarly, one has that

$$U^T \Gamma M^{-1} LU = \begin{bmatrix} 0 & 0^T \\ 0 & \widetilde{L} \end{bmatrix} \tag{24}$$

$$U^T \Gamma M^{-1} HU = \begin{bmatrix} 0 & \widetilde{\boldsymbol{h}} \\ 0 & \widetilde{H} \end{bmatrix} \tag{25}$$

Define $\xi = col(\xi_1, \xi_e)$, where $\xi_e = col(\xi_2, \xi_3, \ldots, \xi_n)$. System (22) is equivalent to the system as follows:

$$\begin{bmatrix} 1 & 0^T \\ 0 & \widetilde{\Gamma} \end{bmatrix} \otimes I_{N\times N} \begin{bmatrix} \ddot{\xi}_1 \\ \ddot{\xi}_e \end{bmatrix} + b \begin{bmatrix} 0 & 0^T \\ 0 & \widetilde{L} \end{bmatrix} \otimes I_{N\times N} \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_e \end{bmatrix} + k \begin{bmatrix} 0 & \widetilde{\boldsymbol{h}} \\ 0 & \widetilde{H} \end{bmatrix} \otimes I_{N\times N} \begin{bmatrix} \xi_1 \\ \xi_e \end{bmatrix} = 0 \tag{26}$$

From system (26), together with (5), one has that

$$\begin{aligned} \dot{\xi}_e(t) &= \eta_e(t) \\ \dot{\eta}_e(t) &= [F_1 \quad F_2] \otimes I_N \varepsilon(t_k) \qquad t \in [t_k, t_{k+1}) \end{aligned} \tag{27}$$

where $F_1 = -k\widetilde{\Gamma}^{-1}\widetilde{H}$, $F_2 = -b\widetilde{\Gamma}^{-1}\widetilde{L}$, and $\varepsilon(t) = col(\xi_e(t), \eta_e(t))$. Therefore, the event-triggered consensus protocol in this paper can be written as:

$$u(t) = [F_1, F_2] \otimes I_{N\times N}\varepsilon(t_k) \tag{28}$$

According to (6), (7), and (8), Converting system (27) to the form in continuous time gives:

$$\begin{gathered}\dot{\varepsilon}(t) = F \otimes I_{N\times N}\varepsilon(t) + J \otimes I_{N\times N}e(t)\\ F = \begin{bmatrix} 0 & I \\ F_1 & F_2 \end{bmatrix} \quad J = \begin{bmatrix} 0 & 0 \\ F_1 & F_2 \end{bmatrix}\end{gathered} \tag{29}$$

where $e(t)$ is defined as

$$e(t) = \begin{bmatrix} e_x(t) \\ e_v(t) \end{bmatrix} = \begin{bmatrix} \xi_e(t_k) - \xi_e(t) \\ \eta_e(t_k) - \eta_e(t) \end{bmatrix} \tag{30}$$

Thus, the proof of consensus in system (1) is transformed into the proof of stability of system (27).

Remark 4. The stability of system (27) implies that the state errors between the UAVs are zero. According to Definition 1, these two propositions are equivalent.

3.2. Analysis of stability

Now, the main result of this paper can be given as follows.

Theorem 1. Consider system (27) and event-triggered consensus protocol (28), sufficient conditions for the stability of the system are given as follows:

$$(\widetilde{H} + \widetilde{H}^T) - \frac{k}{b^2}\widetilde{\Gamma} > 0 \tag{31}$$

$$(\widetilde{L} + \widetilde{L}^T) - \frac{2k}{b^2}\widetilde{\Gamma} - b\Psi^T(\widetilde{H} + \widetilde{H}^T)\Psi > 0 \tag{32}$$

$$\| e \| \leq \sigma \frac{\lambda_{\min}(Q)}{\lambda_{\max}(\bar{P})} \| \varepsilon \| \tag{33}$$

