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Dual-channel event-triggered consensus of multi-agent systems under DoS attacks over switching topologies

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Abstract

In this paper, a dual-channel event-triggered control (ETC) protocol with a state estimator is designed in multi-agent systems with switching topologies under denial-of-service attacks. Firstly, an ETC protocol and a dynamic ETC protocol are designed in the communication channel and the controller–actuator channel, respectively. Different from the traditional single-channel ETC, the dual-channel ETC is designed to further save resources. Second, an estimator is introduced to avoid continuous communication between agents. The sufficient conditions for realizing consensus are obtained under denial-of-service attacks. Moreover, due to the unstable communication topology of multi-agent systems, we designed a distributed controller based on switching topologies. Finally, the feasibility of the proposed method is verified through numerical simulations.

Keywords: Dual-channel event-triggered, DoS attacks, MASs, switching topologies

1. INTRODUCTION

Cluster behaviors formed by multiple individuals in nature through self-organization and mutual collaboration can exhibit capabilities that surpass individual functions. Inspired by this, researchers had proposed the concept of multi-agent systems (MASs). In recent years, MASs have received more attention from researchers due to the wide range of applications in military, transportation and agriculture [1]. In the course of the study of the MAS system, consensus is one of the most important research elements for MASs. Consensus means that agents can communicate in network and achieve consensus. In the past few years, many researchers had studied and explored the consensus in MASs. In reference [2], the stable problem of finite-time of MASs was studied by analyzing the transmission control protocol over active queue management topologies. In reference [3], an adaptive dynamic protocol was designed, which achieves the consensus under the effects of uncertain delays. In reference [4], the sufficient condition of achieving prescribed-time consensus was extended to a bipartite cooperative–antagonistic network.

In general, the goal of consensus is to design appropriate protocols to achieve consensus in communication networks with limited communication resources. In practical engineering, each agent carries limited energy resources when performing certain functions (e.g., collecting information, exchanging communication information with neighboring agents, *etc.*) and the communication channel can only transmit a limited number of data bits. Therefore, designing control strategies to reduce resource consumption so that agents can operate stably for a longer period of time is a research direction of many researchers. Traditional solutions are through time-triggered control, such as sampling control [5] and pulse control [6]. In periodic sampling control strategies, systems sample data at fixed time intervals, and this design method saves resources, but lacks the flexibility to adjust the sampling moment spontaneously. For this reason, event-triggered control (ETC) has garnered the interest of researchers. In reference [7], a second-order dynamic system was studied; the ETC strategy of the system was decided by the states of each agent and its neighbor agent. In references [8,9], model-based ETC strategies were designed in MASs where the continuity of adjacent states was no longer required. In references [10,11], an ETC protocol was designed in the systems, which was only updated at the instants of triggering. In reference [12], a hybrid event triggering mechanism combining three dynamic triggering conditions, namely local state error, velocity-dependent triggering condition, and dynamic maximum triggering interval, is designed to achieve efficient utilization of communication resources. Above research on ETC employs multiple parameters to optimize communication efficiency, imposing increased demands on estimators or samplers. In references [13–15], a dual-channel ETC protocol was designed for agent communication and controller–actuator channels, and consensus could be achieved while saving controller resources. In references [16], an ETC protocol was designed for nonlinear systems.

In earlier work, event generation relies on continuous information from the agents themselves and their neighbors; the control protocols designed using this approach are still continuous. To solve these problems, some researchers proposed the concept of an estimator, which is to estimate the state of the agents instead of the actual state. In reference [17], resilience control under denial-of-service (DoS) attacks is achieved by estimating the dynamics of leaders and followers. In reference [18], consensus control is achieved in cooperative communication networks by combining estimators.

The above results regarding consensus are mostly obtained in a secure network environment. MASs may be affected by DoS attacks in practical applications. DoS attacks usually inject large amounts of data into the communication channel, paralyzing the channel. In order to solve the problem of the impact of DoS attacks, researchers had proposed many control schemes. In reference [19], directed spanning trees are utilized to achieve security consensus under DoS attacks. In reference [20], the timing of DoS attacks is not consistent for each communication channel which is a more realistic scenario. Besides, switching topology is widely used in MASs which also attracted the attention of scholars. In reference [21], the problem of the system reaching a consistent rate of convergence when each link is established randomly with independent probability cycles was analyzed. In reference [22], the consensus of MASs alternately switched by discrete and continuous under the same topological conditions was studied. Dual-channel ETC saves communication activity to necessary events which can significantly reduce communication traffic. It helps reduce the risk of communication resources being exhausted under DoS attacks, thus ensuring that the transmission of critical information is not compromised.

Inspired by the above discussions, this paper studied the consensus problem in MASs with switching topologies under DoS attacks; the major contributions are as follows.

- This paper investigates the MAS consensus under switching communication topologies, which allow agents to have more complex interaction patterns. By rigorously analyzing the relationship between event-triggered feedback parameters and DoS attack dynamics, we establish sufficient conditions for MAS consensus. The derived criteria systematically integrate topology switching constraints, and security thresholds against DoS attacks, ensuring both communication efficiency and robustness in dynamic networked environments.
- Based on the dual-channel framework, ETC protocols are developed for the controller–actuator channel and communication channel to save resources. By incorporating an exponential parameter, the Zeno behavior is eliminated in the controller–actuator channel.

Notations: $\Re^n$, $\Re^{n\times n}$ denote the n-dimensional space, $n\times n$ real matrices, respectively. $|\cdot|$ and $\|\cdot\|$ represent the absolute value and 2-norm, respectively. $\otimes$ denotes the Kronecker product. $\mathrm{abs}(\cdot)$ denotes absolute values of elements of a matrix. $\lambda(\cdot)$ and $\bar{\lambda}(\cdot)$ represent the eigenvalue and singular eigenvalues of a matrix, respectively. $\mathbf{1}_N$ denotes the matrix $N\times 1$ with all ones. $\mathrm{diag}\{\cdot\}$ represents a diagonal matrix.

2. MODEL AND PROBLEM DESCRIPTION

2.1. Graph theory

The communication topology between N agents can be represented by a leader-following undirected digraph $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$, where $\mathcal{V}=\{v_1,v_2,\ldots,v_N\}$ denotes the set of nodes, $\mathcal{E}=\mathcal{V}\times\mathcal{V}$ represents the set of edges and an adjacency matrix $\mathcal{A}$. The edge e_{ij} is represented by a pair of vertices $\left(v_j,v_i\right)$. If $a_{ij}=1$ and $e_{ij}\in\mathcal{E}$, agent j can exchange information with agent i; otherwise, $a_{ij}=0$. If $a_{ij}=1$, agents i and j are neighboring nodes of each other. The set of neighboring agents i is denoted by $N_i=\left\{j\in\mathcal{V}:e_{ij}\in\mathcal{E}\right\}$. If $i\neq j$, the Laplace matrix of $\mathcal{G}$ can be defined as $l_{ii}=\sum_{j=1,j\neq i}^{N}a_{ij}$, $l_{ij}=-a_{ij}$. The real part of the eigenvalues of the L matrix satisfies $0{=}\lambda_1(L)\leq\lambda_2(L)\leq\cdots\leq\lambda_N(L)$. In particular, $\lambda_1=0$ corresponds to a feature vector $\mathbf{1}_N$. If $m_i=1$, the follower agent i can exchange information with the leader; otherwise, $m_i=0$. Define $H=L+M$, $M=diag\{m_1,m_2,...,m_N\}$.

Existence of all possible communication topologies for multi-agent systems can be represented by a set of connected undirected graphs $\mathcal{G}_\rho=\left\{\mathcal{G}_1,\mathcal{G}_2\cdots\mathcal{G}_O\right\}$, where $O>0$ is the number of switching topologies for the entire system. Considering the case of random switching topology, there exists a continuous edge probability matrix $\aleph=\left(\ell_{ij}\right)_{N\times N}$ and $0\leq\ell_{ij}\leq1$. The probability of a connection between every two points is independent of each other. Since there are only two possibilities of connection or non-connection between two nodes, a sequence of $N(N-1)$ independent Bernoulli trials is used to describe it. Consider a non-empty set of continuous time intervals $\{[t_k,t_{k+1})\mid k=0,1,\ldots,M\}$, where $t_0=0,\tau_*<t_{k+1}-t_k\leq\tau^*,\tau_*>0$ is a positive constant. The topology is invariant within each interval $[t_k,t_{k+1})$; the switching signals Υ and L_Υ are constant within this interval.

