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Dynamic event-triggered practical stabilization of random suspension system based on immersion and invariance

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Abstract

This article investigates the practical stabilization problem of random quarter-car active suspension systems. An adaptive dynamic event-trigger strategy is proposed to stabilize the states of vehicle suspension in response to system uncertainty and controller area network resource constraints. Moreover, the model of random active suspension systems is extended to the general random robot systems; the controller is developed with the aid of a double dynamic surface filter, immersion and invariance (I&I) techniques, and event-triggered mechanisms. The results show that the semi-global stability of error systems is achieved, and there are some improvements in triggering times and adaptive estimation performance under the control framework. Finally, simulation comparison results are provided to prove the advantages of the proposed scheme.

Keywords: Random active suspension system, immersion and invariance, double dynamic surface filter, dynamic event-triggered control

1. INTRODUCTION

With the rapid development of science and technology, vehicles have become a commonly used means of transportation. The suspension system is a force transmission connection device between the vehicle body and the wheels. Due to their ability to effectively alleviate impacts and body vibrations caused by uneven road surfaces to ensure ride comfort, the suspension systems have received great attention from researchers [1,2]. As a type of suspension system, active suspension systems (ASS) were often used to improve vehicle damping characteristics. Subsequently, some novel controllers were designed to achieve the expected performance of ASS, such as robust sampled-data H_∞ Control [3], adaptive fuzzy sliding mode control [4], saturated control [5], fault-tolerant control [6,7], performance constraint control [8–10]. The adaptive finite time filtering control problem of a nonlinear quarter ASS subjected to actuator failure was studied in [7]. A new integral barrier Lyapunov function is proposed in [10]; on the one hand, it satisfies the vertical displacement constraint condition, and on the other hand, it stabilizes the suspension position within the neighborhood of the expected position in a finite time.

Due to the lack of accurate identification of model parameters and partial measurement of the system states, nonlinear adaptive control must cope with high levels of uncertainty. For ASS with uncertain parameters, adaptive schemes are proposed in [8,11,12]. As is well known, the two basic methods for dealing with nonlinearity are certain equivalence (CE) [13] and immersion and invariance (I&I) [14]. In principle, CE is to design Lyapunov functions for the error dynamic equation and obtain the update law of parameter estimation, which takes the form of an error nonlinear integrator, while I&I indirectly introduced unknown parameters into the estimation with the aid of state correction terms, which means incorporating system dynamics into lower order expected behavior to achieve control objectives [15]. Whereafter, the I&I technology has been verified in practical robot systems, such as quadrotors [16–18], balls, beam systems [19], and so on. In addition, as a typical nonlinear system, ASS inevitably suffers from the problem of explosion of terms caused by the analytic calculation on the command derivative of stabilizing function. To overcome this difficulty, the command derivative was approximated by a dynamic surface filter (DSF) [20], and then a command filter with compensated dynamics was proposed by [21], which revealed the relation of command filter control with traditional backstepping approaches. This technology is further applied to theoretical development [22–24] and practical systems [25–27].

With the continuous deepening of research on network systems and electronic control components, saving limited bandwidth resources has become an important topic. Usually, it is required that the sensors, controllers, and actuators of the suspension system can continuously obtain information from each other. It is worth mentioning that event-triggered control (ETC) is an effective way to reduce the communication burden on the controller area network [28] proposed a new adaptive event-triggered tracking scheme that can not only offset severe uncertainty but also ensure any pre-set tracking accuracy. Nowadays, a number of results have been presented on ASS under the ETC condition [29–31].

In addition, the event-triggered communication mechanism is widely used in multi-agent systems [32–35]. Only when the event-triggering conditions are satisfied the information of each agent will propagate to adjacent agents, greatly reducing the communication burden [34] proposed two different position controllers to handle the time-varying formation problem of multi-rotor systems based on an event-triggered integral sliding mode method. Further consideration still needs to be given to the collaborative control problem of unmanned aerial vehicle systems in the case of hybrid active interactions between humans [36]. This is a question worth exploring, once again discussing the suspension system.

In real life, the impact of rough roads on vehicles cannot be ignored. In order to improve passenger comfort, the suspension system must absorb road vibrations and prevent them from being transmitted to the vehicle body. Therefore, there are sufficient reasons to consider the control of random nonlinear ASS on rough road surfaces. The random model of ASS was given in [37]; however, it does not consider the issues of unknown parameters and reduced system signal transmission frequency. Motivated by the aforementioned papers, the dynamic event-triggered practical stabilization of uncertain random quarter-car ASS is devoted and even extended to general random nonlinear systems to deal with a class of robot control problems.

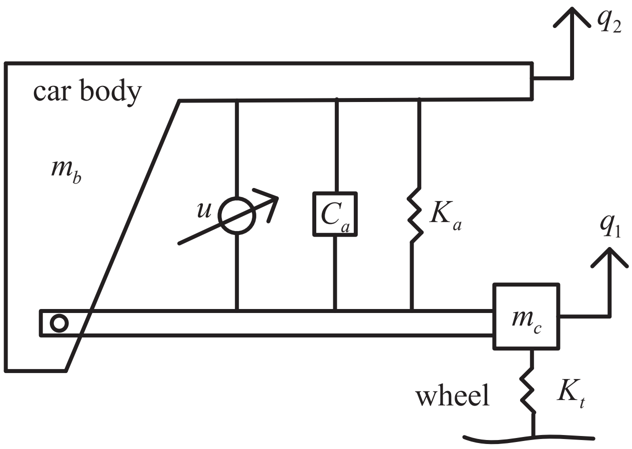

Figure 1. Quarter active suspension model.

The main contributions of this paper are as follows: First, the application of I&I techniques and dynamic event-trigger mechanisms (DETM) to random nonlinear systems has achieved practical stabilization of random ASS.

Compared with[11], it is not necessary to invoke CE and Lyapunov functions. I&I indirectly introduced unknown parameters into the estimation by state correction terms, avoiding the coupling between estimation law and error terms from the perspective of nonlinear regulation, which, to some extent, improves estimation performance.

Second, the double DSF proposed in this paper removes the compensation signals and achieves awesome properties calculated by double integration. Another advantage of DSF is that compared to[38,39], it eliminates the boundedness assumption of prior filter errors and provides a reasonable stability analysis process.

The paper is divided into six parts. The first section is the introduction of relevant background knowledge of the research content. In the second section, the random suspension system and problem formulation are presented. The adaptive dynamic event-trigger controller is designed in the third section, and the performance analysis is discussed in the fourth section. The simulation results are provided in the fifth section, and the last section is a conclusion.

Notations: $\mathbb{R}^n$ denotes the real n-dimensional space. $|x| = (\sum_i x_i^2)^{\frac{1}{2}}$ is the distance norm of vector x. The partial differentiation of the variable is $\frac{\partial y}{\partial x}$. X^T represents the transposition of a vector or matrix. $B(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| \leq r\}$ stands for a ball with x_0 as the center and r as radius.