where

$$\Psi = (\widetilde{L} - \widetilde{H}) \tag{34}$$

$$\bar{P} = \begin{bmatrix} -\frac{k^2}{b}\widetilde{H} & -k\widetilde{L} \\ -k\widetilde{H} & -b\widetilde{L} \end{bmatrix} \tag{35}$$

$$\sigma \in (0,1) \tag{36}$$

Proof: Choose the Lyapunov function as

$$V = \varepsilon^T (P \otimes I_N)\varepsilon \tag{37}$$

in which

$$P = \begin{bmatrix} k(\widetilde{H} + \widetilde{H}^T) & \frac{k\widetilde{\Gamma}}{b} \\ \frac{k\widetilde{\Gamma}}{b} & \widetilde{\Gamma} \end{bmatrix} \tag{38}$$

Define $\widetilde{P} = T^T PT$, where

$$T = \begin{bmatrix} I & 0 \\ -\frac{kI}{b} & I \end{bmatrix} \tag{39}$$

The contract transformation does not change the positivity of the matrix, and

$$\widetilde{P} = \begin{bmatrix} k(\widetilde{H} + \widetilde{H}^T) - \frac{k^2\widetilde{\Gamma}}{b^2} & 0 \\ 0 & \widetilde{\Gamma} \end{bmatrix} \tag{40}$$

Since condition (31) implies that Γ is a positive definite matrix, one has that $\widetilde{P} > 0$. Therefore, P is a positive definite matrix.

Differentiate V to obtain that

$$\begin{aligned} \dot{V} &= \varepsilon^T (F^T P + PF) \otimes I_N \varepsilon + 2\varepsilon^T PJ \otimes I_N e \\ &= -\varepsilon^T Q \otimes I_N \varepsilon + 2\varepsilon^T PJ \otimes I_N e \end{aligned} \tag{41}$$

in which

$$\begin{aligned} Q &= -(F^T P + PF) \\ &= \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{k^2}{b}(\widetilde{H} + \widetilde{H}^T) & k(\widetilde{L} - \widetilde{H}) \\ k(\widetilde{L}^T - \widetilde{H}^T) & b(\widetilde{L} + \widetilde{L}^T) - \frac{2k}{b}\widetilde{\Gamma} \end{bmatrix} \end{aligned} \tag{42}$$

And $Q = Q^T$. Performing the contract transformation on matrix Q, one can obtain that

$$Q = T_1 Q T_1^T = \begin{bmatrix} \widetilde{Q}_1 & 0 \\ 0 & \widetilde{Q}_4 \end{bmatrix} \tag{43}$$

where

$$T_1 = \begin{bmatrix} I & 0 \\ -Q_2^T Q_1^{-T} & I \end{bmatrix} \tag{44}$$

$$\widetilde{Q}_1 = Q_1 \tag{45}$$

$$\widetilde{Q}_4 = b(\widetilde{L} + \widetilde{L}^T) - \frac{2k}{b}\widetilde{\Gamma} - b\Psi^T(\widetilde{H} + \widetilde{H}^T)\Psi \tag{46}$$

According to conditions (31) and (32), matrix is a positive definite matrix. One has that

$$\begin{aligned} \dot{V} &= -\varepsilon^T Q \otimes I_N \varepsilon + 2\varepsilon^T PJ \otimes I_N e \\ &\leq -\lambda_{\min}(Q) \parallel \varepsilon \parallel^2 + 2 \parallel \bar{P} \parallel \parallel \varepsilon \parallel \parallel e \parallel \end{aligned} \tag{47}$$

in which

$$\begin{aligned} \bar{P} &= PJ \\ &= \begin{bmatrix} -\frac{k^2}{b}\widetilde{H} & -k\widetilde{L} \\ -k\widetilde{H} & -b\widetilde{L} \end{bmatrix} \end{aligned} \tag{48}$$