Remark 1 *According to the definition of the lower bound of switching time τ_*, the number of topology switches must be kept within a certain range. If switching occurs too frequently, the excessive switching may negatively affect both the dynamic performance and the stability of the system. This paper utilizes Bernoulli random switching, leveraging multiple iteration step sizes. By modulating the size of these multiples, one can regulate the frequency of random transitions between different communication topologies.*

2.2. DoS attacks model

The DoS attacks usually involve injecting large amounts of data into a communication channel to paralyze the channel, with the main aim of preventing information interaction and affecting monitoring equipment. Some useful assumptions and common lemmas are presented for further analysis.

Assumption 1 [23] *The number of DoS attacks in the* $[t_0, t]$ *is* $N_0(t_0, t)$. *We assume the constants* $N_1 > 0$ *and* $T^f > 0$ *such that*

$$N_0(t_0, t) \leqslant N_1 + \frac{t - t_0}{T^f}, \tag{1}$$

Assumption 2 [23] *The total time of DoS attacks in the* $[t_0, t]$ *is* $\Xi_a(t_0, t)$. *We assume the constants* $N_2 > 0$ *and* $T^d > 0$ *such that*

$$\Xi_a(t_0, t) \leqslant N_2 + \frac{t - t_0}{T^d}. \tag{2}$$

Remark 2 *Assumptions 1 and 2 limit the frequency and duration of attacks to make control meaningful. When the average attack frequency is greater than the sampling frequency of the system, the system is uncontrollable. In addition, these assumptions are justified by the fact that the duration is limited by energy.*

2.3. Problem statement

First, consider a MAS that has one leader and N followers. In this system, the state of the leader X_0 is proposed as

$$\dot{x}_0(t) = Ax_0(t), \tag{3}$$

and followers' dynamics is

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i \in \mathbb{S}, \tag{4}$$

where $\mathbb{S} = 1, 2, ..., N$; $x_0(t) \in \Re^n$ is the states of the leader. The $x_i(t) \in \Re^n$ and $u_i(t) \in \Re^p$ are the states and control input of agent i. The $A \in \Re^{n\times n}$ and $B \in \Re^{n\times p}$ are the constant matrices of the corresponding dimension. The following assumption is considered.

Assumption 3 *The topology graph of MASs* $\mathcal{G}$ *is undirected and connected; the pair of* (A, B) *is stabilisable.*

Lemma 1 [24] *Under Assumption 3, a unique solution denoted as P which satisfies the algebraic Riccati equation as follows*

$$A^T P + PA - \kappa PBB^T P + P = 0, \tag{5}$$

where $\kappa > 0$.

Definition 1 *Given initial conditions, the consensus of MASs is realized if*

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, i \in \mathbb{S}. \tag{6}$$

Lemma 2 [25] $\mathcal{G}$ *is structurally balanced; thus, there is a matrix* $\Delta = diag\{\Delta_1, \Delta_2, \ldots, \Delta_N\} > 0$ *such that* $\Delta H + H^T\Delta > 0$.

3. MAIN RESULT

The research aims to design a controller and dual-channel ETC protocol to achieve consensus of MASs under DoS attacks over switching topologies.

3.1. Controller design

Firstly, the follower's controller is designed as:

$$u_i(t) = \begin{cases} K\Gamma_i(t), & t \in [t_k^i, t_{k+1}^i) \subset \Xi_s(t_0, t), \\ 0, & t \in \Xi_a(t_0, t), \end{cases} \tag{7}$$

$$\Gamma_i(t) = \sum_{\text{j=1}}^{n} a_{ij} \left(\hat{x}_i(t) - \hat{x}_j(t)\right) + m_i \left(\hat{x}_i(t) - x_0(t)\right), \tag{8}$$

where K is a constant gain matrix; $\Xi_s(t_0, t)$ is the period when the MASs without DoS attacks. In certain studies, the controller relied on the state value at the triggering instants. However, in this research, the controller employs an estimated value instead. The design of the estimator is outlined below:

$$\begin{aligned} &\dot{\hat{x}}_j(t) = A\hat{x}_j(t) + \omega_j Bu_j(t), \quad t \in \left[t_k^j, t_{k+1}^j\right), \\ &\hat{x}_j\left(t_k^j\right) = x_j\left(t_k^j\right), \quad j = 1, 2, \ldots N. \end{aligned} \tag{9}$$

where $\omega_j > 0$ is a constant; $\hat{x}_j$ is the estimated state of jth agent in $[t_k^j, t_{k+1}^j)$. When $t = t_k^j$, there is $\hat{x}_j = x_j$. The error variable $e_i(t)$ of the ith agent is

$$e_i(t) = \hat{x}_i(t) - x_i(t). \tag{10}$$

The time derivative of the error variable $e_i(t)$ is

$$\dot{e}_i(t) = \dot{\hat{x}}_i(t) - \dot{x}_i(t) = Ae_i(t) - (1 - \omega_i) BK\Gamma_i(t). \tag{11}$$

Consider a positive definite matrix Q that fulfills the given Riccati inequality

$$QA + A^T Q - 2rQBB^T Q + rI_n < 0, \tag{12}$$

where r is minimum eigenvalue of matrix H. And matrix K is calculated by $K = B^T Q$ in Equation (7). According to the tracking error $\psi_i(t) = x_i(t) - x_0(t)$. The time derivative of tracking error $\psi_i(t)$ as

$$\dot{\psi}_i(t) = \dot{x}_i(t) - \dot{x}_0(t) = Ax_i(t) - Ax_0(t) + BK\Gamma_i(t). \tag{13}$$

Then we can obtain

$$\Gamma(t) = -(H \otimes I_n)(\hat{x}(t) - \mathbf{1}_N \otimes x_0(t)) = -(H \otimes I_n)(e(t) + \Psi(t)), \tag{14}$$

where $\Gamma(t) = (\Gamma_1^T(t), \Gamma_2^T(t), \ldots, \Gamma_N^T(t))^T$. Thus, Equation (13) can be rewritten in Kronecker forms as

$$\dot{\Psi}(t) = (I_N \otimes A)\Psi(t) - (H \otimes BK)\big(e(t) + \Psi(t)\big). \tag{15}$$

where $\Psi(t) = (\Psi_1^T(t), \Psi_2^T(t), \ldots, \Psi_N^T(t))^T$, $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^{\mathrm{T}}(t))^T$.

3.2. Dynamic event-triggered design

Considered a dynamic ETC protocol for agent i with a time-varying parameter threshold $\theta_i(t)$

$$\begin{aligned} &t_1^i = 0, \\ &t_{k+1}^i = \inf_{l > t_k^i}\{l : \gamma_i f(e_i(t), \Gamma_i(t)) \geqslant \theta_i(t), \forall t \in (t_k^i, l]\}, \end{aligned} \tag{16}$$

where $f(e_i(t), \Gamma_i(t)) = (\frac{\varepsilon_2}{\tau} + \varepsilon_3)\|e_i(t)\|^2 - \sigma_i(\varepsilon_1 - \tau\varepsilon_2)\|\Gamma_i(t)\|^2$; $(\varepsilon_1 - \tau\varepsilon_2) > 0$, $\tau > 0$, $\gamma_i > 0$ and $\sigma_i \in (0, 1]$. Other parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are as follows:

$$\begin{aligned} &\varepsilon_1 = r_1 \lambda_{\min}\left(H^{-2}\right), \\ &\varepsilon_2 = \lambda_{\max}\left((I_N \otimes 2QBB^T Q) - 2r_1(H^{-1} \otimes I_N)\right), \\ &\varepsilon_3 = \lambda_{\max}(H \otimes 2QBB^T Q) - r_1, \end{aligned} \tag{17}$$

the constant $r_1 > 0$ satisfies $0 < r_1 < r$ and $\varepsilon_2 > 0, \varepsilon_3 > 0$. $\theta_i(t)$ is designed as:

$$\dot{\theta}_i(t) = -\alpha_i\theta_i(t) - f(e_i(t), \Gamma_i(t)), \tag{18}$$

where $\alpha_i > 0$. $\theta_i(t)$ represents a time-varying parameter that updates are contingent upon self-feedback, measurement discrepancies, neighbor errors, and leader errors. It is noteworthy that $\theta_i(t)$ possesses a property crucial to the subsequent process of proving or deriving the conclusion, as outlined in

Lemma 3 *For predefined scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \alpha_i, \theta_i(0)$ we can obtain*

$$\theta_i(t) > 0, i = 1, 2, ..., N. \tag{19}$$

Proof: Consider the system (14) with (16)(18), when $t \in \cup_{k=1}^{\infty}(t_k^i, t_{k+1}^i] = (0, \infty)$ there exists k', $t \in (t_{k'}^i, t_{k'+1}^i]$. The following formula holds: when the required parameter is not satisfied, the inequality

$$\gamma_i f\left(e_i(t), \Gamma_i(t)\right) < \theta_i(t),$$

applies. When $t = t_{k'+1}^i$, one has

$$\dot{\theta}_i(t) \geqslant -\alpha_i\theta_i(t) - \frac{1}{\gamma_i}\theta_i(t),$$

and thus,

$$\theta_i(t) \geqslant \theta_i(t_k^i)e^{-(\alpha_i+\frac{1}{\gamma_i})(t-t_k^i)} > 0, t \in (t_{k'}^i, t_{k'+1}^i],$$

from $t_1^i = 0$, one has

$$\begin{aligned}\theta_i(t) &\geq \theta_i(t_k^i)e^{-(\alpha_i+\frac{1}{\gamma_i})(t-t_k^i)} \geq \theta_i(t_{k-1}^i)e^{-(\alpha_i+\frac{1}{\gamma_i})(t-t_{k-1}^i)} \\ &\geq \cdots \geq \theta_i(0)e^{-(\alpha_i+\frac{1}{\gamma_i})t} > 0, t \in \left(t_{k'}^i, t_{k'+1}^i\right].\end{aligned} \tag{20}$$

It is detailed that $\theta_i(0) > 0$. Thus, $\theta_i(t) > 0$ holds for $[0, \infty)$.

