

Review

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# Machine learning assisted crystal structure prediction made simple

Chuan-Nan Li<sup>1,2,3</sup> , Han-Pu Liang<sup>2</sup> , Bai-Qing Zhao<sup>2</sup>, Su-Huai Wei<sup>4,\*</sup>, Xie Zhang<sup>1,\*</sup> 

<sup>1</sup>School of Materials Science and Engineering, Northwestern Polytechnical University, Xi'an 710072, Shaanxi, China.

<sup>2</sup>Materials and Energy Division, Beijing Computational Science Research Center, Beijing 100193, China.

<sup>3</sup>Department of Physics, University of Science and Technology of China, Hefei 230026, Anhui, China.

<sup>4</sup>School of Physics, Eastern Institute of Technology, Ningbo 315200, Zhejiang, China.

\***Correspondence to:** Prof. Su-Huai Wei, School of Physics, Eastern Institute of Technology, No. 568, Tongxin Road, Zhuangshi Sub-district, Zhenhai District, Ningbo 315200, Zhejiang, China. E-mail: suhuaiwei@eitech.edu.cn; Prof. Xie Zhang, School of Materials Science and Engineering, Northwestern Polytechnical University, No. 127, Youyi West Road, Beilin District, Xi'an 710072, Shaanxi, China. E-mail: xie.zhang@nwpu.edu.cn

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## Abstract

Crystal structure prediction (CSP) plays a crucial role in condensed matter physics and materials science, with its importance evident not only in theoretical research but also in the discovery of new materials and the advancement of novel technologies. However, due to the diversity and complexity of crystal structures, trial-and-error experimental synthesis is time-consuming, labor-intensive, and insufficient to meet the increasing demand for new materials. In recent years, machine learning (ML) methods have significantly boosted CSP. In this review, we present a comprehensive review of the ML models applied in CSP. We first introduce the general steps for CSP and highlight the bottlenecks in conventional CSP methods. We further discuss the representation of crystal structures and illustrate how ML-assisted CSP works. In particular, we review the applications of graph neural networks (GNNs) and ML force fields in CSP, which have been demonstrated to significantly speed up structure search and optimization. In addition, we provide an overview of advanced generative models in CSP, including variational autoencoders (VAEs), generative adversarial networks (GANs), and diffusion models. Finally, we discuss the remaining challenges in ML-assisted CSP.

**Keywords:** Crystal structure prediction, machine learning, structure representation, graph neural network, machine learning force field, generative model



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## INTRODUCTION

With the rapid development of artificial intelligence, we are now in the so-called Big Data Era, a time when vast amounts of data are generated and collected from various sources at an unprecedented pace<sup>[1,2]</sup>. In this context, the data-driven research paradigm has become mainstream in modern materials science<sup>[3-7]</sup>. This paradigm leverages big data and machine learning (ML) technologies to accelerate the discovery and design of new materials, marking a shift from traditional research methods that rely on experiments and theory to more efficient and automated methodologies. The data-driven research paradigm focuses on extracting features and patterns from a large database to guide the design and property prediction of new materials<sup>[8,9]</sup>. The typical workflow of data-driven materials science research includes: (i) collecting data; (ii) building ML models; and (iii) using ML models for rapid computation and data analysis<sup>[10]</sup>. Obviously, data collection constitutes the basis of this workflow<sup>[11,12]</sup>, with materials science data originating from two primary sources: experimental results and theoretical predictions.

Crystal structure prediction (CSP) serves as a vital data source for modern data-driven materials science research, providing structural information crucial for understanding the electronic, optical, and magnetic properties of materials<sup>[13,14]</sup>. The goal of CSP is to determine the most stable arrangement of atoms solely based on chemical composition<sup>[15,16]</sup>. CSP also explores metastable states that might possess unique properties<sup>[12,17]</sup> and examines all possible compositions to discover new compounds<sup>[18,19]</sup>. When the temperature drops to 0 K, the free energy transforms into enthalpy, consistent with the total energies calculated by most first-principle calculation software [e.g., the Vienna Ab-initio Simulation Package (VASP)<sup>[20-22]</sup>]. Therefore, in most cases, we are looking for the global minimum on the potential energy surface.