From condition (33), one has that

$$\begin{aligned} \dot{V} &\leq -\lambda_{\min}(Q) \parallel \varepsilon \parallel^2 + 2 \parallel \bar{P} \parallel \parallel \varepsilon \parallel \parallel e \parallel \\ &\leq -\lambda_{\min}(Q) \parallel \varepsilon \parallel^2 + \sigma\lambda_{\min}(Q) \parallel \varepsilon \parallel^2 \\ &= -(1-\sigma)\lambda_{\min}(Q) \parallel \varepsilon \parallel^2 \\ &< 0 \end{aligned} \tag{49}$$

Define an event-triggered function as follows:

$$f(e) = \parallel e \parallel - \frac{\sigma}{2} \frac{\lambda_{\min}(Q)}{\parallel \bar{P} \parallel} \parallel \varepsilon \parallel \tag{50}$$

At an event time $t \in \{t_1, \ldots, t_k\}$, one can obtain:

$$e(t_k) = \begin{bmatrix} e_x(t_k) \\ e_v(t_k) \end{bmatrix} = \begin{bmatrix} \xi(t_k) - \xi(t_k) \\ \eta(t_k) - \eta(t_k) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{51}$$

which denotes that condition (33) can always be satisfied.

Therefore, based on Lyapunov stability principles, for an arbitrary initial state $\xi(0)$ and $\eta(0)$ of system (27), one can achieve that

$$\begin{aligned} \lim_{t\to\infty} \xi(t) = 0 \\ \lim_{t\to\infty} \eta(t) = 0 \end{aligned} \tag{52}$$

which are equivalent to that

$$\begin{aligned} \lim_{t\to\infty} x_i - x_j = 0 \\ \lim_{t\to\infty} v_i - v_j = 0 \end{aligned} \tag{53}$$

where $i, j \in \{1, 2, \ldots, n\}$. The stability of system (27) can be achieved, which denotes that the consensus of system (1) can be achieved.

The proof is thus completed.

Remark 5. According to the event-triggered function (50), the event-triggered condition is met when the error $\| e \|$ exceeds the threshold, which means that the accumulated error in the information received at the previous triggering time has exceeded the stable range, and the control input needs to be updated immediately. Apparently, this event-triggered condition is easier to achieve in emergency situations due to the rapid and drastic state changes of neighbors.

Theorem 2. Consider system (27) and event-triggered consensus protocol (28), the system will not exhibit the Zeno behavior, which means that the time interval between any two events will not be less than

$$\tau = \frac{1}{\| F \| - \| J \|} \ln \frac{1+\beta}{1+\frac{\|J\|}{\|F\|}\beta} \tag{54}$$

in which

$\beta = \phi(\tau, 0) = \frac{\sigma}{2} \frac{\lambda_{\min}(Q)}{\|\bar{P}\|}$

Proof: Similar to the proof in [32], we define

$y = \frac{\|e\|}{\|\varepsilon\|}$

And one has that

$\dot{y} = (\frac{-e^T}{\|e\|} - \frac{\varepsilon^T \|e\|}{\|\varepsilon\|^2}) \frac{\dot{\varepsilon}}{\|\varepsilon\|}$
$\leq (1 + \frac{\|e\|}{\|\varepsilon\|}) \frac{\|\dot{\varepsilon}\|}{\|\varepsilon\|}$
$= (1 + y)(\| F \| + \| J \| y)$

and y satisfies that

$y(t) \leq \phi(t, \phi_0)$

in which $\phi(t, \phi_0)$ is the solution of

$$\dot{\phi} = (1+\phi)(\| F \| + \| J \| \phi)$$
$$\phi(0, \phi_0) = \phi_0$$

From (33), the solution of the equation above also satisfies that

$$\phi(\tau, 0) = \frac{\sigma}{2} \frac{\lambda_{\min}(Q)}{\|\bar{P}\|}$$

so that

$$\tau = \frac{1}{\|F\| - \|J\|} \ln \frac{1+\phi(\tau,0)}{1+\frac{\|J\|}{\|F\|}\phi(\tau,0)}$$

The proof is, thus, completed.