Theorem 1 *Considering the MASs (3)(4), consensus among the MASs can be achieved if there exists*

$$\Im = \mu_1 - \frac{\mu_1 + \mu_2}{T^d} - \frac{\ln(\rho_1\rho_2)}{T^f} > 0,$$

where $\mu_1 = \min\left\{\frac{1}{N}\sum\limits_{i=1}^{N}\alpha_i, r_2\right\}$, $\mu_2 > 0$ is a constant. $\rho_1 = \frac{\lambda_{\max}(I_N\otimes Q)}{\lambda_{\min}(I_N\otimes P)}$, and $\rho_2 = \frac{\lambda_{\max}(I_N\otimes P)}{\lambda_{\min}(I_N\otimes Q)}$, $PA + A^TP < \mu_2 P$ and $P > 0$.

Proof: Consider the MASs (3)(4) and design the following Lyapunov function

$$V(t) = \begin{cases} V_s(t), t \in \Xi_s(t_0, t), \\ V_a(t), t \in \Xi_a(t_0, t), \end{cases} \tag{21}$$

when $t \in \Xi_s(t_0, t)$, the system is free from attacks. During this period, design $V_s(t) = V_1(t) + V_2(t)$ and

$$V_1(t) = \Psi^T(t)\left(I_N \otimes Q\right)\Psi(t), \tag{22}$$

$$V_2(t) = \sum_{i=1}^{N}\theta_i(t). \tag{23}$$

The time derivative of $V_1(t)$ is

$$\begin{aligned}\dot{V}_1(t) &= 2\Psi^T(t)\left(I_N \otimes Q\right)\dot{\Psi}(t) \\ &\leq \Psi^T(t)\left(I_N \otimes \left(QA + A^TQ\right) - \left(H \otimes 2QBB^TQ\right)\right)\Psi(t) \\ &\quad - \Psi^T(t)\left(H \otimes 2QBB^TQ\right)e(t) \\ &\leq \Psi^T(t)\left((rI_N - H) \otimes 2QBB^TQ\right)\Psi(t) - r\Psi^T(t)\left(I_N \otimes Q\right)\Psi(t) \\ &\quad - \Psi^T(t)\left(H \otimes 2QBB^TQ\right)e(t) \\ &\leq -r\Psi^T(t)\left(I_N \otimes Q\right)\Psi(t) - e^T(t)\left(H \otimes 2QBB^TQ\right)\Psi(t) \\ &\leq -(r_1 + r_2)(\Gamma^T(t)(H^{-2} \otimes Q)\Gamma(t) + 2\Gamma^T(t)(H^{-1} \otimes Q)e(t) + e^T(t)\left(I_N \otimes Q\right)e(t)) \\ &\quad + e^T(t)\left(H \otimes 2QBB^TQ\right)\Gamma(t) + e^T(t)\left(I_N \otimes 2QBB^TQ\right)e(t) \\ &\leq -r_1\Gamma^T(t)(H^{-2} \otimes Q)\Gamma(t) - r_1e^T(t)\left(I_N \otimes Q\right)e(t) - 2r_1\Gamma^T(t)(H^{-1} \otimes Q)e(t) \\ &\quad + e^T(t)\left(H \otimes 2QBB^TQ\right)\Gamma(t) + e^T(t)\left(I_N \otimes 2QBB^TQ\right)e(t) \\ &\quad - r_2\Psi^T(t)\left(I_N \otimes Q\right)\Psi(t),\end{aligned} \tag{24}$$

where $r = r_1 + r_2$. According to Young's inequality $pq \leq \frac{1}{2k}\|p\|^2 + \frac{k}{2}\|q\|^2$, we can rewrite $\dot{V}_1(t)$ that

$$\begin{aligned}\dot{V}_1(t) &\leq -\varepsilon_1\|\Gamma(t)\|^2 + 2\varepsilon_2\|e(t)\|\|\Gamma(t)\| + \varepsilon_3\|e(t)\|^2 - r_2\Psi^T(t)\Psi(t)\\ &\leq -\varepsilon_1\|\Gamma(t)\|^2 + 2\varepsilon_2\left(\frac{1}{2\tau}\|e(t)\|^2 + \frac{\tau}{2}\|\Gamma(t)\|^2\right) + \varepsilon_3\|e(t)\|^2\\ &\quad - r_2\Psi^T(t)(I_N\otimes Q)\Psi(t)\\ &\leq -\sum_{i=1}^{N}\left[(\varepsilon_1-\varepsilon_2\tau)\|\eta_i(t)\|^2 - \left(\frac{\varepsilon_2}{\tau}+2\varepsilon_3\right)\|e_i(t)\|^2\right]\\ &\quad - r_2\Psi^T(t)(I_N\otimes Q)\Psi(t).\end{aligned} \tag{25}$$

The time derivative of $V_2(t)$ is

$$\begin{aligned}\dot{V}_2(t) &= \sum_{i=1}^{N}\dot{\theta}_i(t)\\ &= \sum_{i=1}^{N}\left(-\alpha_i\theta_i(t) - \left(\frac{\varepsilon_2}{\tau}+\varepsilon_3\right)\|e_i(t)\|^2 + \sigma_i(\varepsilon_1-\varepsilon_2\tau)\|\Gamma_i(t)\|^2\right).\end{aligned} \tag{26}$$

To sum up, we can obtain that

$$\begin{aligned}\dot{V}_s(t) &= \dot{V}_1(t) + \dot{V}_2(t)\\ &\leq -\sum_{i=1}^{N}\left[(\varepsilon_1-\varepsilon_2\tau)\|\Gamma_i(t)\|^2 - \left(\frac{\varepsilon_2}{\tau}+\varepsilon_3\right)\|e_i(t)\|^2\right]\\ &\quad + \sum_{i=1}^{N}\left(-\alpha_i\theta_i(t) - \left(\frac{\varepsilon_2}{\tau}+\varepsilon_3\right)\|e_i(t)\|^2\right.\\ &\quad \left. + \sigma_i(\varepsilon_1-\varepsilon_2\tau)\|\Gamma_i(t)\|^2\right) - r_2\Psi^T(t)(I_N\otimes Q)\Psi(t)\\ &\leq \sum_{i=1}^{N}\left(-\alpha_i\theta_i(t) + (\sigma_i-1)(\varepsilon_1-\varepsilon_2\tau)\|\Gamma_i(t)\|^2\right) - r_2\Psi^T(t)(I_N\otimes Q)\Psi(t)\\ &\leq -\mu_1 V_s(t).\end{aligned} \tag{27}$$

For $t \in \Xi_a(t_0,t)$, there exist DoS attacks among the agents' communication. We choose the following Lyapunov function

$$V_a(t) = \Psi^T(t)(I_N\otimes P)\Psi(t), \tag{28}$$

and its derivative is

$$\begin{aligned}\dot{V}_a(t) &= 2\Psi^T(t)(I_N\otimes P)(I_N\otimes A)\Psi(t)\\ &= \Psi^T(t)(I_N\otimes(PA+A^TP))\Psi(t)\\ &\leq \mu_2 V_a(t).\end{aligned} \tag{29}$$

From (27) and (29), we obtain

$$V(t) \leq \begin{cases} e^{-\mu_1\left(t-t_{2(l-1)}\right)}V\left(t_{2(l-1)}\right), t\in\left[t_{2(l-1)}, t_{2l-1}\right),\\ e^{\mu_2(t-t_{2l-1})}V(t_{2l-1}), t\in[t_{2l-1}, t_{2l}).\end{cases} \tag{30}$$