2. MODEL DESCRIPTION AND PROBLEM SETUP

2.1. Random active suspension model

As presented in Figure 1, the quarter-car active suspension in a random environment is shown in this paper. The masses of the wheel and car body are m_c and m_b, respectively, with displacements of q_1 and q_2. The hydraulic controller between the wheel and the body is u, and the linear spring and nonlinear damper are in parallel, where the spring coefficient is K_a and the damping coefficient is C_a. The wheel is regarded as a linear spring with a spring coefficient of K_t.

Borrowed from[37], the random model of ASS by the aid of Lagrangian principles and relative motions is constructed as

$$
\begin{aligned}
m_c\ddot{q}_1 - K_a(q_2 - q_1) + K_t q_1 &= C_a(\dot{q}_2 - \dot{q}_1) - u - m_c\xi_1, \\
m_b\ddot{q}_2 + K_a(q_2 - q_1) &= -C_a(\dot{q}_2 - \dot{q}_1) + u - m_b\xi_2.
\end{aligned}
\tag{1}
$$

Let $x_1 = m_c q_1 + m_b q_2$, $x_2 = \dot{x}_1$, $x_3 = q_2 - q_1$, $x_4 = \dot{x}_3$, note that C_a is an unknown coefficient, denoted as $\theta = C_a$. The impact of rough road surface on the wheels and body is considered as a stationary process ξ_1 and ξ_2, respectively.

Then, the random model (1) with an unknown damping coefficient is shown as

$$
\begin{aligned}
\dot{x}_1 = &\quad x_2, \\
\dot{x}_2 = &\quad -\tfrac{K_t}{m_c+m_b}x_1 + \tfrac{K_t m_b}{m_c+m_b}x_3 - m_c\xi_1 - m_b\xi_2, \\
\dot{x}_3 = &\quad x_4, \\
\dot{x}_4 = &\quad f_4 + \phi\theta + (\tfrac{1}{m_c} + \tfrac{1}{m_b})u - \xi_2 + \xi_1,
\end{aligned}
\tag{2}
$$

where $f_4 = -K_a\frac{m_b+m_c}{m_b m_c}x_3 + K_t\frac{(x_1 - m_b x_3)}{m_c(m_b+m_c)}$, $\phi = -\frac{m_b+m_c}{m_b m_c}x_4$.

2.2 Problem setup

In response to the problems of unknown parameters and high communication requirements in traditional robot control systems, the objective of this paper is to design an adaptive DETM for random suspension systems (2) to achieve semi-global practical stabilization. In order to solve a type of control problem similar to a random suspension system, system (2) is organized into the following general forms of random systems for controller design, that is

$$
\begin{aligned}
\dot{x}_i = &\quad f_i(\bar{x}_i) + x_{i+1} + \phi_i(\bar{x}_i)\theta_i + g_i(\bar{x}_i)\xi_i,\ i = 1, \cdots, n-1, \\
\dot{x}_n = &\quad f_n(x) + u + \phi_n(x)\theta_n + g_n(x)\xi_n,
\end{aligned}
\tag{3}
$$

where $\bar{x}_i = [x_1, \ldots, x_i]^T$, $u \in \mathbb{R}$, and $x_1 \in \mathbb{R}$ are the state, input, and output of the system (3), respectively. $\theta_i \in \mathbb{R}$ denotes an unknown parameter. The corresponding functions $f_i(\bar{x}_i), g_i(\bar{x}_i), \phi_i(\bar{x}_i)$ are locally Lipschitz in x. Before achieving the objective of this paper, several assumptions on random disturbance $\xi(t) = [\xi_1, \cdots, \xi_n]^T$ and functions $g_i(x), \phi_i(x)$ need to be given.

Assumption 1 For any $\varepsilon > 0$ and $M > 0$, the random disturbance $\xi(t)$ satisfies

$$
\sup_{t_0 \le s \le t} E|\xi(t)|^2 \le M,\ \forall t \ge t_0. \tag{4}
$$

Assumption 2 There exist constants $d_g, d_\phi > 0$ such that

$$
g_i(x)^2 \le d_g,\ \phi_i(x)^2 \le d_\phi. \tag{5}
$$

The purpose of introducing (4) is to consider that the disturbance energy is bounded in practical situations[40]. In order to better select parameters in stability analysis, (5) was cited, and similar considerations were also used in coordinated control systems[41]. To facilitate the controller design, the following inequalities are presented.

Lemma 1[32] For any vectors $x, y \in \mathbb{R}^n$, any scalars $\varepsilon > 0$, $p > 1$ and $q = \frac{p}{p-1}$, there holds

$$
x^T y \le \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q.
$$

Lemma 2[10] For all $z \in \mathbb{R}$, $\varepsilon > 0$ and $\zeta = 0.2785$, the inequality of hyperbolic tangent holds

$$
0 < |z| - z\tanh\left(\frac{z}{\varepsilon}\right) \leqslant e^{-(\zeta+1)}\varepsilon.
$$

3. ADAPTIVE EVENT-TRIGGERED CONTROLLER DESIGN

Compared with traditional adaptive laws, adaptive techniques based on I&I do not require linear parameterization conditions or CE and can better compensate for parameter uncertainty. In order to avoid explosive terms generated by recursive processes, inspired by[20,21], a novel double DSF is proposed:

$$\begin{aligned} \dot{\eta}_{i1} &= k_i(-\eta_{i1}+\eta_{i2}), \\ \dot{\eta}_{i2} &= k_i(-\eta_{i2}+\alpha_i), \\ \alpha_i^c &= \eta_{i1},\ \ i=1,\cdots,n-1 \end{aligned} \tag{6}$$

with filter time constant $k_i > 0$, which means that $\dot{\alpha}_i^c = k_i(\eta_{i2}-\eta_{i1})$ can be served as a substitute of $\dot{\alpha}_i$. Referring to coordinate transformation, the tracking error can be rewritten as

$$z_i = x_i - \alpha_{i-1}^c = x_i - \alpha_{i-1} + \Delta_{i-1},\ i=1,\cdots,n \tag{7}$$

where $\alpha_0^c = 0$, $\Delta_i = \alpha_i - \alpha_i^c$, and $\Delta_0 = 0$ are specified for the convenience of subsequent discussions. In order to deal with unknown items, we define estimation errors based on I&I as

$$\tilde{\theta}_i = \hat{\theta}_i - \theta_i + \rho_i(x_i),\ i=1,\cdots,n \tag{8}$$

where $\rho_i(x_i)$ is a smooth function that will be designed later.