Theorem 3. Consider system (27) and event-triggered consensus protocol (28), for any positive definite matrix Q, if there exists a positive definite matrix P satisfying $Q = -(F^T P + PF)$, suitable parameter σ and trigger functions $f(e)$ can always be designed so that the system achieves consensus under the event-triggered conditions based on the above proof process.

Theorem 4. For the multi-UAV system, appropriate distance should be maintained between individuals. Consensus protocol (2) can be transformed into

$$u_i(t) = -\sum_{j \in N_i} \left\{ k w_{ij} [(x_i - d_i) - (x_j - d_j)] + b v_{ij} [(\dot{x}_i - \dot{d}_i) - (\dot{x}_j - \dot{d}_j)] \right\}$$
$$= -\sum_{j \in N_i} [k w_{ij} (x_i - x_j - d_{ij}) + b v_{ij} (\dot{x}_i - \dot{x}_j)]$$

in which d_i is the constant offset. We define $x_i^{'} = x_i - d_i$ and $\dot{x}_i^{'} = \dot{x}_i$ so that

$$u_i(t) = -\sum_{j \in N_i} \left\{ k w_{ij} [(x_i - d_i) - (x_j - d_j)] + b v_{ij} [(\dot{x}_i - \dot{d}_i) - (\dot{x}_j - \dot{d}_j)] \right\}$$
$$= -\sum_{j \in N_i} [k w_{ij} (x_i^{'} - x_j^{'}) + b v_{ij} (\dot{x}_i^{'} - \dot{x}_j^{'})]$$

Therefore, offset d_i will not affect the consensus of the system.

4. SIMULATION

According to the scenario described in Remark 2, we consider a UAV group consisting of five individuals whose dynamics are described by (1). The information interaction topologies of their velocity and position are described in Figure 1A and B, respectively. The position information of other UAVs cannot be sensed by individual 1 due to the damage of its position sensor.

One can obtain a Laplacian matrix of interaction topologies of velocity and position, respectively, as follows:

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Consider $M = diag(1, 1, 1, 0.5, 0.25)$. According to Theorem 1 and Theorem 3, the system parameters can be selected as follows: $k = 3$, $b = 12$, and $\sigma = 0.99$. The sampling period T is set to 0.01. The initial state of the system is set as follows:

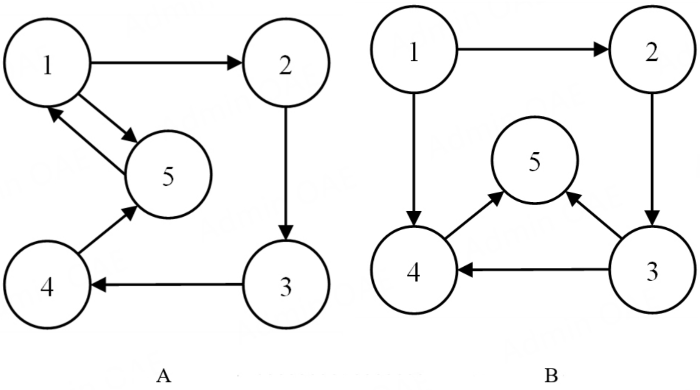

Figure 1. The interaction topologies of velocity (A) and position (B).

