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# Adaptive neural control for delayed discrete-time switched systems under deception attacks

**Dongke Zhao, Yongjie Tian, Ning Zhao**

College of Control Science and Engineering, Bohai University, Jinzhou 121013, Liaoning, China.

**Correspondence to:** Dr. Ning Zhao, College of Control Science and Engineering, Bohai University, 19 Keji Road, Songshan New District, Jinzhou 121013, Liaoning, China. E-mail: zhaoning_hrbeu@163.com

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## Abstract

This paper focuses on the issue of adaptive neural control for discrete-time switched systems with time delay and deception attacks. Firstly, the switching signal is constrained by the dwell time. Considering that the deception attacks are unknown, the neural network technique is employed to approximate the attack signals. Then, an adaptive state feedback controller is established to compensate for the adverse effects of deception attacks for switched systems. Meanwhile, sufficient conditions for the boundedness of the switched system are given through the Lyapunov functional method, and the controller gains can be obtained by resolving the linear matrix inequality. Finally, the feasibility of the proposed method is illustrated via a numerical example.

**Keywords:** Switched system, deception attacks, adaptive control

## 1. INTRODUCTION

Switched systems consist of some subsystems and a switching rule that manages these subsystems. Because of their complexity and practicality, switched systems have been used in a wide variety of applications, such as manipulator robots [1], aircraft control systems [2], circuit systems [3], and so on. The switched systems are featured as the stability of each subsystem is not necessarily related to the stability of the closed-loop system. Therefore, designing a correct switching strategy is very significant. In recent years, the approach to constrain a switching strategy has included dwell time (DT) [4] and average DT (ADT) [5].

With the popularity of networking, network control systems (NCSs) have been studied and applied by scholars. In NCSs, control and feedback signals are transmitted through communication networks between sensors, controllers, and actuators. It has the characteristics of high reliability and low cost. Therefore, combining the advantages of NCSs and switched systems is a valuable issue. Despite the evident advantages of NCSs, there are also unavoidable disadvantages such as communication resources and network security problems. In particular, network security is an important problem considered in the open network environment. At present, network attacks can be mainly divided into deception attacks[6,7] and denial-of-service (DoS) attacks. The purpose of the DoS attacks is to prevent the transmission of data. The deception attacks aim to modify real signals and substitute them with false signals. Therefore, the research on how to compensate for the adverse effects of deception attacks on the system is of extraordinary significance. The discrete nonlinear systems with deception attacks were considered, in which Bernoulli distributions were used to describe the adverse effects of deception attacks on the systems[8]. The Takagi-Sugeno fuzzy systems were considered, in which the deception attacks existed in the feedforward and feedback channels, and were modeled independent Bernoulli processes[9]. The problem of network security and communication resources of vehicles under deception attacks were investigated, in which deception attacks were considered as constrained bounded energy signals[10]. In the above articles, deception attacks are supposed to be known or bounded, which is obviously a limitation. Therefore, the deception attacks considered in this paper involve no assumptions but rather focus on the identification and estimation of unknown attack signals. According to the practical situation, deception attacks have been designed as system state-dependent nonlinear functions. Therefore, the deception attack signals can be identified and estimated by applying neural network (NN) techniques, and the approximation function can be employed to design the controller in this article. Currently, the applications of NN techniques to approximate nonlinear functions have yielded numerous research results[11]. In nonlinear systems, these techniques were employed to approximate nonlinear functions that construct adaptive models to save communication resources and address stabilization problems. For the nonlinear switched systems, error terms were approximated by NN technology and compensated with nonlinear filters to solve the adaptive tracking control problem[12]. The adaptive NN observer problem was studied, in which the NN observer is designed to compensate for quantization errors and accurately estimate the state of the actuator[13]. In addition, some existing research results have approximated deception attack signals with NN techniques. The authors investigated the filtering problem of networked switched systems by approximating deception attacks with these techniques[14]. The event-triggered mechanism was designed based on NN, and the deception attacks were approximated by NN, which solved the security control problem[15]. It should be noted that the NN technique has been widely applied to nonlinear systems, but there is little attention paid to the processing of deception attacks as nonlinear functions, which motivates our research. Therefore, the challenge addressed in this paper is how to employ NN techniques to estimate deception attacks occurring in the controller-actuator (C-A) transmission channel, where deception attacks are considered as nonlinear functions.

In this paper, the control problem of discrete-time switched systems with time delay and deception attacks is considered, and a controller design scheme is proposed by using NN techniques to approximate the deception attack signals. The main contributions are as follows:

(1) Different from bounded deception attacks[16], the deception attacks in this paper are approximated using NN techniques, which can better defend against deception attacks.

(2) The switching signals are constrained by the DT, and a new Lyapunov functional method is constructed to ensure the boundedness of the switched systems.