Step 1. From (3), (7), and (8), the dynamic of z_1-subsystem is expressed as

$$\dot{z}_1 = f_1 + z_2 + \alpha_1 - \Delta_1 + g_1\xi_1 - \dot{\alpha}_0^c + \phi_1(\hat{\theta}_1 - \tilde{\theta}_1 + \rho_1).$$

In light of (3) and (8), one obtains

$$\dot{\tilde{\theta}}_1 = \dot{\hat{\theta}}_1 + \frac{\partial\rho_1}{\partial x_1}(f_1 + x_2 + g_1\xi_1 + \phi_1(\hat{\theta}_1 - \tilde{\theta}_1 + \rho_1)).$$

Select the fictitious control and adaptive law as

$$\begin{aligned} \alpha_1 &= -c_1z_1 - f_1 - \phi_1(\hat{\theta}_1 + \rho_1), \\ \dot{\hat{\theta}}_1 &= -\frac{\partial\rho_1}{\partial x_1}(f_1 + x_2 + \phi_1(\hat{\theta}_1+\rho_1)) - \lambda_1\delta_1(\hat{\theta}_1+\rho_1), \end{aligned} \tag{9}$$

where $\rho_1 = \frac{\lambda_1}{d_o}\int_0^{x_1}\phi_1(s)\mathrm{d}s$, and $\delta_1, c_1, d_o, \lambda_1 > 0$. Substituting (9) into the derivative of the Lyapunov function $V_1 = \frac{1}{2}z_1^2 + \frac{1}{2\lambda_1}\tilde{\theta}_1^2$ yields:

$$\begin{aligned} \dot{V}_1 =& z_1(-c_1z_1 + z_2 - \Delta_1 + g_1\xi_1 - \phi_1\tilde{\theta}_1) + \tilde{\theta}_1(-\frac{1}{d_0}\phi_1(g_1\xi_1 - \phi_1\tilde{\theta}_1) - \delta_1(\hat{\theta}_1+\rho_1)) \\ \le& -(c_1 - \frac{d_o}{4} - \frac{d_od_g}{4})z_1^2 + z_1(z_2-\Delta_1) + \frac{2}{d_o}\xi_1^2 - (\frac{\delta_1}{2} - \frac{d_\phi d_g}{4d_o})\tilde{\theta}_1^2 + \frac{1}{2}\delta_1\theta_1^2, \end{aligned} \tag{10}$$

Which used Young inequality (see Lemma 1) and (5):

$$\begin{aligned} z_1g_1\xi_1 \le& \frac{d_od_g}{4}z_1^2 + \frac{1}{d_o}\xi_1^2, \\ -z_1\phi_1\tilde{\theta}_1 \le& \frac{d_o}{4}z_1^2 + \frac{1}{d_o}\phi_1^2\tilde{\theta}_1^2, \\ \frac{1}{d_0}\phi_1\tilde{\theta}_1g_1\xi_1 \le& \frac{d_\phi d_g}{4d_o}\tilde{\theta}_1^2 + \frac{1}{d_o}\xi_1^2, \\ -\tilde{\theta}_1\delta_1(\hat{\theta}_1+\rho_1) \le& -\frac{1}{2}\delta_1\tilde{\theta}_1^2 + \frac{1}{2}\delta_1\theta_1^2. \end{aligned} \tag{11}$$

Step i. ($i = 2,\cdots,n-1$) Following the above design process, we can get the virtual control and adaptive law

$$\begin{aligned} \alpha_i &= -c_iz_i - z_{i-1} - f_i - \phi_i(\hat{\theta}_i+\rho_i) + \dot{\alpha}_{i-1}^c, \\ \dot{\hat{\theta}}_i &= -\frac{\partial\rho_i}{\partial x_i}(f_i + x_{i+1} + \phi_i(\hat{\theta}_i+\rho_i)) - \lambda_i\delta_i(\hat{\theta}_i+\rho_i), \end{aligned} \tag{12}$$

where $\rho_i = \frac{\lambda_i}{d_o}\int_0^{x_i}\phi_i(s)\mathrm{d}s$ and $\delta_i, c_i, \lambda_i > 0$. Together with (12) and Lyapunov function $V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2\lambda_i}\tilde{\theta}_i^2$, similar to (10), the dynamic of $\dot{V}_i$ can be formulated as

$$\begin{aligned}\dot{V}_i \leq &-\sum_{j=1}^{i}(c_j - \frac{d_o}{4} - \frac{d_o d_g}{4})z_j^2 + \sum_{j=1}^{i} z_{j-1}\Delta_{j-1} + z_i(z_{i+1} - \Delta_i)\\ &+\sum_{j=1}^{i}\frac{2j}{d_o}\xi_j^2 - \sum_{j=1}^{i}(\frac{\delta_j}{2} - \frac{d_\phi d_g}{4d_o})\tilde{\theta}_j^2 + \sum_{j=1}^{i}\frac{1}{2}\delta_j\theta_j^2.\end{aligned} \tag{13}$$

Step n. Construct the Lyapunov function

$$V_{z\theta} = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2\lambda_n}\tilde{\theta}_n^2, \tag{14}$$

where $\lambda_n > 0$, whose derivative can be organized as

$$\dot{V}_{z\theta} = \dot{V}_{n-1} + z_n(f_n + u + g_n\xi_n + \phi_n\theta_n - \dot{\alpha}_{n-1}^c) + \frac{1}{\lambda_n}\tilde{\theta}_n(\dot{\hat{\theta}}_n + \frac{\partial\rho_n}{\partial x_n}(f_n + u + g_n\xi_n + \phi_n\theta_n)). \tag{15}$$

Naturally, the adaptive law is given as

$$\dot{\hat{\theta}}_n = -\frac{\partial\rho_n}{\partial x_n}(f_n + u + \phi_n(\hat{\theta}_n + \rho_n) - \lambda_n\delta_n(\hat{\theta}_n + \rho_n), \tag{16}$$

where $\rho_n = \frac{\lambda_n}{d_o}\int_0^{x_n}\phi_n(s)\mathrm{d}s$ and $\delta_n > 0$. Drawing on the idea in [10,42], the following DETM is given

$$\begin{aligned}u(t) &= \nu(t_k), \forall t \in [t_k, t_{k+1}),\\ t_{k+1} &= \inf\{t > t_k \big| |h(t)| \geq \beta(t) + \epsilon\},\\ \dot{\beta}(t) &= -b\beta(t) - |h(t)| + \epsilon,\end{aligned} \tag{17}$$

where $\nu(t)$ is the control auxiliary function to be given later, and $\beta(t)$ is an internal dynamic variable. $h(t) = \nu(t) - u(t)$ is measurement errors between the sampling control and current control, the parameters $\epsilon, b > 0$. Once the mechanism (17) is triggered, the control is updated to $\nu(t_{k+1})$; otherwise, it will remain at a constant value $\nu(t_k)$.