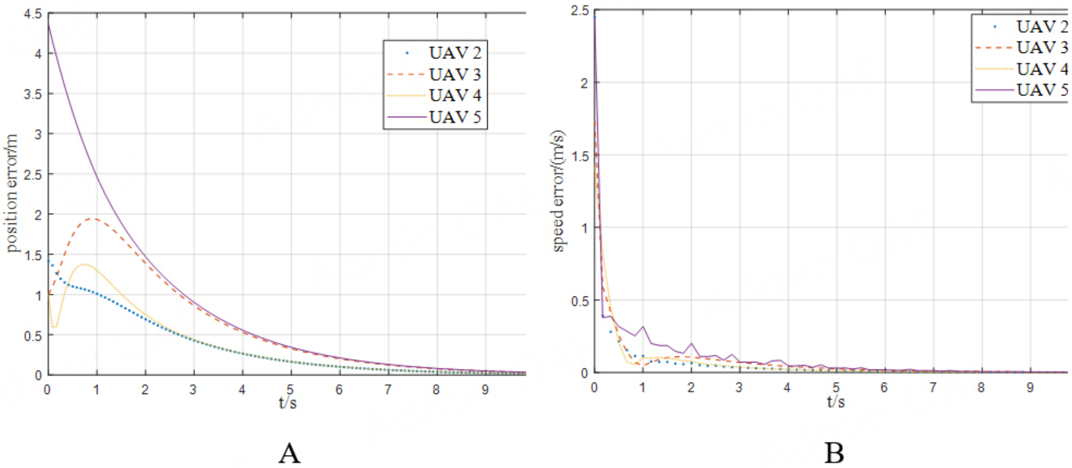

Figure 2. Position error (A) and speed error (B).

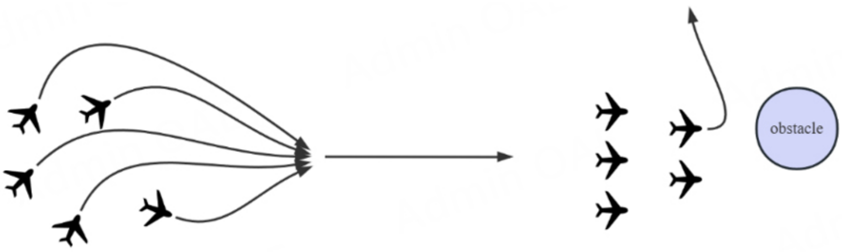

Figure 3. Task flow diagram of UAV groups.

$$x_0 = [(3,2,1)^T, (3,3,2)^T, (4,2,1)^T, (3,2,2)^T, (4,5,-2)^T]^T$$

$$v_0 = [(1,2,4)^T, (2,4,3)^T, (2,1,3)^T, (1,1,5)^T, (2,4,3)^T]^T$$

The evolution of position error and velocity error is shown in Figure 2A and B, respectively.

Through available sensors, UAV groups reach consensus within 10 s. Suppose that the UAV group encounters an emergency at 10 s that causes a sudden change of velocity in an individual.The overall task flow of UAV groups is shown in Figure 3.

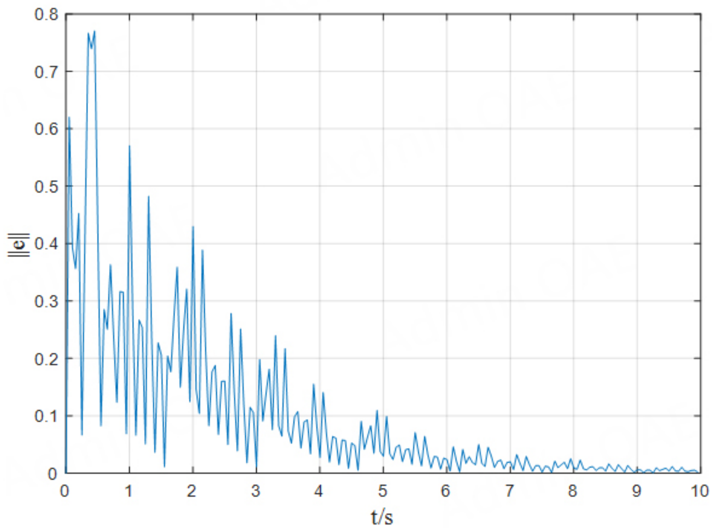

Figure 4. The evolution of $\| e \|$.