We can calculate that $\mu_1\mu_2 > 1$. For $t \in \left[t_{2(l-1)}, t_{2l-1}\right)$, it has

$$\begin{aligned}V(t) &\leq e^{-\mu_1\left(t-t_{2(l-1)}\right)}V\left(t_{2(l-1)}\right)\\ &\leq \rho_1 e^{-\mu_1\left(t-t_{2(l-1)}\right)}V\left(t_{2(l-1)}^-\right)\\ &\leq \rho_1 e^{-\mu_1\left(t-t_{2(l-1)}\right)}e^{\mu_2\left(t_{2(l-1)}-t_{2l-3}\right)}V(t_{2l-3})\\ &\leq \cdots\\ &\leq {\rho_1}^{N(t_0,t)}{\rho_2}^{N(t_0,t)}e^{\mu_2\Xi_a(t_0,t)}\\ &\quad\times e^{-\mu_1(t-t_0-\Xi_a(t_0,t))}V(t_0).\end{aligned} \tag{31}$$

Under Assumptions 1-2, there is

$$(\rho_1\rho_2)^{N(t_0,t)} \leq (\rho_1\rho_2)^{N_1}e^{\frac{\ln(\rho_1\rho_2)}{T^f}(t-t_0)}, \tag{32}$$

and

$$e^{\mu_2 \Xi_a(t_0,t)} e^{-\mu_1(t-t_0-\Xi_a(t_0,t))} \leq e^{\mu_2(N_2+\frac{t-t_0}{T^d})} e^{-\mu_1(t-t_0)} e^{\mu_1(N_2+\frac{t-t_0}{T^d})} \\ \leq e^{(\mu_1+\mu_2)N_2} e^{\frac{\mu_1+\mu_2}{T^d}(t-t_0)} e^{-\mu_1(t-t_0)}. \tag{33}$$

The (31) can be scaled to

$$V(t) \leq (\rho_1\rho_2)^{N_1} e^{\frac{\ln(\rho_1\rho_2)}{T^f}(t-t_0)} \times e^{(\mu_1+\mu_2)N_2} e^{\frac{\mu_1+\mu_2}{T^d}(t-t_0)} e^{-\mu_1(t-t_0)} V(t_0) \\ \leq (\rho_1\rho_2)^{N_1} e^{(\mu_1+\mu_2)N_2} \times e^{(\frac{\ln(\rho_1\rho_2)}{T^f}+\frac{\mu_1+\mu_2}{T^d}-\mu_1)(t-t_0)} V(t_0). \tag{34}$$

Therefore, as $t \in [t_{2(l-1)}, t_{2l-1})$, we can obtain $V(t) \leq \varsigma_1 e^{-\beta(t-t_0)} V(t_0)$ where $\varsigma_1 = (\rho_1\rho_2)^{N_1} e^{(\mu_1+\mu_2)N_2}$. And as $t \in [t_{2(l-1)}, t_{2l-1})$, it has $V(t) \leq \varsigma_2 e^{-\beta(t-t_0)} V(t_0)$ as above, where $\varsigma_2 = {\rho_1}^{N(t_0,t)-1} \rho_2^{N(t_0,t)} e^{(\mu_1+\mu_2)N_2}$. By analyzing the above two situations, we can obtain

$$V(t) \leq \varsigma e^{-\beta(t-t_0)} V(t_0), \tag{35}$$

where $\varsigma = \max\{\varsigma_1, \varsigma_2\}$. Similar to Theorem 1, the consensus of MASs was realized.

3.3. Zeno behavior analysis

In this chapter, we will analyze and confirm that Zeno behavior is excluded in the systems under two specific scenarios. There exists inequality

$$D^+\|e_i(t)\| \leq \|\dot{x}_i(t)\| + \|\dot{\hat{x}}_i(t)\| \\ \leq \|Ax_i(t)\| + \|(1+\omega_i)Bu_i(t)\| + \|A\hat{x}_i(t)\| \\ = \|A\|\|e_i(t)\| + Z_k^i, \tag{36}$$

where $Z_k^i = 2\|\dot{\hat{x}}_i(t)\| + \|(1+\omega_i)Bu_i(t)\|$. The proof that follows will be presented in two scenarios.

1. If $\|A\| \neq 0$, from Equation (36), one can get

$$\|e_i(t)\| \leq \frac{Z_k^i}{\|A\|}\left(e^{\|A\|(t-t_k^i)} - 1\right). \tag{37}$$

For the designed ETC protocol, the triggering instants t_{k+1}^i satisfies

$$(\frac{\varepsilon_2}{\tau} + \varepsilon_3) \parallel {e^T}_i(t)\|^2 \geq \sigma_i(\varepsilon_1 - \tau\varepsilon_2) \parallel {\Gamma_i}^T(t_{k+1}^i)\|^2 + \frac{\theta_i(0)}{\gamma_i} e^{-(\alpha_i+\frac{1}{\gamma_i})t_{k+1}^i}. \tag{38}$$

Then,

$$t_{k+1}^i - t_k^i \geq \frac{1}{\parallel A \parallel} \ln\left(\begin{array}{c} 1 + \frac{\|A\|}{\sqrt{(\frac{\varepsilon_2}{\tau}+\varepsilon_3)Z_k^i}} \\ \times\sqrt{\sigma_i(\varepsilon_1 - \tau\varepsilon_2)\left\|\Gamma_i^T(t_{k+1}^i)\right\|^2 \frac{\theta_i(0)}{\gamma_i} e^{-(\alpha_i+\frac{1}{\gamma_i})t_{k+1}^i}} \end{array} \right). \tag{39}$$

Subsequently, we employ proof by contradiction to demonstrate that Zeno behavior does not occur. First, we suppose that the Zeno behavior of agent i will occur, indicating that $\sum_{k=0}^{\infty} \Delta_k^i$ must converge, where $\Delta_k^i = t_{k+1}^i - t_k^i$ is a positive sequence, and then $\lim_{m\to\infty} \sum_{k=0}^{m} \Delta_k^i = \lim_{k\to\infty} t_{k+1}^i - t_0 = \infty$, which is divergent. Thus, Zeno behavior does not occur.

2.if $\|A\| = 0$, then $\left\|e_i^T(t)\right\| \leq Z_k^i(t - t_k^i)$ The proof process for this case is analogous to the first cases and therefore will not be reiterated here. It can be seen that the Zeno behavior of agent i will be excluded from the above two situations. If the MASs are subjected to DoS attacks, $u_i(t) = 0$; the Zeno behavior of the agent i also will not occur.

3.4. Dual-channel event-triggered consensus

In this part, based on the idea of dual channels, the ETC protocol between the controller and actuator channel with the communication channel is designed to save resources. The Zeno behavior in the controller and actuator channel can be eliminated by introducing the exponential parameter, and the ETC protocol in the controller and actuator channel can still work stably when the input signal is very small. In addition, the ETC protocol in the controller and actuator channel is designed before the communication channel, and it is designed to depend only on the input signal and does not require neighbor information. First, an equivalent controller $\tilde{u}_i(t)$ is designed as

$$\tilde{u}_i(t) = \begin{cases} u_i(t_k), & t \in \Xi_s(t_0, t), \\ 0, & t \in \Xi_a(t_0, t). \end{cases} \tag{40}$$

For $t \in \Xi_s(t_0, t)$, the controller update time depends on the following ETC protocol

$$\begin{aligned} &\tilde{u}_i(t) = u_i\left(t_k^i\right), \forall t \in [t_k^i, t_{k+1}^i), \\ &t_{k+1}^i = \inf_{l > t_k^i} \left\{ l : \left|\wp_i^u(t)\right| - z_i \left|\tilde{u}_i(t)\right| \geq e^{-\upsilon_i t}, \forall t \in \left(t_k^i, l\right] \right\}, \end{aligned} \tag{41}$$

where $\wp_i^u(t) = u_i(t) - \tilde{u}_i(t)$, $0 < z_i < 1$ and $\upsilon_i > 0$. Then, there exist $|\tilde{\alpha}_i(t)| \leq 1$ and $|\tilde{\beta}_i(t)| \leq 1$. Thus, we have

$$\tilde{u}_i(t) = \frac{u_i(t) - \tilde{\beta}_i(t) e^{-\upsilon_i t}}{1 + \tilde{\alpha}_i(t) z_i}. \tag{42}$$