The rest of this paper is structured as follows. In Section 2, the system description and controller design strategy are proposed. The main results of adaptive neural controllers are introduced in Section 3. A numerical example is applied to demonstrate the feasibility of the method in Section 4. Finally, the conclusion is presented in

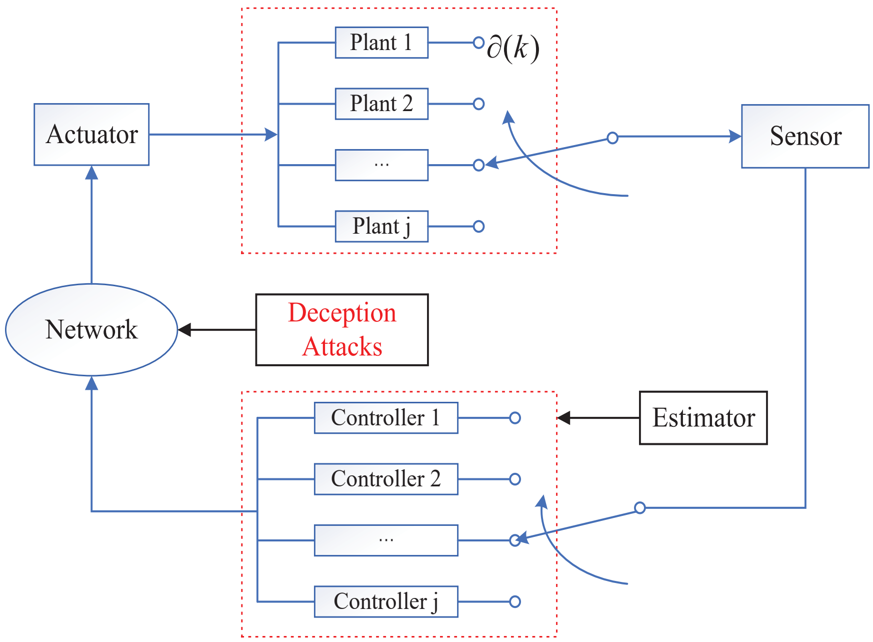


**Figure 1.** Framework of a switched system under deception attacks.

Section 5.

*Notation:* $\mathbb{N}$ signifies the set of integers, and the space of $n \times m$ real matrices is represented as $\mathbb{R}^{n\times m}$. In a symmetric block matrix, the symbol $*$ represents the symmetric term. The symbol $\|\cdot\|$ stands for the Euclidean vector norm. The $\|\cdot\|_F$ is Frobenius norm.

## 2. PROBLEM FORMULATION AND PRELIMINARIES

### 2.1. System description

Consider a class of discrete-time switched systems with time delay given as

$$\begin{cases} x(k+1) = A_{\partial(k)}x(k) + A^l_{\partial(k)}x(k-l) + B_{\partial(k)}u(k), \\ x(k) = \wp(k), \quad -l \le k \le 0, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^p$ denote the state vector and the control input, respectively. $l > 0$ is the time delay of the switched system, $\wp(k)$ is the initial condition for $x(k)$. $\partial\,[0,\infty) \longrightarrow \mathcal{K} = \{1, 2, \cdots, j\}$ is the switching signal, in which $j \in \mathbb{N}$ is the quantity of subsystems. $A_j$, $A^l_j$ and $B_j$ are known constant matrices with appropriate dimensions.

Let the switching instants be $\{k_d, d \in \mathbb{N}\}$, then the time interval between two adjacent switching instants is depicted as $k_{d+1} - k_d \ge \varphi$, where $\varphi$ is the minimum DT.

### 2.2. Adaptive neural state feedback controller

The C-A transmission channel is influenced by deception attacks, and the authenticity of the data transmission is modified, as shown in Figure 1. Therefore, the actuator input is established as follows:

$$u(k) = \tilde{u}(k) + h(x(k)), \tag{2}$$

where $\tilde{u}(k)$ denotes the control signal, and $h(x(k))$ indicates the signal of deception attacks.

Assuming the signal of deception attacks is the nonlinear function that depends on the system state, then the nonlinear function $h(x(k))$ is expressed as

$$h(x(k)) = M^T L(x(k)) + \gamma(x(k)),\ x(k) \in \Lambda \subset \mathbb{R}^n, \tag{3}$$

where $\Lambda$ expresses a compact set, $M \in \mathbb{R}^{m\times p}$ is a weight matrix in which $m$ denotes the number of nodes, and $\gamma(x(k))$ stands for the approximation error with $\|\gamma(x(k))\| \le \bar{\gamma}\,\|x(k)\|$. Then, the basis function

$$L(x(k)) = [\ \ell_1(x(k)) \quad \ell_2(x(k)) \quad \cdots \quad \ell_m(x(k))\ ]^T,$$

satisfies $\|L(\cdot)\| \le L_{\max}$ in which $L_{\max}$ is a positive constant. $\ell_i(x(k))$ has the form as follows:

$$\ell_i(x(k)) = \exp\left(-\frac{\|x(k) - f_i\|^2}{\upsilon_i}\right),$$

where $\upsilon_i$ and $f_i$ denote the width and center of function $\ell_i(x(k))$, respectively. And $f_i = [f_{i1} \ \ f_{i2} \ \ \cdots \ \ f_{in}]^T$.

The challenge of defending against deception attacks is recognizing unknown attack signals. In this article, the NN technique is applied to approximate the deception attacks, thereby compensating for the adverse effect of deception attacks on the switched system. Therefore, the adaptive neural controller can be constructed as follows:

$$\tilde{u}(k) = K_{\partial(k)}x(k) - \hat{h}(x(k)), \tag{4}$$

where $K_{\partial(k)}$ is the controller gains to be determined, and $\hat{h}(x(k))$ is the estimation of the deception attack signal $h(x(k))$.