With the aid of (17), it can come to a conclusion that $|\nu(t) - u(t)| \leq \beta(t) + \epsilon$ is always held on intervals $[t_k, t_{k+1})$, it is easy to find two continuous functions $\omega(t) \in [-1, 1]$ to satisfy

$$u(t) = \nu(t) - \omega(t)(\beta(t) + \epsilon), \tag{18}$$

where $\omega(t_k) = 0, \omega(t_{k+1}) = \pm 1$. The auxiliary control input function is selected as

$$\nu(t) = \alpha_n - (\beta(t) + \bar{\epsilon})\tanh(\frac{z_n(\beta(t) + \bar{\epsilon})}{\sigma}), \tag{19}$$

where $\alpha_n = -c_n z_n - z_{n-1} - f_n - \phi_n(\hat{\theta}_n + \rho_n) + \dot{\alpha}_{n-1}^c$, parameters $c_n, \sigma > 0$ and $\bar{\epsilon} > \epsilon$. Replacing (16)-(19) in (15) yields

$$\begin{aligned}\dot{V}_{z\theta} = &\dot{V}_{n-1} + z_n(-c_n z_n - (\beta(t) + \bar{\epsilon})\tanh(\frac{z_n(\beta(t) + \bar{\epsilon})}{\sigma}) - \omega(t)(\beta(t) + \epsilon) + g_n\xi_n - \phi_n\tilde{\theta}_n)\\ &+ \tilde{\theta}_n(-\frac{1}{d_o}\phi_n(g_n\xi_n - \phi_n\tilde{\theta}_n) - \delta_n(\hat{\theta}_n + \rho_n)),\end{aligned}$$

note that the item based on Lemma 2 satisfies

$$-z_n(\beta(t) + \bar{\epsilon})\tanh(\frac{z_n(\beta(t) + \bar{\epsilon})}{\sigma}) \leq 0.2785\sigma - |z_n(\beta(t) + \bar{\epsilon})|,$$

and $|z_n(\beta(t)+\bar{\epsilon})| > |z_n\omega(t)(\beta(t)+\epsilon)|$. Following the calculation process in (11) and combining (13), the dynamic of $V_{z\theta}$ turns to

$$\dot{V}_{z\theta} \leq -\sum_{j=1}^{n}(c_j - \frac{d_o}{4} - \frac{d_o d_g}{4})z_j^2 + \sum_{j=1}^{n-1} z_j\Delta_j + \sum_{j=1}^{n}\frac{1}{2}\delta_j\theta_j^2 - \sum_{j=1}^{n}(\frac{\delta_j}{2} - \frac{d_\phi d_g}{4d_o})\tilde{\theta}_j^2 + \sum_{j=1}^{n}\frac{2}{d_o}\xi_j^2 + 0.2785\sigma. \quad (20)$$

Remark 1 The DETM (17) adjusts the update interval and control accuracy based on a control signal. Compared with the static event-triggered mechanism (SETM), the proposed controller has a dynamically adjusted trigger threshold and a shorter execution interval[42]. Due to the presence of the internal dynamic variable $\beta(t)$, triggers will not infinitely activate at $u = 0$.

4. PERFORMANCE ANALYSIS

The stability properties of the random nonlinear system (3) with adaptive DETM controllers (17) and DSF techniques are summarized in this section. Let us make some preparations for stability analysis. For $i = 1, \cdots, n-1$, define the transform

$$\begin{aligned} e_{i1} &= \eta_{i1} - \alpha_i, \\ e_{i2} &= \eta_{i2} - \alpha_i. \end{aligned} \quad (21)$$

Denote $e = [e_1^T, \cdots, e_{n-1}^T]^T$, and $e_i = [e_{i1}, e_{i2}]^T$. From (6) and (21), we have

$$\begin{aligned} \dot{e}_{i1} &= k_i(e_{i2} - e_{i1}) - \dot{\alpha}_i, \\ \dot{e}_{i2} &= -k_i e_{i2} - \dot{\alpha}_i. \end{aligned} \quad (22)$$

Unfolding $\dot{\alpha}_i$ gives

$$\begin{aligned} \dot{\alpha}_i =& -c_i\dot{z}_i - \dot{z}_{i-1} + \ddot{\alpha}_{i-1}^c - \dot{f}_i + \dot{\phi}_i(\hat{\theta}_i + \rho_i) + \phi_i(\dot{\hat{\theta}}_i + \dot{\rho}_i) \\ =& -c_i\dot{z}_i - \dot{z}_{i-1} - \sum_{j=1}^{i}\frac{\partial f_i}{\partial x_j}\dot{x}_j + \phi_i(\dot{\hat{\theta}}_i + \dot{\rho}_i) + \ddot{\alpha}_{i-1}^c + \sum_{j=1}^{i}\frac{\partial \phi_i}{\partial x_j}(\hat{\theta}_i + \rho_i)\dot{x}_j, \end{aligned} \quad (23)$$

where $z_0 = 0$, $\dot{z}_i = -c_i z_i - z_{i-1} + z_{i+1} + e_{i1} + g_i\xi_i - \phi_i\tilde{\theta}_i$, $\dot{x}_j = -c_j z_j - z_{j-1} + z_{j+1} + e_{j1} - \phi_j\tilde{\theta}_j + g_j\xi_j + k_{j-1}(e_{j-1,2} - e_{j-1,1})$, $\dot{\hat{\theta}}_i = \frac{\lambda_i}{d_o}\phi_i(f_i + x_{i+1} + \phi_i(\hat{\theta}_i + \rho_i) - \lambda_i\delta_i(\hat{\theta}_i + \rho_i)$, $\dot{\rho}_i = \frac{\lambda_i}{d_o}\phi_i(x_i)\dot{x}_i$, and

$$\ddot{\alpha}_{i-1}^c = \begin{cases} 0, & i = 1, \\ -k_{i-1}^2(2e_{i-1,2} - e_{i-1,1}), & i = 2, \cdots, n. \end{cases}$$

For the e-subsystem, select Lyapunov function $V_e = \frac{1}{2}\sum_{i=1}^{n-1} e_i^T e_i$, combining with (14), the following whole Lyapunov function is obtained:

$$V(\chi) = V_{z\theta}(z, \theta) + V_e(e), \quad (24)$$

which satisfies $a_1|\chi|^2 \leq V(\chi) \leq a_2|\chi|^2$ with $\chi = [z^T, e^T, \tilde{\theta}]^T$, $a_1 = \frac{1}{2}\min\{1, \frac{1}{\lambda_i}\}$, and $a_2 = \frac{1}{2}\max\{1, \frac{1}{\lambda_i}\}$.

For any constant $v_0 > 0$, define $r_0 = \sqrt{\frac{v_0}{a_2}}$, the initial value contained within a compact set $B(0, r_0) = \{\chi : |\chi| \leq r_0\}$. Since f_i, ϕ_i, ρ_i are smooth, then $\frac{\partial f_i}{\partial x_j}, \frac{\partial \phi_i}{\partial x_j}, \frac{\partial \rho_i}{\partial x_j}$ are bounded in the ball $B(0, r_0)$, which, along with (23), implies that constants $b_{ij} > 0 (j = 1, \cdots, 4)$ can always be found such that

$$|\dot{\alpha}_i| \leq b_{i1}|\bar{e}_i| + b_{i2}|\bar{z}_{i+1}| + b_{i3}|\tilde{\theta}_i| + b_{i4}|\bar{\xi}_i|,$$

where $b_{ij} (j = 1, \cdots, 4)$ depends on r_0, and only b_{i1} depends on $\bar{k}_{i-1}$. With the aid of Young inequality, for any constant $d_o > 0$, there holds

$$|(e_{i1} + e_{i2})\dot{\alpha}_i| \leq \kappa_i|e_i|^2 + \frac{1}{d_o}(|\bar{e}_i|^2 + |\bar{z}_{i+1}|^2 + |\bar{\xi}_i|^2 + |\tilde{\theta}_i|),$$

where $\kappa_i = d_o(b_{i1}^2 + b_{i2}^2 + b_{i3}^2 + b_{i4}^2)$, which results in