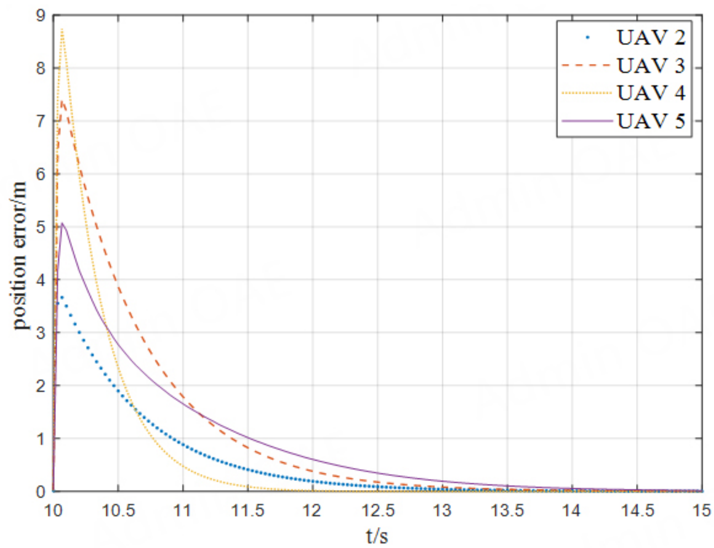

Figure 5. The evolution of position error after encountering the emergency situation.

Simulation results show that the system can reach consensus under the event-triggered protocol proposed in this paper. The variation curve of $\| e \|$ is shown in Figure 4.

The evolution of $\| e \|$ indicates that the event-trigger frequency of the system is less than the set sampling frequency (100 Hz). According to Remark 5, the error $\| e \|$ accumulates continuously within two adjacent event-triggered moments until reaching the threshold (33) and entering the next triggering moment (the local maximum points of $\| e \|$ in Figure 4).

Figure 5, Figure 6, and Figure 7 show the simulation results of the UAV group encounter the emergency situation.

The simulation results above demonstrate the effectiveness of the proposed protocol in this paper with communication faults, even in case of unexpected situations. The protocol proposed in this paper has a broader

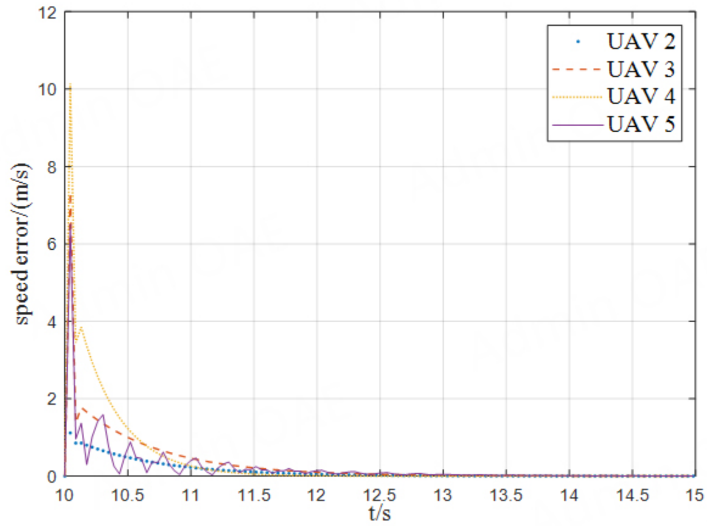

Figure 6. The evolution of speed error after encountering the emergency situation.

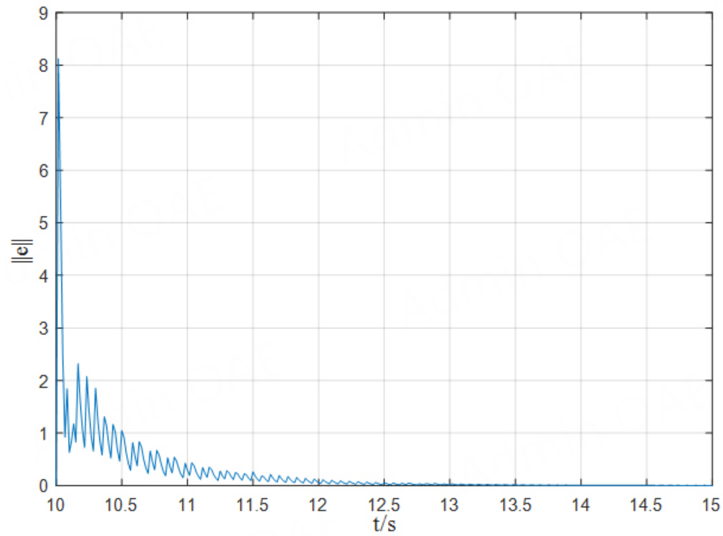

Figure 7. The evolution of $\| e \|$ after encountering the emergency situation.