Therefore, the estimation $\hat{h}(x(k))$ of the deception attack signal $h(x(k))$ is constructed as

$$\hat{h}(x(k)) = \hat{M}^T(k)L(x(k)), \tag{5}$$

where $\hat{M}(k)$ is the estimation of $M$. Then, $\hat{M}(k)$ is designed as

$$\hat{M}(k+1) = -\lambda\hat{M}(k) + \hat{M}(k) - \omega\eta(k)x^T(k)S^T, \tag{6}$$

where $\lambda > 0, \omega > 0$ are two constants, $S \in \mathbb{R}^{n\times p}$ is an adjustment parameter and

$$\eta(k) = L(x(k)) \Big/ (1 + \|L(x(k))\|^2\|Sx(k)\|^2). \tag{7}$$

Let the error $\tilde{M}(k) = \hat{M}(k) - M$, then we have

$$\tilde{M}(k+1) = -\lambda\hat{M}(k) + \tilde{M}(k) - \omega\eta(k)x^T(k)S^T. \tag{8}$$

By means of the above analysis, the system is represented as

$$\begin{cases} x(k+1) = A_{\partial(k)}x(k) + A^l_{\partial(k)}x(k-l) + B_{\partial(k)}K_{\partial(k)}x(k) - B_{\partial(k)}\tilde{M}^T(k)L(x(k)) + B_{\partial(k)}\gamma(x(k)), \\ x(k) = \wp(k), \quad -l \le k \le 0. \end{cases} \tag{9}$$

**Remark 1** *Different from the result[17], the NN technique is employed to estimate and approximate the attack signal rather than constrain it to a bounded signal. The approximation capability of the NN is effectively utilized to estimate the attack signal, counteracting the malicious effects brought about by attacks.*

## 3. MAIN RESULTS

In this section, the sufficient conditions for the uniform and ultimately bounded of the switched system (9) are given subject to deception attacks. Meanwhile, the controller gains are designed.

**Theorem 1** *For given constants $0 < \varpi < 1$, $\lambda > 0$, $h > 0$, $\tau > 0$, $\varsigma > 0$, $\bar{\gamma} > 0$ and $\kappa > 1$, if there exist matrices $P_j \in \mathbb{R}^{n\times n} > 0$, $Q_j \in \mathbb{R}^{n\times n} > 0$, $F_{j1} \in \mathbb{R}^{n\times n} > 0$,$F_{j2} \in \mathbb{R}^{n\times n} > 0$, such that*

$$\varpi\kappa^{\frac{1}{\varphi}} < 1, \tag{10}$$

$$\vartheta < 0, \tag{11}$$

$$\begin{bmatrix} \Psi_{j1} & \Psi_{j2} \\ * & -I_p \end{bmatrix} < 0, \tag{12}$$

$$P_j \le \kappa P_i, \; Q_j \le \kappa Q_i, \tag{13}$$

*where*

$$\begin{aligned}
\Psi_{j1} =& e_1^T(Q_j - \varpi P_j)e_1 + e_2^T P_j e_2 - e_3^T \varpi^l Q_j e_3 - \tau e_4^T e_4 + (e_1^T F_{j1} + e_2^T F_{j2})(F_{j1}e_1 + F_{j2}e_2) \\
&+ \mathrm{He}\{[e_1^T F_{j1} + e_2^T F_{j2}][(A_j + B_j K_j)e_1 - e_2 + A_j^l e_3 - B_j e_4]\}, \\
\Psi_{j2} =& \bar{\gamma} e_1^T B_j, \\
e_1 =& \begin{bmatrix} I_n & 0_{n\times n} & 0_{n\times n} & 0_{n\times m} \end{bmatrix}, \\
e_2 =& \begin{bmatrix} 0_{n\times n} & I_n & 0_{n\times n} & 0_{n\times m} \end{bmatrix}, \\
e_3 =& \begin{bmatrix} 0_{n\times n} & 0_{n\times n} & I_n & 0_{n\times m} \end{bmatrix}, \\
e_4 =& \begin{bmatrix} 0_{m\times n} & 0_{m\times n} & 0_{m\times n} & I_m \end{bmatrix}, \\
\vartheta =& h(1 - \varpi + \frac{1}{4\varsigma} - \lambda) + \tau L_{\max}^2,
\end{aligned}$$

*Then, the switched system (9) is bounded.*

**Proof 1** *Construct the Lyapunov functional method as*

$$V_{\partial(k)}(k) = V_{\partial(k)}^1(k) + hV^2(k), \tag{14}$$

*where $V_{\partial(k)}^1(k) = x^T(k)P_{\partial(k)}x(k) + \sum_{d=k-l}^{k-1} \varpi^{(k-d-1)}x^T(d)Q_{\partial(k)}x(d)$ and $V^2(k) = \mathrm{Tr}\{\tilde{M}^T(k)\tilde{M}(k)\}$. For convenience, let $V_{\partial(k)}^1(k) = V_j^1(k)$. Then the difference of $V_j^1(k)$ along the trajectories of the switched system (9) can be obtained as*

$$\begin{aligned}
V_j^1(k+1) - \varpi V_j^1(k) =& x^T(k+1)P_j x(k+1) + \sum_{d=k+1-l}^{k} \varpi^{k-d}x^T(d)Q_j x(d) \\
&- \varpi x^T(k)P_j x(k) - \varpi \sum_{d=k-l}^{k-1} \varpi^{k-d-1}x^T(d)Q_j x(d) \\
=& x^T(k)(Q_j - \varpi P_j)x(k) + x^T(k+1)P_j x(k+1) - \varpi^l x^T(k-l)Q_j x(k-l).
\end{aligned} \tag{15}$$