In that case, the tracking error $\psi_i(t)$ is

$$\dot{\psi}_i(t) = A\psi_i(t) + \hat{\alpha}_i(t) BK\hat{\Gamma}_i(t) - \hat{\beta}_i(t) B, \tag{43}$$

where $\hat{\alpha}_i(t) = \frac{1}{1+\tilde{\alpha}_i(t)z_i}$, $\hat{\beta}_i(t) = \frac{\tilde{\beta}_i(t)e^{-\upsilon_i t}}{1+\tilde{\alpha}_i(t)z_i}$ and $\hat{\alpha}_i(t) < \frac{1}{1-z_i} = \bar{\alpha}_i$, $\hat{\beta}_i(t) \leqslant \frac{1}{1-z_i}e^{-\upsilon_i t} = \bar{\beta}e^{-\upsilon_i t}$. The tracking error ψ is written in the form as

$$\dot{\Psi}(t) = (I_N \otimes A)\Psi(t) + \left(\hat{\alpha}(t) H \otimes B\hat{K}\right)(\Psi(t) + e(t)) - \hat{\beta}(t) \otimes B, \tag{44}$$

where $\hat{\alpha}(t) = diag\{\hat{\alpha}_1(t), \hat{\alpha}_2(t), \ldots, \hat{\alpha}_N(t)\}$ and $\hat{\beta}(t) = (\hat{\beta}_1(t), \hat{\beta}_2(t), \ldots, \hat{\beta}_N(t))^T$. In the context of Equation (40), the following ETC protocol can be obtained

$$\begin{aligned} &t_1^i = 0, \\ &t_{k+1}^i = \inf_{l > t_k^i} \{l : \hat{\gamma}_i \hat{f}\left(e_i(t), \hat{\Gamma}_i(t)\right) \geqslant \hat{\theta}_i(t), \forall t \in (t_k^i, l]\}, \end{aligned} \tag{45}$$

where $\hat{f}\left(e_i(t), \hat{\Gamma}_i(t)\right) = \left(\frac{(\hat{\varepsilon}_2+\delta_1)}{k_1} + \hat{\varepsilon}_3\right)\|\Gamma_i(t)\|^2 - \hat{\sigma}_i(\hat{\varepsilon}_1 - k_2\hat{\varepsilon}_4 - k_1(\hat{\varepsilon}_2 + \delta_1))\left\|\hat{\Gamma}_i(t)\right\|^2 + \delta_3 e^{-2\upsilon_i t}$, $\hat{\sigma}_i(\hat{\varepsilon}_1 - k_2\hat{\varepsilon}_4 - k_1(\hat{\varepsilon}_2 + \delta_1)) > 0$ with $k_1, k_2 > 0$, $\hat{\sigma}_i \in (0, 1]$. Other parameters are designed as follows:

$$\begin{aligned} &\hat{\varepsilon}_1 = \bar{\lambda}_{\min}\left(\tilde{H}\Delta\tilde{H}^T \otimes P\right), \\ &\hat{\varepsilon}_2 = \bar{\lambda}_{\max}\left(\tilde{H}\Delta \otimes P - \delta_2 k_2 \tilde{H} \otimes I_n\right), \\ &\hat{\varepsilon}_3 = \bar{\lambda}_{\max}\left((\Delta\bar{\hat{\alpha}}H + H^T\bar{\hat{\alpha}}\Delta) \otimes \hat{W} + \delta_2 k_2 I_N \otimes I_n - \Delta \otimes P\right), \\ &\hat{\varepsilon}_4 = \bar{\lambda}_{\max}\left(\delta_2 \tilde{H}\tilde{H}^T \otimes I_n\right), \\ &\delta_1 = \bar{\lambda}_{\max}\left(\tilde{H}\Delta\bar{\hat{\alpha}}H \otimes \hat{W}\right), \\ &\delta_2 = \bar{\lambda}_{\max}(\Delta \otimes P), \\ &\delta_3 = \frac{\delta_2}{k_2}\left(\overline{\beta}\|B\|\right)^2, \end{aligned} \tag{46}$$

where $\hat{W} = PBB^TP$, $\dot{\hat{\theta}}_i(t) = -\widehat{\alpha}_i\hat{\theta}_i(t) - \hat{f}\left(e_i(t), \hat{\Gamma}_i(t)\right)$, $\widehat{\alpha}_i > 0$ and $\hat{\theta}_i(0) > 0$.

Remark 3 *Dual-channel ETC ensures system stability by triggering actions only when necessary and adjusting parameters based on communication topology and agent dynamics to achieve system security consensus. Dual-channel ETC enhances the efficiency of the system by separating communication and control channels, reducing unnecessary communication. It also offers flexible triggering mechanisms, optimizing resource utilization and boosting overall system performance. Thus, while the dual-channel triggering protocol designed here is similar to the triggering form of the existing work, the design idea is different and from a practical point of view, the approach in this section is simpler and does not require additional wiring.*

Remark 4 *In order to avoid continuous communication and further save communication resources, an estimator is introduced in this chapter. However, the use of the estimator leads to the inability to exclude Zeno behavior. A common approach is to use a constant trigger threshold to exclude Zeno behavior. However, this design will make $\tilde{u}_i(t)$ can not reach $u_i(t)$. Inspired by the references[26,27], an exponential threshold $e^{-\upsilon_i t}$ is designed to solve problems caused by the estimator in this section. The designed controller update protocol still works properly when the input signal is extremely small.*

Theorem 2 *Considering MASs (3)(4), with the control protocol (45), consensus among the MASs can be achieved if there exist*

$$\hat{\Im} = \hat{\mu}_1 - \frac{\hat{\mu}_1 + \hat{\mu}_2}{T^d} - \frac{\ln(\hat{\rho}_1\hat{\rho}_2)}{T^f} > 0. \tag{47}$$

Proof: for $t \in \Xi_s(t_0, t)$, choose $\hat{V}_s(t) = \hat{V}_1(t) + \hat{V}_2(t)$, and we have

$$\begin{aligned} \hat{V}_1(t) &= e^T(t)\,(\Delta \otimes P)\,e(t), \\ \hat{V}_2(t) &= \sum_{i=1}^{N} \theta_i(t). \end{aligned} \tag{48}$$

Then, applying (48), $\hat{V}_1(t)$ designed as

$$\begin{aligned} \dot{\hat{V}}_1(t) &= 2\Psi^T(t)\,(\Delta \otimes P)\,\dot{\Psi}(t) \\ &= \Psi^T(t)(\Delta \otimes (A^T P + PA) + (\Delta\hat{\alpha}(t)H + H^T\hat{\alpha}(t)\Delta) \otimes PB\hat{K})\Psi(t) \\ &\quad + 2\Psi^T(t)\left(\Delta\hat{\alpha}(t)H \otimes PB\hat{K}\right)e(t) - 2\Psi^T(t)\,(\Delta \otimes P)\left(\hat{\beta}(t) \otimes B\right), \end{aligned} \tag{49}$$

where $\hat{K} = -B^T P$. The time derivative of $\hat{V}_1(t)$ is

$$\begin{aligned} \dot{\hat{V}}_1(t) &\le -\Psi^T(t)\,(\Delta \otimes P)\,\Psi(t) - 2\Psi^T(t)\left(\Delta\hat{\alpha}(t)H \otimes \hat{W}\right)e(t) - 2\Psi^T(t)\,(\Delta \otimes P)\left(\hat{\beta}(t) \otimes B\right) \\ &\le -\Psi^T(t)\,(\Delta \otimes P)\,\Psi(t) - \Psi^T(t)\,(\Delta \otimes P)\,\Psi(t) - 2\hat{\Gamma}^T(t)\left(\tilde{H}\Delta\hat{\alpha}(t)H \otimes \hat{W}\right)e(t) \\ &\quad + 2e^T(t)\left(\Delta\hat{\alpha}(t)H \otimes \hat{W}\right)e(t) - 2\Psi^T(t)\,(\Delta \otimes P)\left(\hat{\beta}(t) \otimes B\right). \end{aligned} \tag{50}$$