application scenario and is further promoted compared to traditional consensus proof. In order to cope with communication faults, wireless detection, and other tasks, an event-triggered protocol has been introduced due to the lower frequency and effectiveness of obtaining neighbor information through sensors compared to traditional information exchange based on wireless data transmission, which provides a theoretical basis for further physical verification. At the same time, the protocol allows for damage to some sensors, further improving the fault tolerance range of the system.

5. DISCUSSION

In this paper, an event-triggered consensus protocol of multi-UAVs has been proposed, which is used to solve the consensus problem of systems in normal or emergency situations with communication faults. Compared to traditional protocols, differences in the interaction topologies of speed and location information are allowed. With the help of Lyapunov stability principles, sufficient conditions to achieve system consensus are given. We have also presented simulation results to illustrate the effectiveness of our approach.

In future work, how to obtain more generalized and sufficient consensus conditions will be considered. Further, we will extend the results presented in this paper to complex inertial systems and topological networks, including random and time-delay networks.

DECLARATIONS

Authors' contributions

Made significant contributions to the research direction and design and conducted theoretical analysis, proof, and explanation: Guo Z, Wei C, Shen Y
Providing administrative, technical, and material support: Yuan W

Availability of data and materials

Not applicable.

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Conflicts of interest

All authors declared that there are no conflicts of interest.

Ethical approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

REFERENCES

1. Vinyals O, Babuschkin I, Czarnecki WM, et al. Grandmaster level in starcraft II using multi-agent reinforcement learning. *Nature* 2019;575:350-4. DOI
2. Thunberg J, Goncalves J, Hu X. Consensus and formation control on SE(3) for switching topologies. *Automatica* 2016;66:109-21. DOI
3. Botts CH, Spall JC, Newman AJ. Multi-agent surveillance and tracking using cyclic stochastic gradient. In: Proceedings of the American Control Conference. Boston, MA, USA; 2016. p. 270-5. DOI
4. Ren W, Beard RW, Atkins EM. A survey of consensus problems in multi-agent coordination. In: Proceedings of the American Control Conferenc. Portland, OR, USA. IEEE; 2005. p. 1859-64. DOI
5. Heylighen F. The science of self-organization and adaptivity. In: Kiel LD, Elliott EW, editors. Complexity and the human sciences. Routledge; 2001. p. 57-85. Available from: https://www.eolss.net/sample-chapters/c15/E1-29-01-05.pdf. [Last accessed on 6 Nov 2023]
6. Yang D, Ren W, Liu X, Chen W. Decentralized event-triggered consensus for linear multi-agent systems under general directed graphs. *Automatica* 2016;69:242-9. DOI
7. Ren W, Atkins E. Second-order consensus protocols in multiple vehicle systems with local interactions. In: Proceedings of the AIAA Guidance, Navigation, and Control Conference. San Francisco, California. AIAA; 2005. p. 1-14. DOI
8. Huang N, Duan Z, Zhao Y. Consensus of multi-agent systems via delayed and intermittent communications. *IET Control Theory Appl* 2015;9:1-8. DOI
9. Zhao X, Shi P, Zheng X, Zhang J. Intelligent tracking control for a class of uncertain high-order nonlinear systems. *IEEE Trans Neural Netw Learn Syst* 2016;27:1976-82. DOI
10. Huang S, Liao F, Teo RSH. Fault tolerant control of quadrotor based on sensor fault diagnosis and recovery information. *Machines* 2022;10:1088. DOI