$$\begin{aligned}\dot{V}_e &= \sum_{i=1}^{n-1}(e_{i1}^T\dot{e}_{i1} + e_{i2}^T\dot{e}_{i2}) \\ &\leq -\sum_{i=1}^{n-1}(\tfrac{1}{2}k_i e_i^T e_i + (e_{i1}+e_{i2})\dot{\alpha}_i) \\ &\leq -\sum_{i=1}^{n-1}\left(\tfrac{1}{2}k_i - \kappa_i\right)|e_i|^2 + \sum_{i=1}^{n-1}\tfrac{1}{d_o}(|\bar{e}_i|^2 + |\bar{z}_{i+1}|^2 + |\bar{\xi}_i|^2 + |\tilde{\theta}_i|^2) \\ &\leq -\sum_{i=1}^{n-1}(\tfrac{1}{2}k_i - \kappa_i)|e_i|^2 + \tfrac{n-1}{d_o}(|e|^2 + |z|^2 + |\xi|^2 + |\tilde{\theta}|^2),\end{aligned} \tag{25}$$

where κ_i, depending on $\bar{k}_{i-1}$, is independent of k_i. Substituting (20) and (25) into the derivative of (24) yields that for $V \leq v_0$, one has

$$\begin{aligned}\dot{V} \leq\ & -\sum_{i=1}^{n}(c_i - \tfrac{d_o}{4} - \tfrac{d_o d_g}{4})z_i^2 + \sum_{i=1}^{n-1}z_i\Delta_i - \sum_{i=1}^{n-1}(\tfrac{1}{2}k_i - \kappa_i)|e_i|^2 + \tfrac{n-1}{d_o}(|e|^2 + |z|^2 + |\xi|^2 + |\tilde{\theta}|^2) + \tfrac{2n}{d_o}|\xi|^2 \\ & -\sum_{i=1}^{n}(\tfrac{\delta_i}{2} - \tfrac{d_\phi d_g}{4d_o})|\tilde{\theta}_i|^2 + \sum_{i=1}^{n}\tfrac{1}{2}\delta_i\theta_i^2 + 0.2785\sigma \\ \leq\ & -\sum_{i=1}^{n}(c_i - \tfrac{d_o}{2} - \tfrac{n-1}{d_o} - \tfrac{d_o d_g}{4})|z_i|^2 - \sum_{i=1}^{n-1}(\tfrac{1}{2}k_i - \kappa_i - \tfrac{n}{d_o})|e_i|^2 - \sum_{i=1}^{n}(\tfrac{\delta_i}{2} - \tfrac{d_\phi d_g}{4d_o} - \tfrac{n-1}{d_o})|\tilde{\theta}_i|^2 + \tfrac{n+1}{d_o}|\xi|^2 \\ & + \sum_{i=1}^{n}\tfrac{1}{2}\delta_i\theta_i^2 + 0.2785\sigma.\end{aligned} \tag{26}$$

Let $c_i = c_{i1} + c_{i2}$, $k_i = k_{i1} + k_{i2}$, $\delta_i = \delta_{i1} + \delta_{i2}$, and define $c = \min\{2c_{i1}, k_{i1}, \delta_{i1}\}$ and $\bar{c} = 2\min\{c_{i2} - \frac{d_o}{2} - \frac{n-1}{d_o} - \frac{d_o d_g}{4}, \frac{1}{2}k_{i2} - \kappa_i - \frac{n}{d_o}, \frac{1}{2}\delta_{i2} - \frac{d_\phi d_g}{4d_o} - \frac{n-1}{d_o}\}$. Denote $\zeta_1 = \frac{n+1}{d_o}$ and $\zeta_2 = \sum_{i=1}^{n}\frac{1}{2}\delta_i\theta_i^2 + 0.2785\sigma$. To let $c > 0, \bar{c} \geq 0$, requirements on parameters are

$$\begin{aligned}& c_{i1} > 0,\ c_{i2} \geq \tfrac{d_o}{2} + \tfrac{n-1}{d_o} + \tfrac{d_o d_g}{4}, \\ & k_{i1} > 0,\ k_{i2} \geq 2\kappa_i + \tfrac{2n}{d_o}, \\ & \delta_{i1} > 0,\ \delta_{i2} \geq \tfrac{d_\phi d_g}{2d_o} + \tfrac{2(n-1)}{d_o},\end{aligned} \tag{27}$$

where κ_i depends on $b_{ij}(j = 1, \cdots, 5)$, and b_{ij} depends on v_0. When $c > 0$ and $\bar{c} \geq 0$ are required, it is concluded from (26) that

$$V \leq v_0 \Rightarrow \dot{V} \leq -cV + \zeta_1|\xi|^2 + \zeta_2. \tag{28}$$

The error system is summarized as follows

$$\begin{cases}\dot{e}_{i1} &= k_i(e_{i2} - e_{i1}) - \dot{\alpha}_i, \\ \dot{e}_{i2} &= -k_i e_{i2} - \dot{\alpha}_i, \\ \dot{z}_i &= -c_i z_i - z_{i-1} - \phi_i\tilde{\theta}_i + z_{i+1} + e_{i1} + g_i\xi_i, \\ \dot{\tilde{\theta}}_1 &= \frac{\partial\rho_1}{\partial x_1}(g_i\xi_i - \phi_1\tilde{\theta}_i) - \lambda_i\delta_i(\hat{\theta}_i + \rho_i).\end{cases} \tag{29}$$

Based on the above argument, we intend to summarize the following results.

Theorem 1 Consider the random ASS system described by (3), under Assumption 1 and Assumption 2, with fictitious control law and adaptive update law in (9), (12), (16), the event-triggering rule (17), satisfying parameter requirements (27), the closed-loop system (29) has the following performance:

(1). the system (29) is semi-globally noise to state practically stable in probability (SGNSpS-P);
(2). all signals in (29) are bounded in probability;
(3). the desired performance of the stabilization error z_1 can be made arbitrarily small by adjusting parameters;
(4). the inter-execution intervals $(t_{k+1} - t_k)$ have lower bounds.

Proof It is clear that the closed-loop system is SGNSpES-P along (24) and (28), which means that z, e, and $\tilde{\theta}$ are all bounded in probability. From (7) and (9), the signals $x_1, \alpha_1, \hat{\theta}_1$ are bounded in probability. Recalling $x_i = z_i + \alpha_{i-1} + e_{i-1,1}$ and (12), (16), the signals $x_i, \alpha_i, \hat{\theta}_i$ are got. Based on (17) and (19), the boundedness in probability of the final control u is obtained.

Regarding (28), with the aid of Gronwall inequality, we arrive at

$$P\{z_1^2 \leq |\chi_0|^2 e^{-c(t-t_0)} + 2d_M\} \geq 1 - \varepsilon,\ |\chi_0| \leq r_0,$$

for any $\varepsilon > 0$, where $d_M = \frac{1}{c}(\zeta_1 M + \zeta_2)$ can be adjusted arbitrarily for any ε by tuning d_o, c large enough.