From

$$\begin{aligned} -\hat{\Gamma}^T(t)\left(\tilde{H}\Delta\hat{\alpha}(t)H \otimes \hat{W}\right)e(t) &\le \left\|\hat{\Gamma}^T(t)\left(\tilde{H}\Delta\hat{\alpha}(t)H \otimes \hat{W}\right)e(t)\right\| \\ &\le \left\|\hat{\Gamma}^T(t)\right\|\left\|\tilde{H}\Delta\bar{\hat{\alpha}}H \otimes \hat{W}\right\|\|e(t)\| \\ &\le \delta_1\left\|\hat{\Gamma}(t)\right\|\|e(t)\|, \end{aligned} \tag{51}$$

$$\begin{aligned} -2\Psi^T(t)\,(\Delta \otimes P)\left(\hat{\beta}(t) \otimes B\right) &\le 2\delta_2\left\|\Psi^T(t)\right\|\left\|\hat{\beta}(t) \otimes B\right\| \\ &\le 2\delta_2\left(\frac{k_2}{2}\|\Psi(t)\|^2 + \frac{1}{2k_2}\|\hat{\beta}(t) \otimes B\|^2\right) \\ &\le \delta_2 k_2\|\Psi(t)\|^2 + \delta_3\sum_{i=1}^{N} e^{-2\upsilon_i t}, \end{aligned} \tag{52}$$

we obtain

$$\begin{aligned}
\dot{\hat{V}}_1(t) \leq & -\Psi^T(t)\,(\Delta\otimes P)\,\Psi(t)+e^T(t)\left((\Delta\overline{\tilde{\alpha}}H+H^T\overline{\tilde{\alpha}}\Delta)\otimes\hat{W}\right)e(t)+2\delta_1\|\hat{\Gamma}(t)\|e(t)\| \\
& +\delta_2k_2e^T(t)e(t)+\delta_2k_2\hat{\Gamma}^T(t)\left(\tilde{H}\tilde{H}^T\otimes I_n\right)\hat{\Gamma}(t)-2\delta_2k_2\hat{\Gamma}^T(t)\left(\tilde{H}\otimes I_n\right)e(t)+\delta_3\sum_{i=1}^{N}e^{-2\upsilon_it} \\
\leq & -\hat{\Gamma}^T(t)\left(\tilde{H}\Delta\tilde{H}^T\otimes P\right)\hat{\Gamma}(t)-e^T(t)\,(\Delta\otimes P)\,e(t) \\
& +2\hat{\Gamma}^T(t)\left(\tilde{H}\Delta\otimes P\right)e(t)+2\delta_1\left\|\hat{\Gamma}(t)\right\|\|e(t)\|+e^T(t)\left((\Delta\overline{\tilde{\alpha}}H+H^T\overline{\tilde{\alpha}}\Delta)\otimes\hat{W}\right)e(t) \\
& +\delta_2k_2\hat{\Gamma}^T(t)\left(\tilde{H}\tilde{H}^T\otimes I_n\right)\hat{\Gamma}(t)+\delta_3\sum_{i=1}^{N}e^{-2\upsilon_it}-2\delta_2k_2\hat{\Gamma}^T(t)\left(\tilde{H}\otimes I_n\right)e(t)+\delta_2k_2e^T(t)e(t) \\
\leq & \,(-\hat{\varepsilon}_1+k_2\hat{\varepsilon}_4)\left\|\hat{\Gamma}(t)\right\|^2+2(\hat{\varepsilon}_2+\delta_1)\left\|\hat{\Gamma}(t)\right\|\|e(t)\|+\hat{\varepsilon}_3\|e(t)\|^2+\delta_3\sum e^{-2\upsilon_it}-\Psi^T(t)\,(\Delta\otimes P)\,\Psi(t).
\end{aligned} \tag{53}$$

Applying Young's inequality, we get

$$\begin{aligned}
\dot{\hat{V}}_1(t) \leq & \,(-\hat{\varepsilon}_1+k_2\hat{\varepsilon}_4)\left\|\hat{\Gamma}(t)\right\|^2+k_1(\hat{\varepsilon}_2+\delta_1)\left\|\hat{\Gamma}(t)\right\|^2+\frac{(\hat{\varepsilon}_2+\delta_1)}{k_1}\|e(t)\|^2+\hat{\varepsilon}_3\|e(t)\|^2+\delta_3\sum_{i=1}^{N}e^{-2\upsilon_it} \\
\leq & \,(-\hat{\varepsilon}_1+k_2\hat{\varepsilon}_4+k_1(\hat{\varepsilon}_2+\delta_1))\left\|\hat{\Gamma}(t)\right\|^2+\left(\frac{(\hat{\varepsilon}_2+\delta_1)}{k_1}+\hat{\varepsilon}_3\right)\|e(t)\|^2+\delta_3\sum_{i=1}^{N}e^{-2\upsilon_it} \\
\leq & \sum_{i=1}^{N}\left((-\hat{\varepsilon}_1+k_2\hat{\varepsilon}_4+k_1(\hat{\varepsilon}_2+\delta_1))\|\hat{\Gamma}_i(t)\|^2+(\frac{(\hat{\varepsilon}_2+\delta_1)}{k_1}+\hat{\varepsilon}_3)\|e_i(t)\|^2+\delta_3e^{-2\upsilon_it}\right).
\end{aligned} \tag{54}$$

Applying the derivatives of $\hat{V}_2(t)$, then

$$\begin{aligned}
\hat{V}_s(t) &= \dot{\hat{V}}_1(t)+\dot{\hat{V}}_2(t) \\
&\leq -\sum_{i=1}^{N}\widehat{\alpha}_i\hat{\theta}_i(t)-\sum_{i=1}^{N}\left((1-\hat{\sigma}_i)(\hat{\varepsilon}_1-k_2\hat{\varepsilon}_4-k_1(\hat{\varepsilon}_2+\delta_1))\left\|\hat{\Gamma}_i(t)\right\|^2\right) \\
&\leq -\hat{\mu}_1\hat{V}_s(t).
\end{aligned} \tag{55}$$

The subsequent process of proving the consensus is similar to Theorem 1 and is not given to avoid redundancy. Next, Zeno behavior exclusion is divided into two categories: the actuator channel and the communication channel. 1. For the controller–actuator channel, we have

$$\left|\wp_i^u(t)\right| \leq z_i\left|\tilde{u}_i(t)\right|+e^{-\upsilon_it}, \tag{56}$$

where $\wp_i^u(t)=u_i(t)-\tilde{u}_i(t)$ then we have

$$\begin{aligned}
\frac{d}{dt}|\wp_i^u(t)| &= \frac{d}{dt}(\wp_i^u(t)\cdot\wp_i^u(t))^{\frac{1}{2}} \\
&= sgn(\wp_i^u)\dot{\wp}_i^u \\
&\leq |\dot{u}_i(t)|.
\end{aligned} \tag{57}$$

Considering the event-triggered interval $[t_k^i, t_{k+1}^i)$, there exist $|\wp_i^u(t_k)|=0$ and $\lim_{t\to t_{k+1}}|\wp_i^u(t)|=\nu_i|\tilde{u}_i(t)|+e^{-\upsilon it}$; then, we can obtain

$$t_{k+1}^i-t_k^i \geq \left(z_i|\tilde{u}_i(t)|+e^{-\upsilon_it}\right)/\vartheta. \tag{58}$$

If Zeno behavior exists, it means infinite triggers within a limited time T_c, i.e., $\sum_{k=0}^{\infty}\Delta_k^i=T_c$, where $\Delta_k^i=t_{k+1}^i-t_k^i$. Then, we have $\sum_{k=0}^{\infty}\Delta_k^i=\lim_{k\to\infty}t_{k+1}^i-t_0^i=T_c$, indicating $\lim_{k\to\infty}t_{k+1}^i=T_c$. Thus, we have $\sum_{k=0}^{\infty}\Delta_k^i=\infty$ and $\lim_{k\to\infty}t_{k+1}^i=\infty$. This contradicts $\lim_{k\to\infty}t_{k+1}^i=T_c$; thus, Zeno behavior will not occur. For dynamic ETC protocol in the communication channel, similar to chapter 3.4., Zeno behavior can be excluded. This proof is finished.

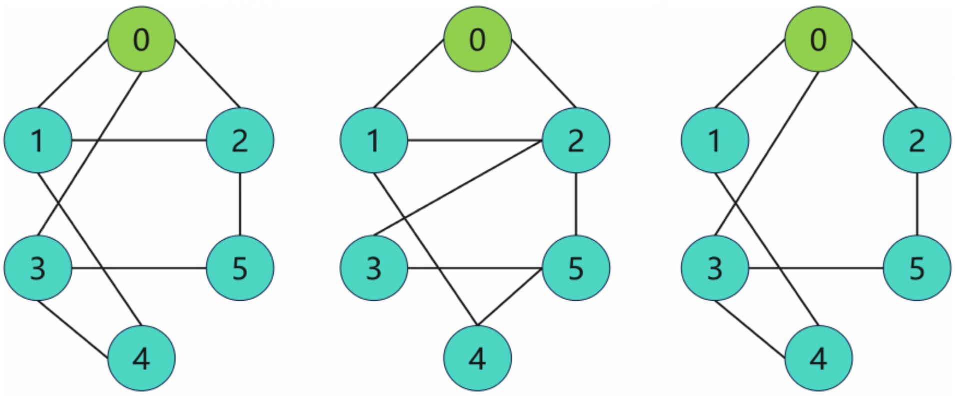

Figure 1. Network topology of MASs.