11. Tsai YH, Chang FR, Yang WC. GPS fault detection and exclusion using moving average filters. *IEE Proc Radar Sonar Navig* 2004;151:240-7. DOI
12. Olfati-saber R, Murray R. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans Automat Contr* 2004;49:1520-33. DOI
13. Xiao F, Wang L. State consensus for multi-agent systems with switching topologies and time-varying delays. *Int J Control* 2006;79:1277-84. DOI
14. Hatano Y, Mesbahi M. Agreement over random networks. *IEEE Trans Automat Contr* 2005;50:1867-72. DOI
15. Cao M, Morse A, Anderson B. Agreeing asynchronously. *IEEE Trans Automat Contr* 2008;53:1826-38. DOI
16. Yan B, Wu C, Shi P. Formation consensus for discrete-time heterogeneous multi-agent systems with link failures and actuator/sensor faults. *J Franklin I* 2019;356:6547-70. DOI
17. Chen C, Lewis FL, Xie S, et al. Resilient adaptive and $H\infty$ controls of multi-agent systems under sensor and actuator faults. *Automatica* 2019;102:19-26. DOI
18. Xiong S, Hou Z. Data-driven formation control for unknown MIMO nonlinear discrete-time multi-agent systems with sensor fault. *IEEE Trans Neural Netw Learn Syst* 2022;33:7728-42. DOI
19. Lee D, Spong MW. Stable flocking of multiple inertial agents on balanced graphs. *IEEE Trans Autom Control* 2007;52:1469-75. DOI
20. Li W, Spong MW. Stability of general coupled inertial agents. *IEEE Trans Autom Control* 2010;55:1411-6. DOI
21. Zheng Y, Zhu Y, Wang L. Consensus of heterogeneous multi-agent systems. *IET Control Theory A* 2011;5:1881-8. DOI
22. Mu B, Shi Y. Distributed LQR consensus control for heterogeneous multiagent systems: theory and experiments. *IEEE/ASME Trans Mechatron* 2018;23:434-43. DOI
23. Ma Q, Miao G. Output consensus for heterogeneous multi-agent systems with linear dynamics. *Appl Math Comput* 2015;271:548-55. DOI
24. Du H, Wen G, Wu D, Cheng Y, Lü J. Distributed fixed-time consensus for nonlinear heterogeneous multi-agent systems. *Automatica* 2020;113:108797. DOI
25. Li X, Yu Z, Li Z, Wu N. Group consensus via pinning control for a class of heterogeneous multi-agent systems with input constraints. *Inf Sci* 2021;542:247-62. DOI
26. Dorf RC, Farren M, Phillips C. Adaptive sampling frequency for sampled-data control systems. *IRE Trans Automat Contr* 1962;7:38-47. DOI
27. Fan Y, Feng G, Wang Y, et al. Distributed event-triggered control of multi-agent systems with combinational measurements. *Automatica* 2013;49:671-5. DOI
28. Kim H, Shim H, Seo JH. Output consensus of heterogeneous uncertain linear multi-agent systems. *IEEE Trans Automat Contr* 2011;56:200-6. DOI
29. Peng C, Han Q. On designing a novel self-triggered sampling scheme for networked control systems with data losses and communication delays. *IEEE Trans Ind Electron* 2016;63:1239-48. DOI
30. Antonio VTJ, Adrien G, Manuel A, et al. Event-triggered leader-following formation control for multi-agent systems under communication faults: application to a fleet of unmanned aerial vehicles. *J Syst Eng Electron* 2021;32:1014-22. DOI
31. Tasooji TK, Khodadadi S, Marquez HJ. Event-based secure consensus control for multirobot systems with cooperative localization against DoS attacks. *IEEE/ASME Trans Mechatron* 2023:1-15. DOI
32. Dimarogonas DV, Johansson KH. Event-triggered control for multi-agent systems. In: Proceedings of the 48th IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference. Shanghai. China: IEEE; 2009. p. 7131-6. DOI