*Calculating the difference of $V^2(k)$ along (8), we have*

$$\begin{aligned}
V^2(k+1) - V^2(k) =& \mathrm{Tr}\{\tilde{M}^T(k+1)\tilde{M}(k+1)\} - \mathrm{Tr}\{\tilde{M}^T(k)\tilde{M}(k)\} \\
=& \mathrm{Tr}\{\lambda^2\hat{M}^T(k)\hat{M}(k) - 2\lambda\hat{M}^T(k)\tilde{M}(k) + 2\lambda\omega\hat{M}(k)\eta(k)Sx(k) - 2\omega\tilde{M}(k)\eta(k)Sx(k)\} \\
&+ \omega^2\|\eta(k)\|^2\|Sx(k)\|^2.
\end{aligned} \tag{16}$$

*According to the (7), we can get*

$$\|\eta(k)\|^2\|Sx(k)\|^2 \leq \frac{1}{4}.$$

*Therefore, it is possible to obtain*

$$\begin{aligned}&\mathrm{Tr}\{2\lambda\omega\hat{M}(k)\eta(k)Sx(k)\} \leq \lambda\mathrm{Tr}\{\hat{M}^T(k)\hat{M}(k)\}+\frac{\lambda\omega^2}{4},\\&-\mathrm{Tr}\{2\omega\tilde{M}(k)\eta(k)Sx(k)\} \leq \frac{\mathrm{Tr}\{\tilde{M}^T(k)\tilde{M}(k)\}}{4\varsigma}+\varsigma\omega^2,\\&\mathrm{Tr}\{-2\lambda\hat{M}^T(k)\tilde{M}(k)\} = \mathrm{Tr}\{\lambda M^TM-\lambda\tilde{M}^T(k)\tilde{M}(k)-\lambda\hat{M}^T(k)\hat{M}(k)\}.\end{aligned} \tag{17}$$

*Combining (16) and (17), it can yield*

$$\Delta V^2(k) \leq (\frac{1}{4\varsigma}-\lambda)\left\|\tilde{M}(k)\right\|_F^2+\delta, \tag{18}$$

*where* $\delta = \lambda\,\|M\|_F^2 + (\varsigma + \frac{1+\lambda}{4})\omega^2$.

*For the free-weighting matrices* $F_{j1}$ *and* $F_{j2}$*, we can deduce that*

$$\begin{aligned}0 =&2[x^T(k)F_{j1}+x^T(k+1)F_{j2}]\\&\times[-x(k+1)+A_jx(k)+A_j^lx(k-l)+B_jK_jx(k)-B_j\tilde{M}^T(k)L(x(k))+B_j\gamma(x(k))]\\=&2\xi^T(k)[e_1^TF_{j1}+e_2^TF_{j2}][(A_j+B_jK_j)e_1-e_2+A_j^le_3-B_je_4]\xi(k)\\&+2\xi^T(k)[e_1^TF_{j1}+e_2^TF_{j2}]B_j\gamma(x(k)),\end{aligned} \tag{19}$$

*where* $\xi(k) = [x^T(k), x^T(k+1), x^T(k-l), L^T(x(k))\tilde{M}(k)]^T$*, we have*

$$\begin{aligned}0 =&\xi^T(k)\mathrm{He}\{[e_1^TF_{j1}+e_2^TF_{j2}][(A_j+B_jK_j)e_1-e_2+A_j^le_3-B_je_4]\}\xi(k)\\&+2\xi^T(k)[e_1^TF_{j1}+e_2^TF_{j2}]B_j\gamma(x(k)),\end{aligned} \tag{20}$$

*in which*

$$\begin{aligned}2\xi^T(k)[e_1^TF_{j1}+e_2^TF_{j2}]B_j\gamma(x(k)) \leq&\xi^T(k)(e_1^TF_{j1}+e_2^TF_{j2})(F_{j1}e_1+F_{j2}e_2)\xi(k)\\&+\xi^T(k)(\bar{\gamma}^2e_1^TB_jB_j^Te_1)\xi(k).\end{aligned} \tag{21}$$

*For any positive constant* $\tau$*, one obtains*

$$-\tau L^T(x(k))\tilde{M}(k)\tilde{M}^T(k)L(x(k))+\tau L_{\max}^2\mathrm{Tr}\{\tilde{M}^T(k)\tilde{M}(k)\} \geq 0. \tag{22}$$

*Combining (14), (15), (18) and (22), we can obtain*

$$V_j^1(k+1)-\varpi V_j^1(k)+h(V^2(k+1)-\varpi V^2(k)) \leq \xi^T(k)[\Psi_{j1}+\Psi_{j2}^T\Psi_{j2}]\xi(k)+\vartheta\left\|\tilde{M}(k)\right\|_F^2+h\delta \tag{23}$$

*Based on the sufficient condition (11) and the Schur complement of sufficient condition (12), we can deduce that*

$$V(k+1) \leq \varpi V(k)+\bar{\delta}, \tag{24}$$

*where* $\bar{\delta} = h\delta$.