In the following, it needs to be demonstrated that the designed trigger control can avoid the Zeno phenomenon. The derivative of (19) becomes

$$\dot{v}(t) = \dot{\alpha}_n - (\dot{\beta}(t) + \bar{\epsilon})\tanh\left(\frac{z_n(\beta(t)+\bar{\epsilon})}{\sigma}\right) - (\beta(t)+\bar{\epsilon})\frac{\dot{z}_n\beta + \dot{\beta}z_n}{\sigma\cosh^2\left(\frac{z_n(\beta(t)+\bar{\epsilon})}{\sigma}\right)},$$

which is bounded in the compact set $B(0, r_0)$, thus, $\dot{v}(t) \leq \bar{v}$ with $\bar{v} > 0$ is established. In time interval $[t_k, t_{k+1})$ of (17), it can obtain that $|h(t)| = |u(t) - v(t)| \leq \beta(t) + \epsilon \leq \bar{h}$. Further, the derivative of $h^2(t)$ becomes $|(h^2(t))'| = 2|h(t)| \cdot |\dot{v}(t)| \leq 2\bar{h}\bar{v}$. When $t \in [t_k, t_{k+1}]$, it can be asserted that $(u(t_k^+) - v(t_k^+))^2 = 0$, $(u(t_{k+1}^-) - v(t_{k+1}^-))^2 \geq \epsilon^2$, therefore, $\epsilon^2 \leq (u(t_{k+1}^-) - v(t_{k+1}^-))^2 - (u(t_k^+) - v(t_k^+))^2 = \int_{t_k^+}^{t_{k+1}^-} |h(s)^2| \mathrm{d}s \leq 2\bar{h}\bar{v}(t_{k+1} - t_k)$, it can come to a conclusion that $t_{k+1} - t_k \geq \frac{\epsilon^2}{2\bar{h}\bar{v}}$, which implies that the inter-execution intervals have lower bounds.

Remark 2 A command filter proposed in [21] is replaced with a double DSF (6) in this paper. This results in a simpler form of Lyapunov functions used in $V_e(e)$, which lightens the burden of stability analysis, while the most important advantage of this change is that filter gains are replaced by k_i to get more freedom in stability analysis; otherwise, the last inequality in (27) may have no solution.

5. SIMULATION

For the random ASS (1), following the previous DETM controller design with I&I in section 3, we can arrive at

$$\begin{aligned}
\alpha_1 =&\ -c_1 z_1,\\
\alpha_2 =&\ \tfrac{m_b+m_c}{K_t m_b}\left(-c_2 z_2 - z_1 + \tfrac{K_t}{m_b+m_c} z_1 + \dot{\eta}_{11}\right),\\
\alpha_3 =&\ -c_3 z_3 - \tfrac{K_t m_b}{m_b+m_c} z_2 + \dot{\eta}_{21},\\
\alpha_4 =&\ \tfrac{m_b m_c}{m_b+m_c}\left(-c_4 z_4 - z_3 - f_4 - \phi(\hat{\theta}+\rho) + \dot{\eta}_{31}\right),\\
\dot{\hat{\theta}} =&\ -\tfrac{\partial\rho}{\partial x_4}\left(f_4 + \phi(\hat{\theta}+\rho)\right) - \lambda\delta(\hat{\theta}+\rho),\\
v(t) =&\ \alpha_4 - (\beta(t)+\bar{\epsilon})\tanh\left(\tfrac{z_4(\beta(t)+\bar{\epsilon})}{\sigma}\right),\\
u(t) =&\ v(t_k), \forall t \in [t_k, t_{k+1}),\\
t_{k+1} =&\ \inf\{t > t_k \big| |h(t)| \geq \beta(t) + \epsilon\},\\
\dot{\beta}(t) =&\ -b\beta(t) - |h(t)| + \epsilon,
\end{aligned} \tag{30}$$

where $\rho = \frac{1}{2}\frac{\lambda}{d_o}\frac{m_b+m_c}{m_b m_c}x_4^2$, $\frac{\partial\rho}{\partial x_4} = \frac{\lambda}{d_o}\frac{m_b+m_c}{m_b m_c}x_4$. By choosing the Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{4} z_i^2 + \frac{1}{2\lambda}\tilde{\theta}^2 + \frac{1}{2}\sum_{i=1}^{3} e_i^T e_i, \tag{31}$$

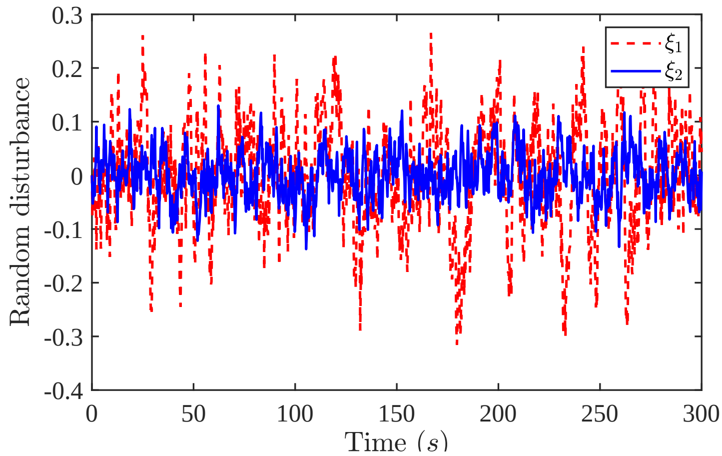

Figure 2. Random disturbances.

the dynamic of (31), along with the design of controllers (30) and the handling of inequalities, leads to

$$\begin{aligned}\dot{V} \leq & -(c_1 - \frac{3}{d_o} - \frac{d_0}{4})|z_1|^2 - (c_2 - \frac{d_c d_o}{4} - \frac{3}{d_o})|z_2|^2 - (c_3 - \frac{3}{d_o} - \frac{d_0}{4})|z_3|^2 - (c_4 - \frac{3}{d_o} - \frac{d_0}{2})|z_4|^2 \\ & -(\frac{\delta}{2} - \frac{d_\phi}{2d_o} - \frac{3}{d_o})|\tilde{\theta}|^2 - \sum_{i=1}^{3}(\frac{1}{2}k_i - \kappa_i - \frac{3}{d_o})|e_i|^2 + \frac{5}{d_o}(|\xi_1|^2 + |\xi_2|^2) + \frac{1}{2}\delta\theta^2 + \frac{0.2785\sigma(m_b+m_c)}{m_b m_c}, \\ \leq & -cV + \zeta_1|\xi|^2 + \zeta_2,\end{aligned}$$