4. NUMERICAL SIMULATION

The matrix parameters are designed as

$$A = \begin{pmatrix} -2 & 1 & -1 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The adjacency matrices L and M are replaced by L_1, L_2, L_3 and M_1, M_2, M_3. The matrices K and Q can also be calculated by

$$K = \begin{pmatrix} 0.3940 & 0.4082 \\ 0.9093 & 0.3940 \\ 0.4288 & 0.2227 \end{pmatrix}^T, \quad Q = \begin{pmatrix} 0.4082 & 0.3940 & 0.2227 \\ 0.3940 & 0.9093 & 0.4288 \\ 0.2227 & 0.4288 & 0.6308 \end{pmatrix}.$$

Choose $\omega_i = 0.1$, $\theta_1(0) = 90$, $\theta_2(0) = 92$, $\theta_3(0) = 70$, $\theta_4(0) = 62$, $\theta_5(0) = 93$. Set $x_0 = [3, -1, 2]^T$, $x_1 = [2.4, 2.3, 0.2]^T$, $x_2 = [-3.2, 1, 1.7]^T$, $x_3 = [2, -3, 1.5]^T$, $x_4 = [-1.5, -0.5, -2.8]^T$, $x_5 = [1.1, 1, -2.2]^T$. The communication structure of the MASs will switch randomly among the three scenarios depicted in Figure 1. Figure 2 illustrates the state difference between each follower and the leader. From Figure 2, we can observe that the state errors between followers and leaders all converge to zero within six seconds. The state trajectories of the leader and followers are displayed in Figure 3, clearly showing that each agent can follow the leader. From Figure 2 and Figure 3, we can deduce that the MASs achieve leader-following consensus in finite time. Figure 4 presents the triggering times of five agents. As the state errors between followers and leader converge to zero, the increase of triggering times of followers slows down. Lastly, Figure 5 shows the network topology of the MASs.

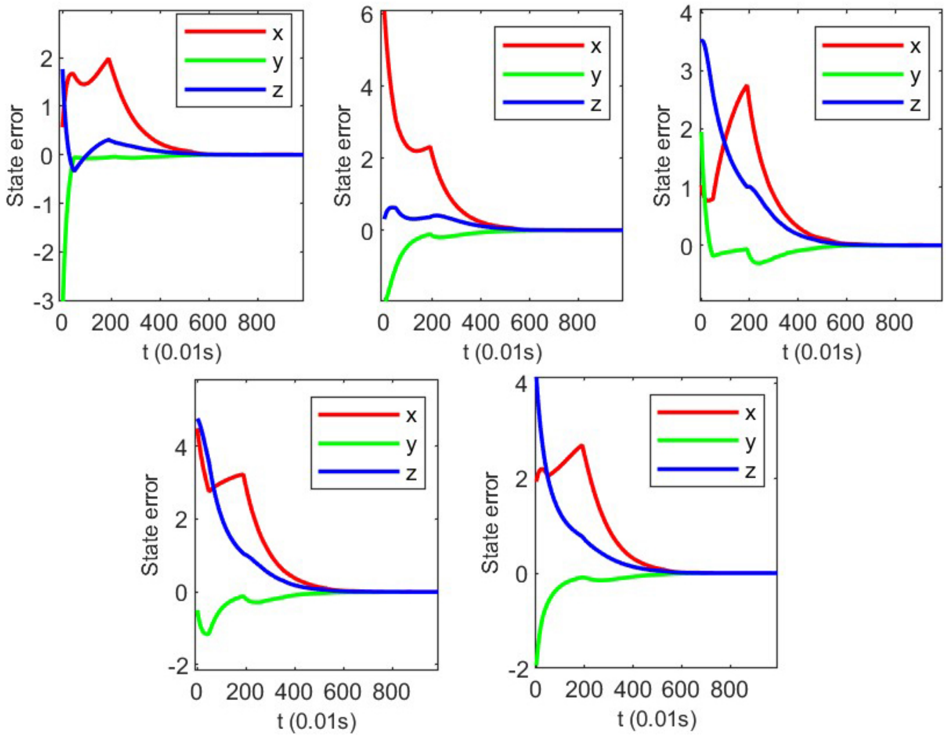

Figure 2. State error of MASs.

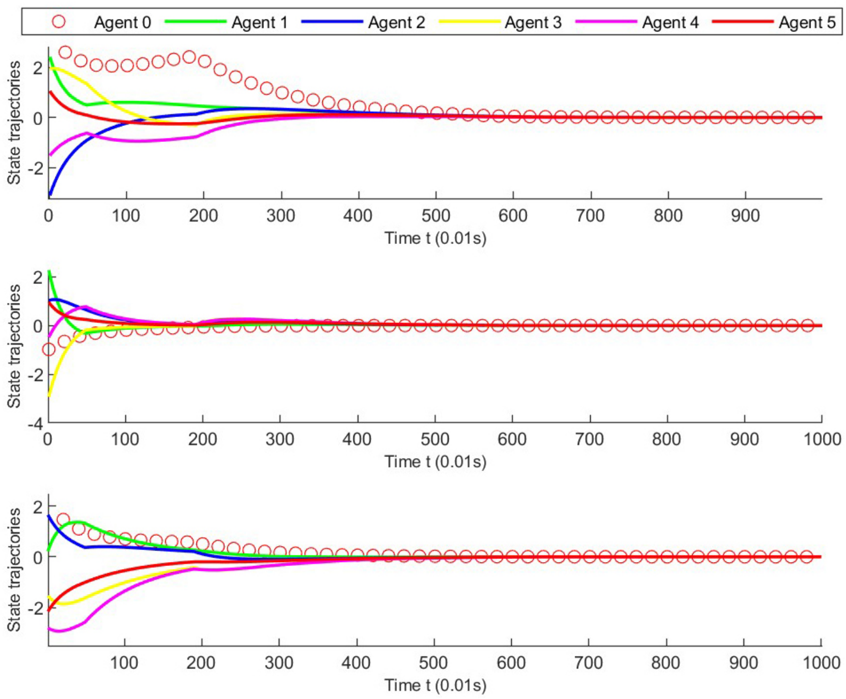

Figure 3. State trajectory of agents.

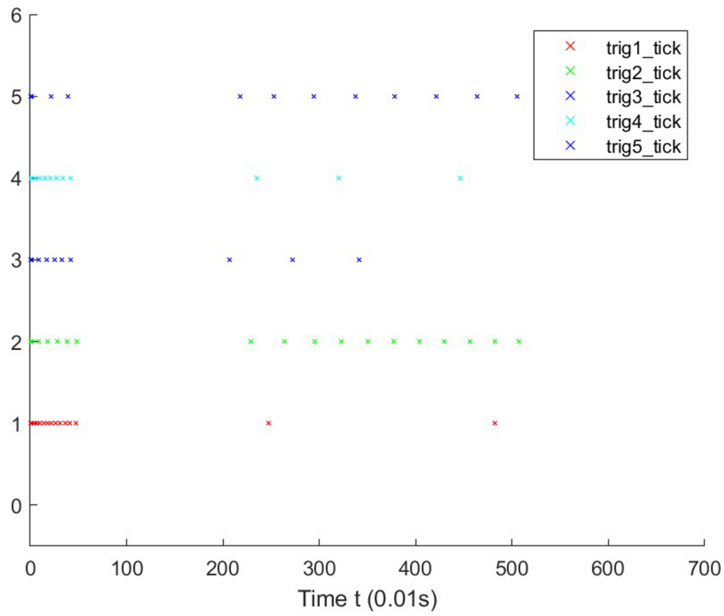

Figure 4. Triggering times of followers.

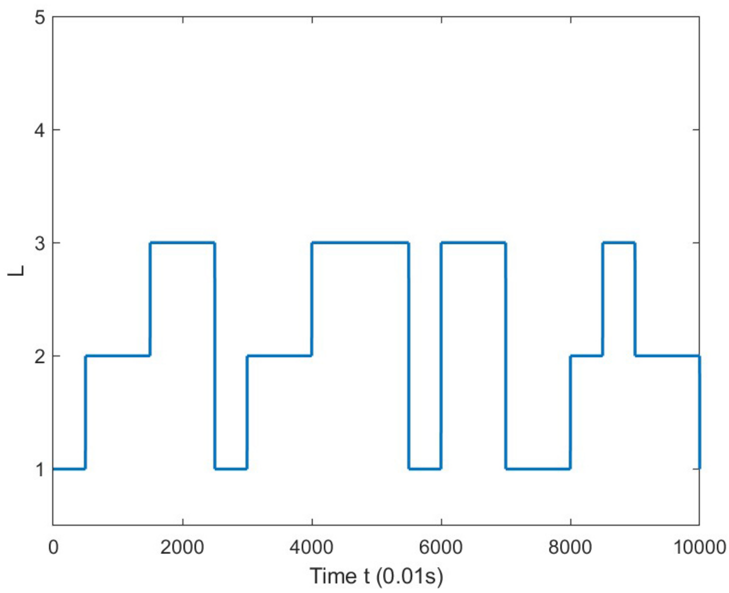

Figure 5. Current topology of MASs.