*For* $k \in [k_d, k_{d+1}]$*, according to condition* (13)*, we can obtain*

$$\begin{aligned}
V(k) \leq & \varpi^{k-k_d} V(k_d) + \bar{\delta}[1 + \varpi + \cdots \varpi^{k-k_d-1}], \\
\leq & \kappa \varpi^{k-k_d} V(k_d^-) + \bar{\delta}[1 + \varpi + \cdots \varpi^{k-k_d-1}], \\
\leq & \kappa \varpi^{k-k_{d-1}} V(k_{d-1}) + \bar{\delta}[1 + \varpi + \cdots \varpi^{k-k_d-1}] \\
& + \kappa \bar{\delta}[\varpi^{k-k_d} + \varpi^{k-k_d+1} + \cdots + \varpi^{k-k_{d-1}+1}], \\
& \vdots \\
\leq & \kappa^{D(0,k)} \varpi^k V(0) + \bar{\delta}[1 + \varpi + \cdots \varpi^{k-k_d-1}] \\
& + \kappa \bar{\delta}[\varpi^{k-k_d} + \varpi^{k-k_d+1} + \cdots + \Omega^{k-k_d-1}] \\
& + \cdots + \kappa^{D(0,k)} \bar{\delta}[\varpi^{k-k_1} + \varpi^{k-k_1+1} + \cdots + \varpi^{k-k_0-1}], \\
\leq & \kappa^{D(0,k)} \varpi^k V(0) + [\bar{\mu}_0 + \bar{\mu}_1 + \cdots \bar{\mu}_{D(0,k)}],
\end{aligned} \tag{25}$$

*in which*

$$\begin{aligned}
\bar{\mu}_0 = & \bar{\delta}[1 + \varpi + \cdots \varpi^{k-k_d-1}] = \frac{\bar{\delta}(1 - \varpi^{k-k_d})}{1 - \varpi} \leq \frac{\bar{\delta}}{1 - \varpi}, \\
\bar{\mu}_1 = & \kappa \bar{\delta}[\varpi^{k-k_d} + \varpi^{k-k_d+1} + \cdots + \varpi^{k-k_{d-1}-1}] \leq \frac{\kappa \bar{\delta} \varpi^{k-k_d}(1 - \varpi^{k_d-k_{d-1}})}{1 - \varpi} \leq \frac{\kappa \bar{\delta} \Omega^{\varphi}}{1 - \varpi}, \\
\bar{\mu}_2 = & \kappa^2 \bar{\delta}[\varpi^{k-k_{d-1}} + \varpi^{k-k_{d-1}+1} + \cdots + \varpi^{k-k_{d-2}-1}] \leq \frac{\kappa^2 \bar{\delta} \varpi^{k-k_{d-1}}(1 - \varpi^{k_{d-1}-k_{d-2}})}{1 - \varpi} \\
\leq & \frac{\kappa^2 \bar{\delta}(1 - \varpi)^{k-k_{d-1}}}{1 - \varpi} \leq \frac{\kappa^2 \bar{\delta}(1 - \varpi)^{2\varphi}}{1 - \varpi}, \\
& \vdots \\
\bar{\mu}_{D(0,k)} = & \kappa^{D(0,k)} \bar{\delta}[\varpi^{k-k_1} + \varpi^{k-k_1+1} + \cdots + \varpi^{k-k_0-1}] \leq \frac{\kappa^{D(0,k)} \bar{\delta} \varpi^{k-k_1}(1 - \varpi^{k-k_1})}{1 - \varpi} \\
\leq & \frac{\kappa^{\frac{k}{\varphi}} \bar{\delta}(1 - \varpi)^{k-k_1}}{1 - \varpi} \leq \frac{\kappa^d \bar{\delta}(1 - \varpi)^{d\varphi}}{1 - \varpi}.
\end{aligned} \tag{26}$$

*Based on* (25) *and* (26)*, we can get*

$$V(k) \leq (\varpi \kappa^{\frac{1}{\varphi}})^k V(0) + \tilde{\delta}, \tag{27}$$

*where* $\tilde{\delta} = \sum_{m=0}^{d} \frac{\kappa^m \bar{\delta}(1-\varpi)^{m\varphi}}{1-\varpi}$.