where $d_c = m_b^2 + m_c^2 + (\frac{K_t m_b}{m_b+m_c})^2$, $d_\phi = (\frac{m_b+m_c}{m_b m_c})^2$, $\xi = \max\{\xi_1, \xi_2\}$. In order to better select parameters and adjust the tracking effect, let $c_i = c_{i1} + c_{i2}$, $k_i = k_{i1} + k_{i2}$, $\delta = \delta_1 + \delta_2$, and define $c = \min\{2c_{i1}, k_{i1}, \delta_1\}$ and $\bar{c} = 2\min\{c_{12} - \frac{d_o}{4} - \frac{3}{d_o}, c_{22} - \frac{d_c d_o}{4} - \frac{3}{d_o}, c_{32} - \frac{d_o}{4} - \frac{3}{d_o}, c_{42} - \frac{d_o}{2} - \frac{3}{d_o}, \frac{1}{2}k_{i2} - \kappa_i - \frac{3}{d_o}, \frac{1}{2}\delta_2 - \frac{d_\phi}{2d_o} - \frac{3}{d_o}\}$. Denote $\zeta_1 = \frac{5}{d_o}$ and $\zeta_2 = \frac{1}{2}\delta\theta^2 + \frac{0.2785\sigma(m_b+m_c)}{m_b m_c}$. To let $c > 0, \bar{c} \geq 0$, requirements on parameters are

$$\begin{aligned} & c_{i1} > 0, k_{i1} > 0, \delta_1 > 0, c_{12} > \frac{d_o}{4} + \frac{3}{d_o}, c_{22} > \frac{d_c d_o}{4} + \frac{3}{d_o}, \\ & c_{32} > \frac{d_o}{4} + \frac{3}{d_o}, c_{42} > \frac{d_o}{2} + \frac{3}{d_o}, k_{i2} > 2\kappa_i + \frac{6}{d_o}, \delta_2 > \frac{d_\phi}{d_o} + \frac{6}{d_o}. \end{aligned} \tag{32}$$

Theorem 2 For random suspension systems (1) with control law (30) satisfying parameter requirements (32), the closed-loop system is SGNSpS-P; the stabilization error $x_1 = z_1$ can be made arbitrarily small, and the triggering mechanism will not cause the Zeno phenomenon.

Following Theorem 1, the previous properties can be obtained directly; the specific proof process was omitted.

Next, the practical stabilization problem is simulated to demonstrate the merit of the obtained feedback controller (30). Choose system parameters as $m_b = m_c = 0.4$, $K_t = 2.5$, $K_a = 4$, $C_a = 0.2$. With the initial conditions $q_1(0) = 0.01$, $q_2(0) = 0.02$, $\dot{q}_1(0) = 0.5$, $\dot{q}_2(0) = 0.4$, $\theta(0) = 0.2$, $\beta(0) = 0.02$, $\eta_{11}(0) = \eta_{21}(0) = \eta_{31}(0) = 0.01$, $\eta_{21}(0) = \eta_{22}(0) = \eta_{32}(0) = 0.1$, we have the following simulation results of (30), as shown in Figure 2-Figure 4.

Followed by section VI of[40], the disturbances ξ_1, ξ_2 can be regarded as zero-mean widely stationary processes resulting from limited bandwidth white noise, and the corresponding parameters are as follows: the sample time is $t_c = 0.2$, the noise powers are $A_1 = 0.2 and A_2 = 0.1$, and corresponding gain coefficients are $b_1 = 0.5 and b_2 = 0.2$. The random disturbances $\xi_i, (i = 1, 2)$ are presented in Figure 2, which shows that the second-order moment of the disturbance is bounded in assumption 1.

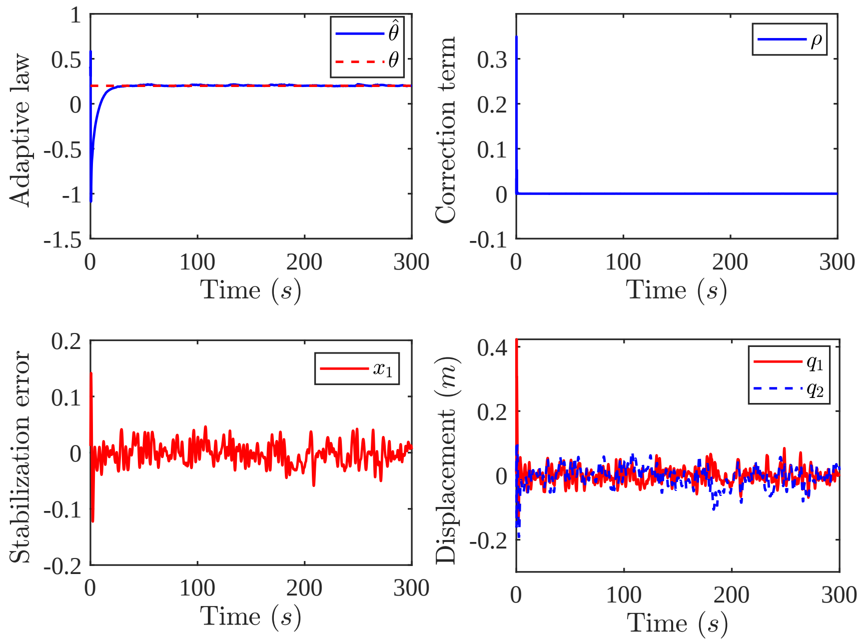

Figure 3. Estimation and stabilization results.

In addition, the parameters in controller are $c_1 = 0.8, c_2 = 0.6, c_3 = 0.4, c_4 = 0.6, \delta = \lambda = 0.4, d_o = 5$, $b = 0.2, \sigma = 4, \epsilon = 0.04, \bar{\epsilon} = 0.05$, the coefficients in DSFs are $k_1 = k_2 = 30, k_3 = 25$. As shown in Figure 3, the adaptive estimation result based on I&I and correction term $\rho(t)$ is presented. Meanwhile, it can be found that the stabilization error $z_1 = x_1$ also converges to a small neighborhood of the origin under the DETM controller, and the states of the suspension system q_1, q_2 are also practically stable. This indicates that the system goals are achieved under the designed controller.

The continuous control $v(t)$ and ETC $u(t)$ are reflected in Figure 4; as time goes by, event triggers continue to occur, and the event-triggered interval is constantly changing, obviously, the designed dynamic event-triggered controller avoids Zeno behavior. Note that the internal dynamic variable $\beta(t)$ is introduced in DETM, which is also shown in Figure 4.

Below, we will conduct a simulation comparison between DETM and SETM under I&I and DSF. As shown in Figure 5, the relative threshold triggering strategy in [43,44] is demonstrated, that is

$$\begin{aligned} v(t) = &\ -(1+\beta)(\alpha_4 \tanh(\tfrac{z_4\alpha_4}{\sigma}) + \bar{\epsilon}\tanh(\tfrac{z_4\bar{\epsilon}}{\sigma})), \\ u(t) = &\ v(t_k), \forall t \in [t_k, t_{k+1}), \\ t_{k+1} = &\ \inf\{t > t_k \big| |u(t) - v(t)| \geq \beta|u(t)| + \epsilon\}, \end{aligned}$$

where $\beta \in (0,1), \epsilon > 0, \bar{\epsilon} > \frac{\epsilon}{1-\beta}$. Comparing Figure 4 and Figure 5, the control inputs of the two types of control strategies fluctuate within the same range, and the adaptive estimation and stabilization effects are both well. However, the DETM gives the next execution time greater than the SETM. Comparing the stem and leaf plots in these two figures, it can draw a conclusion that the DETM has fewer triggering times, which better illustrates the advantages of DETM in Remark 1.