5. CONCLUSIONS

This paper focuses on the consensus problem of MASs. Considering a MAS with a leader and N followers, a distributed controller is designed. To avoid continuous communication, an estimator is used to estimate the state of agents during the trigger interval so that the estimated state is used instead of the actual state to achieve the goal of intermittent communication. In order to further save communication resources, a dual-channel ETC is designed for the controller-actuator channel and communication channel, respectively. The continuous validity of the ETC protocol is ensured by introducing an exponential triggering threshold in the controller-actuator channel. Finally, it is verified through simulation that consensus control can be achieved while effectively saving controller resources and communication resources. Furthermore, applying our findings to address security control challenges in mobile robot systems under DoS attacks represents a crucial research direction. This study will facilitate future research efforts on the consensus of heterogeneous MASs, the implementation of security controls against hybrid cyber attacks, and the development of self-triggered protocols.

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Authors' contributions
Conceptualization, methodology, numerical simulations, writing: Li, W.; Chen, X.
Formal analysis, review, editing: Chen, X.; Cheng, L.; Ma, J.; Zhao, F.
All authors have read and agreed to the published version of the manuscript.

Availability of data and materials
The authors generated all data and materials used in the research as an integral part of the study, with explicit details provided in the methodology section of the manuscript. These are available from the corresponding author upon reasonable request.

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Conflicts of interest
Chen, X. is the Guest Editor of the Special Issue of "Adaptive Event-Triggered Control and Optimization for Complex Systems". He is not involved in any steps of editorial processing, notably including reviewer selection, manuscript handling, or decision-making, while the other authors have declared that they have no conflicts of interest.

Ethical approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

REFERENCES

1. Cheng, W.; Zhang, K.; Jiang, B.; Ding, S. X. Fixed-time fault-tolerant formation control for heterogeneous multi-agent systems with parameter uncertainties and disturbances. *IEEE Trans. Circuits Syst. I Regul. Pap.* **2021**, *68*, 2121-33 DOI
2. Wang, C.; Chen, X.; Cao, J.; Qiu, J.; Liu, Y.; Luo, Y. Neural network-based distributed adaptive pre-assigned finite-time consensus of multiple TCP/AQM networks. *IEEE Trans. Circuits Syst. I Regul. Pap.* **2021**,*68*, 387-95 DOI
3. Zamani, N.; Askari, J.; Kamali, M.; Aghdam, A. Distributed adaptive consensus tracking control for non-linear multi-agent systems with time-varying delays. *IET Control Theory Appl.* **2020**, *14*, 3382-94 DOI
4. Guo, X.; Liang, J. Prescribed-time bipartite consensus for signed directed networks on time scales. *Int. J. Control.* **2021**,*96*, 508-16 DOI

5. He, W.; Zhang, B.; Han, Q. L.; Qian, F.; Kurths, J.; Cao, J. Leader-following consensus of nonlinear multiagent systems with stochastic sampling. *IEEE Trans. Cybern.* **2017**, *47*, 327-38 DOI
6. He, W.; Chen, G.; Han, Q.; Qian, F. Network-based leader-following consensus of nonlinear multi-agent systems via distributed impulsive control. *Inf. Sci.* **2017**, *380*, 145-58. DOI
7. Luo, Y.; Xiao, X.; Cao, J.; Li, A.; Lin, G. Event-triggered guaranteed cost consensus control for second-order multi-agent systems based on observers. *Inf. Sci.* **2021**, *546*, 283–297. DOI
8. Li, W.; Zhang, H.; Sun, S.; Zhang, J. Fully distributed event-triggered consensus protocols for multi-agent systems with physically interconnected network. *Neurocomputing.* **2020**, *418*, 191-9. DOI
9. Li, X.; Tang, Y.; Karimi, H. Consensus of multi-agent systems via fully distributed event-triggered control. *Automatica.* **2020**, *116*, 108898. DOI
10. Hu, A.; Park, J.; Hu, M. Consensus of nonlinear multiagent systems with intermittent dynamic event-triggered protocols. *Nonlinear Dyn.* **2021**, *104*, 1299-313. DOI
11. Chen, X.; Feng, J. E.; Xu, W.; Cao, J.; Qiu, J. Evolution modeling and control of networked dynamic games with event-triggering mechanism. *Chaos.* **2025**, *35*,013102. DOI
12. Chen, C.; Zou, W.; Xiang, Z. Event-triggered consensus of multiple uncertain euler–lagrange systems with limited communication range. *IEEE Trans. Syst. Man Cybern. Syst.* **2023**, *53*,5945-54. DOI
13. Li, K. Z. u; Teel, A. R.; Sun, X. M.; Wang, X. F. Model-based dynamic event-triggered control for systems with uncertainty: a hybrid system approach. *IEEE Trans. Autom. Control.* **2021**, *66*, 444-51 DOI
14. Hu, W.; Yang, C.; Huang, T.; Gui, W. A distributed dynamic event-triggered control approach to consensus of linear multiagent systems with directed networks. *IEEE Trans. Cybern.* **2020**, *50*,869-74 DOI
15. Koike, R.; Endo, T.; Matsuno, F. Output-based dynamic event-triggered consensus control for linear multiagent systems. *Automatica.* **2021**, *133*,109863 DOI
16. Xiang, Z.; Li, P.; Zou, W. Event-triggered optimal control for a class of continuous-time switched nonlinear systems. *IEEE Trans. Autom. Sci. Eng.* **2025**, *22*,1620-30. DOI
17. He, W.; Li, S.; Ahn, C. K.; Guo, J.; Xiang, Z. Sampled-data stabilization of stochastic interconnected cyber-physical systems under DoS attacks. *IEEE Syst. J.* **2022**, *16*,3844-54 DOI
18. Feng, Z.; Hu, G. Secure cooperative event-triggered control of linear multiagent systems under DoS attacks. *IEEE Trans. Control Syst. Technol.* **2020**, *28*,741-52. DOI
19. Guo, X. G.; Liu, P. M.; Wang, J. L.; Ahn, C. K. Event-triggered adaptive fault-tolerant pinning control for cluster consensus of heterogeneous nonlinear multi-agent systems under aperiodic DoS attacks. *IEEE Trans. Netw. Sci. Eng.* **2021**, *8*,1941-56. DOI
20. Yang, Y.; Li, Y.; Yue, D. Event-trigger-based consensus secure control of linear multi-agent systems under DoS attacks over multiple transmission channels. *Sci. China Inf. Sci.* **2020**, *63*,150208. DOI
21. Chen, Z.; Cai, L.; Zhao, C. Convergence time of average consensus with heterogeneous random link failures. *Automatica.* **2021**, *127*,109496. DOI
22. Lin, X.; Zheng, Y.; Wang, L. Consensus of switched multi-agent systems with random networks. *Int. J. Control.* **2016**, *90*,1113-22. DOI
23. De Persis, C.; Tesi, P. Input-to-state stabilizing control under denial-of-service. *IEEE Trans. Automat. Contr.* **2015**, *60*, 2930-44. DOI
24. Qin, J.; Fu, W.; Zheng, W. X.; Gao, H. On the bipartite consensus for generic linear multiagent systems with input saturation. *IEEE Trans. Cybern.* **2017**, *47*,1948-58. DOI
25. Bhowmick, S.; Panja, S. Leader–follower bipartite consensus of linear multiagent systems over a signed directed graph. *IEEE Trans. Circuits Syst. II.* **2019**, *66*, 1436-40. DOI
26. Chen, X.; Hu, S.; Yang, T.; Xie, X.; Qiu, J. Event-triggered bipartite consensus of multiagent systems with input saturation and DoS attacks over weighted directed networks. *IEEE Trans. Syst. Man Cybern. Syst.* **2024**, *54*, 4054-65. DOI
27. Hu, S.; Qiu, J.; Chen, X.; Zhao, F.; Jiang, X. Dynamic event-triggered control for leader-following consensus of multiagent systems with the estimator. *IET Control Theory Appl.* **2022**, *16*, 475-84. DOI