*The sufficient condition* (10) *is satisfied, implying that the switched system* (9) *is bounded. The proof is finished.*

**Theorem 2** *For given constants* $0 < \varpi < 1$, $\lambda > 0$, $h > 0$, $\tau > 0$, $\varsigma > 0$, $\alpha_j > 0$, $\varepsilon_p > 0$, $\varepsilon_q > 0$, $\bar{\gamma} > 0$ *and* $\kappa > 1$*, if there exist matrices* $\tilde{P}_j \in \mathbb{R}^{n \times n} > 0$, $\tilde{Q}_j \in \mathbb{R}^{n \times n} > 0$, $Z_{j1} \in \mathbb{R}^{n \times n} > 0$, $Y_j \in \mathbb{R}^{n \times n}$*, such that*

$$\begin{bmatrix} \tilde{\Psi}_1 & \tilde{\Psi}_2 \\ * & -I_p \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} -\kappa \tilde{P}_i & Z_{i1} \\ * & \bar{P}_j \end{bmatrix} \leq 0, \quad \begin{bmatrix} -\kappa \tilde{Q}_i & Z_{i1} \\ * & \bar{Q}_j \end{bmatrix} \leq 0, \tag{29}$$

*where*

$$\begin{aligned}\tilde{\Psi}_1 =& e_1^T(\tilde{Q}_j - \varpi\tilde{P}_j)e_1 + e_2^T\tilde{P}_je_2 - e_3^T\varpi^l\tilde{Q}_je_3 - \tau e_4^Te_4 + (e_1^T + \alpha e_2^T)(e_1 + \alpha e_2)\\ &+ \mathrm{He}\{[e_1^T + \alpha e_2^T][(A_j + B_jK_j)Z_{j1}e_1 - Z_{j1}e_2 + A_j^lZ_{j1}e_3 - B_je_4]\},\\ \tilde{\Psi}_2 =& \bar{\gamma}e_1^TZ_{j1}B_j,\\ \bar{P}_j =& \varepsilon_p^2\tilde{P}_j - 2\varepsilon_pZ_{j1},\\ \bar{Q}_j =& \varepsilon_q^2\tilde{Q}_j - 2\varepsilon_qZ_{j1},\end{aligned}$$

Then, switched system (9) is bounded. Moreover, the controller gains are designed as

$$K_j = Y_jZ_{j1}^{-1}.$$

**Proof 2** *Let*

$$\begin{cases} Z_{j1} = F_{j1}^{-1},\\ F_{j2} = \alpha_jF_{j1}.\end{cases}$$

*According to Theorem 1, we define* $\Phi_j = [Z_{j1}, Z_{j1}, Z_{j1}, I_p, I_p]$*, pre-multiplying and post-multiplying both sides of inequality* (12) *with* $\Phi_j$*. We assume* $\tilde{P}_j = Z_{j1}P_jZ_{j1}$*,* $\tilde{Q}_j = Z_{j1}Q_jZ_{j1}$*.*

*In addition, it is easy to see that*

$$\begin{aligned}-P_j^{-1} = -Z_{j1}\tilde{P}_j^{-1}Z_{j1} \le \varepsilon_p^2\tilde{P}_j - 2\varepsilon_pZ_{j1},\\ -Q_j^{-1} = -Z_{j1}\tilde{Q}_j^{-1}Z_{j1} \le \varepsilon_q^2\tilde{Q}_j - 2\varepsilon_qZ_{j1}.\end{aligned} \tag{30}$$

*Similarly, pre-multiplying and post-multiplying both sides of inequalities in* (13) *with* $Z_{j1}$ *and using the Schur complement lemma, then* (29) *is ensured by* (13) *holds. The inequality* (12) *can be guaranteed by* (28)*. This proof is completed.*

## 4. NUMERICAL EXAMPLES

Consider a discrete-time switched system with time delay (1) as follows:

$$A_1 = \begin{bmatrix} -0.2 & 1\\ 0.1 & 0.6\end{bmatrix}, A_1^l = \begin{bmatrix} 0.01 & 0.3\\ 0.1 & 0\end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & 0\end{bmatrix}^T,$$
$$A_2 = \begin{bmatrix} -0.2 & 0.5\\ -0.8 & 0.8\end{bmatrix}, A_2^l = \begin{bmatrix} 0.01 & 0\\ 0.1 & 0.3\end{bmatrix}, B_2 = \begin{bmatrix} 0.2 & 0.1\end{bmatrix}^T.$$

When $l = 1$, the parameters are given as $\bar{\gamma} = 1.5$, $\varepsilon_q = 0.4$, $\varepsilon_q = 0.5$, $\alpha_1 = 1.8$, $\alpha_2 = 2$, $\varpi = 0.9$, $\kappa = 1.8$, $\varphi = 6$, $\tau = 0.1$, $L_{\max}^2 = 81$, $\lambda = 0.6$, $\varsigma = 100$, $h = 1400$. Then, $\vartheta = -40.4$ and $\varpi\kappa^{\frac{1}{\varphi}} = 0.9926$ can be obtained through calculations, so conditions (10) and (11) in Theorem 1 are satisfied. By solving Theorem 2, we obtain

$$K_1 = \begin{bmatrix} 1.7615 & -5.4597\end{bmatrix}, \quad K_2 = \begin{bmatrix} 3.8378 & -3.9402\end{bmatrix}.$$

The attack signal is set as $h(x(k)) = \sqrt{x_1^2(k) + x_2^2(k)} - 6\sin(x_1(k))$. The NN technique applied to approximate the attack signal $h(x(k))$ consists of 81 nodes whose centers $f_i$ are evenly spaced in $\underbrace{[-6,6]\times\cdots[-6,6]}_{81}$ and the widths are $\upsilon_i = 1$ with $i \in \langle 81\rangle$.