In order to illustrate the difference in adaptive effects between I&I and CE under DETM and DSF, following[11,12], an adaptive law is designed as

$$\dot{\hat{\theta}} = -\lambda z_4 \phi + \lambda\delta\hat{\theta},$$

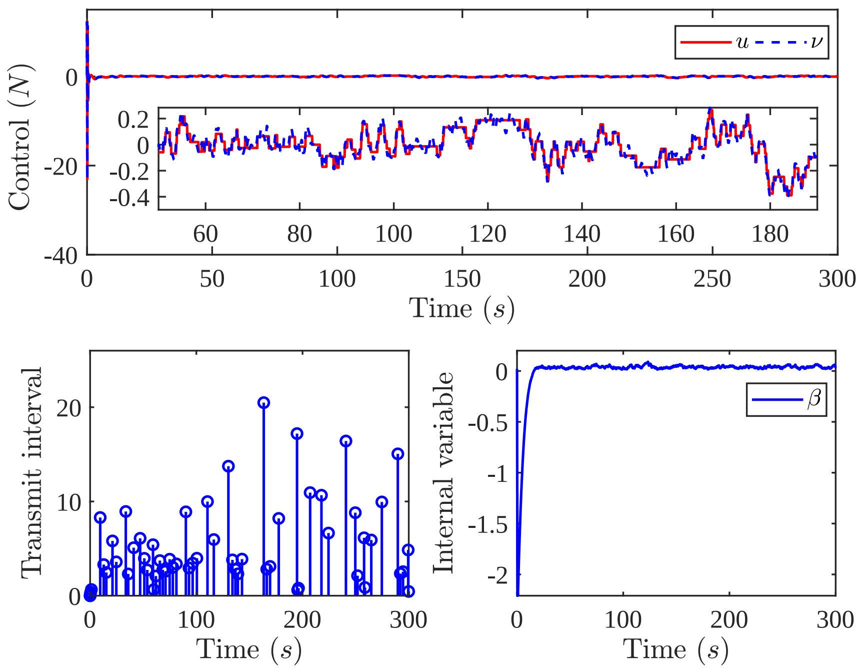

Figure 4. DETM control. DETM: dynamic eventtrigger mechanisms.

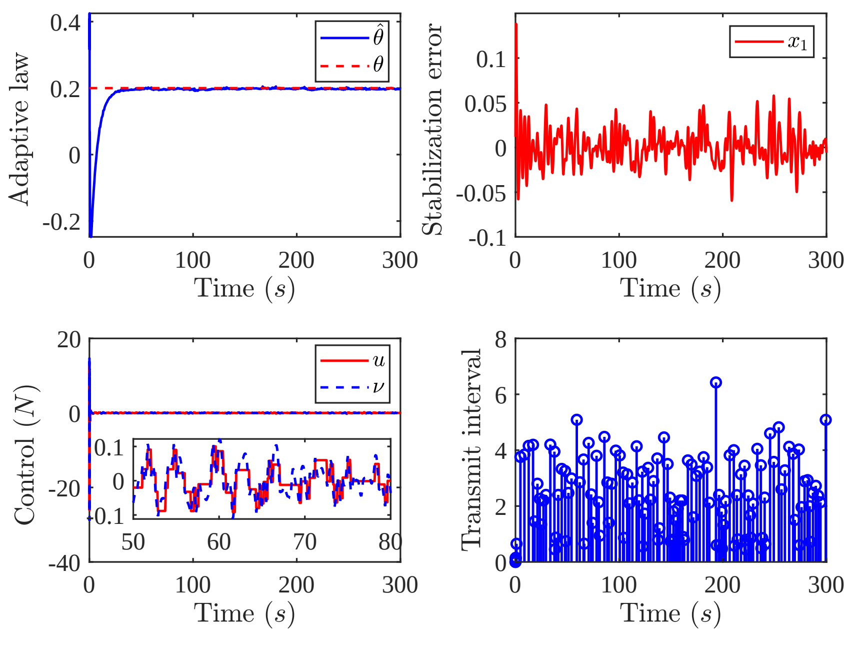

Figure 5. SETM control in [43,44]. SETM: static event-triggered mechanism.

where $\lambda, \delta > 0$, then the estimation effect is shown in Figure 6; by comparison, the adaptive estimation effect is better in I&I in Figure 3 when the control changes are not significantly different.

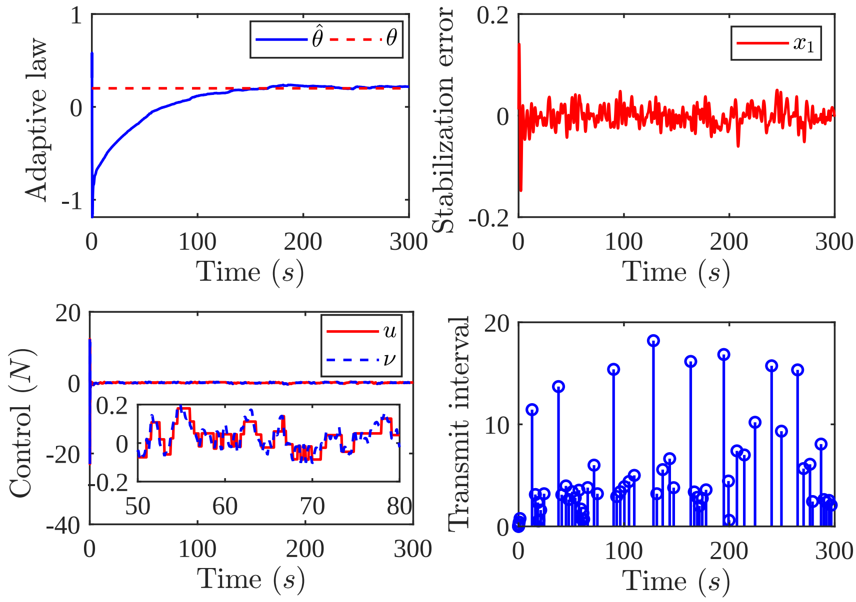

Figure 6. CE in [11,12]. CE: certain equivalence.

6. CONCLUSION

In this work, an adaptive ETC method with double DSF has been presented for random quarter-car active suspension models. Compared with static event-trigger, the designed dynamic event-triggered strategy can effectively reduce communication burden and save network resources. Not only automotive suspension systems but also the research on practical stabilization problems of general random nonlinear systems have also been provided. More importantly, for general random nonlinear systems, tracking controllers can also be designed to achieve the tracking goals. Future work may include vehicle network control issues under network attacks or adaptive ETC issues for stochastic under-actuated systems. The safety issues of multi-agent under-actuated systems, such as unmanned aerial vehicles and surface vessels, are also worth further investigation.

DECLARATIONS

Authors' contributions

Made significant contributions to the conception: Wu Z
Made significant contributions to the writing: Yang C
Made significant contributions to the revision: Feng L

Availability of data and materials

Not applicable.

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Conflicts of interest

All authors declared that there are no conflicts of interest.

Consent for publication

Not applicable.

Ethical approval and consent to participate

Not applicable.

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