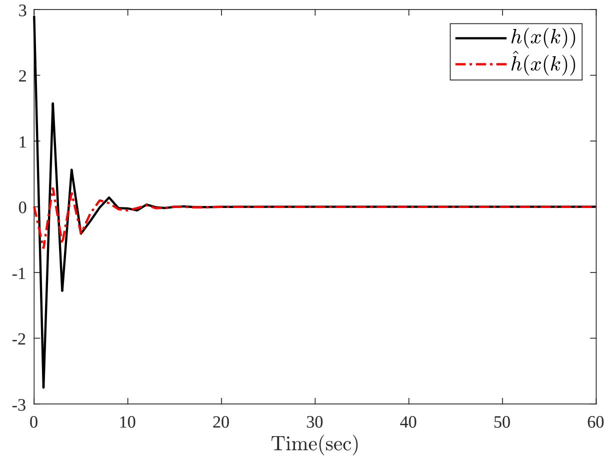


**Figure 2.** Trajectories of the attack signal and its estimation signal where $\lambda = 0.5$ and $\omega = 0.7$.

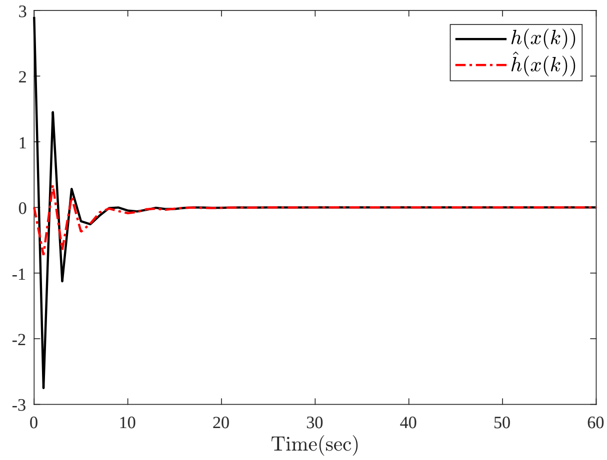


**Figure 3.** Trajectories of the attack signal and its estimation signal where $\lambda = 0.5$ and $\omega = 0.8$.

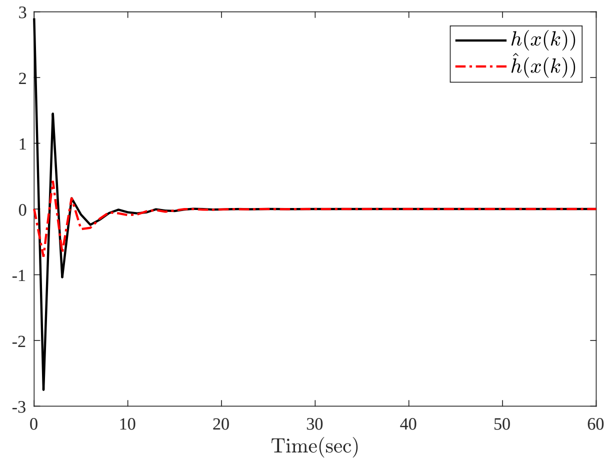


**Figure 4.** Trajectories of the attack signal and its estimation signal where $\lambda = 0.6$ and $\omega = 0.8$.

Let the initial conditions be $x(0) = [-0.4 \;\; 0.4]^T$, $\hat{M}(0) = 0_{81\times 1}$ and $W = [-1 \;\; 1]$. The parameters $\lambda$ and $\omega$ play an important role in the updated law of $\hat{M}$. To demonstrate the impact of parameters on performance,

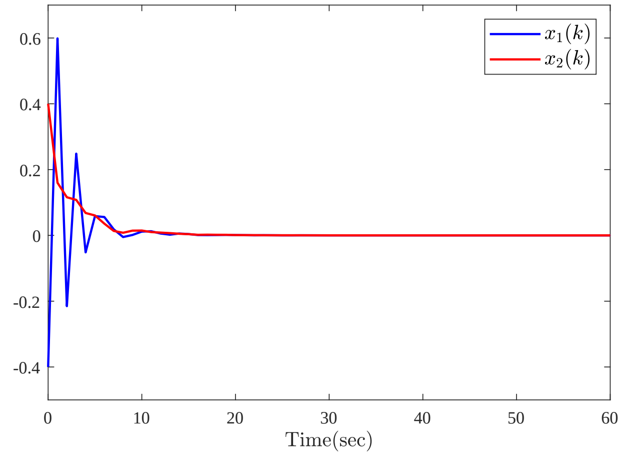


**Figure 5.** The state responses $x(k)$.

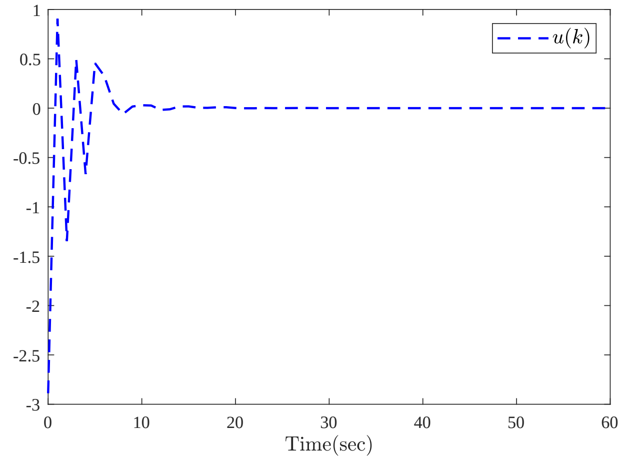


**Figure 6.** The control signal $u(k)$.

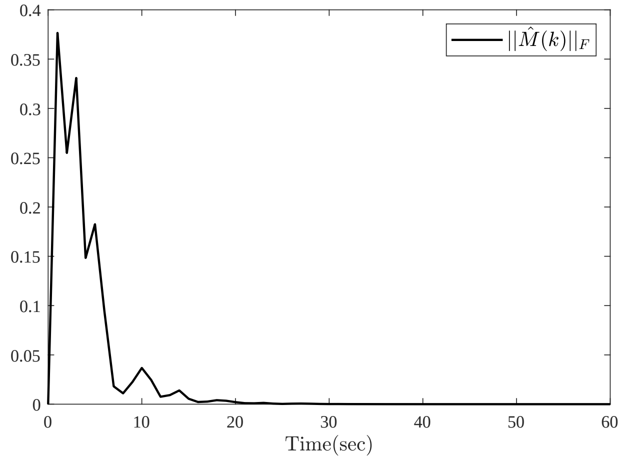


**Figure 7.** The responses of adaptive parameter $\|\hat{M}(k)\|_F$.

we introduce a performance metric $\tilde{h}$, where $\tilde{h} = \sum_{k=1}^{60} \left|h(x(k)) - \hat{h}(x(k))\right|$. Therefore, it is necessary to choose

**Table 1.** $\tilde{h}$ **for different** $(\lambda, \omega)$

| $(\lambda, \omega)$ | $(0.5, 0.8)$ | $(0.5, 0.7)$ | $(0.6, 0.8)$ |
|---|---|---|---|
| $\tilde{h}$ | 7.3253 | 8.2059 | 6.9905 |

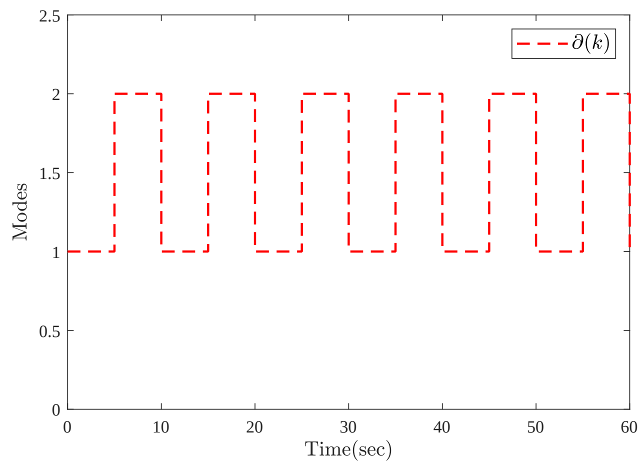


**Figure 8.** The switching modes of the switched system.

appropriate values for $\lambda$ and $\omega$. Trajectories for the attack signal $h(x(k))$ and its estimation $\hat{h}(x(k))$ at $\lambda = 0.5$ and $\omega = 0.7$ are depicted in Figure 2, while the corresponding paths at $\lambda = 0.5$ and $\omega = 0.8$ are shown in Figure 3. Additionally, trajectories for $\lambda = 0.6$ and $\omega = 0.8$ are presented in Figure 4. From Table 1, it can be observed that a smaller value of $\tilde{h}$ corresponds to better parameter convergence. Then, the parameters are selected as $\lambda = 0.6$ and $\omega = 0.8$.

Based on the parameters given above, we have the following simulation results. Figure 5 and Figure 6 depict the state responses $x(k)$ and the control signal $u(k)$, which indicate the ability of the controller to resist the adverse effects of deception attacks to bring the system to stability. The response of adaptive parameter $\|\hat{M}(k)\|_F$ is illustrated in Figure 7. The switching modes of the switched system are shown in Figure 8. Therefore, the feasibility of the proposed method is demonstrated.

## 5. CONCLUSIONS

In this article, we have addressed the problem of adaptive neural control for discrete-time switched systems with time delay and deception attacks. To counteract the impact of deception attacks, the NN technique has been employed to approximate the attack signal. An adaptive state controller, which incorporates the attack estimation signal and the approximation error, effectively eliminating malicious influences, has been proposed. Sufficient conditions for the boundedness of the switched system under deception attacks have been attained by using the Lyapunov functional and the DT methods. Additionally, a design strategy for controller gains has been presented. Finally, the feasibility of the proposed method has been demonstrated through a numerical example. Future work will focus on filtering and memory feedback control for hybrid systems.

## DECLARATIONS

### Authors' contributions

Methodology, Writing - original draft, investigation, and conceptualization: Zhao D
Conceptualization and visualization: Tian Y

Conceptualization, investigation, and visualization: Zhao N

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Not applicable.

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## Conflicts of interest
All authors declared that there are no conflicts of interest.

## Ethical approval and consent to participate
Not applicable.

## Consent for publication
Not applicable.



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