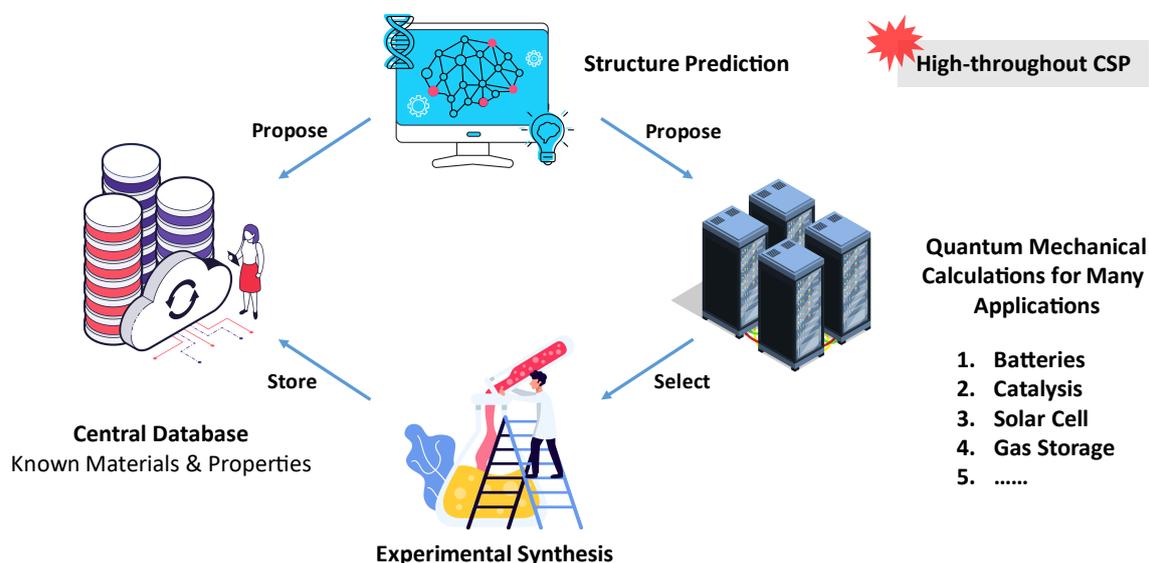
As illustrated in [Figure 1](#), ML-based CSP can continuously supply structural data for databases or practical applications. This ML-driven materials design process is highly efficient and facilitates the discovery of new materials through high-throughput CSP. Additionally, CSP involves exploring potential material structures under extreme conditions and environments, and identifying materials that may emerge from experiments but are costly to synthesize or require numerous attempts<sup>[23-26]</sup>. In short, compared to data collection that solely relies on trial-and-error experimental synthesis - which is time-consuming and labor-intensive - CSP is more economical, environmentally friendly, and safer<sup>[27-29]</sup>. CSP can be transformed into a combinatorial problem, with general steps including<sup>[30]</sup>: (i) space gridding; (ii) atom arrangement; and (iii) energy evaluation. By extensively repeating the last two steps, we can find the low-energy arrangements of atoms. However, this exhaustive structure search method is suitable when the number of structures is small, but faces significant challenges when the number of structures explodes.

The main difficulty in CSP is that the number of possible structures increases explosively as the number of atoms in a unit cell increases<sup>[14,31]</sup>. If we use the general steps mentioned above, the number of possible structures can be estimated using<sup>[14]</sup>:

$$C = \frac{1}{(V/\delta^3)} \frac{(V/\delta^3)!}{[(V/\delta^3) - N]!N!}, \quad (1)$$

where  $\delta$  is the grid resolution (e.g.,  $\delta = 1 \text{ \AA}$ ),  $V$  is the volume of the unit cell, and  $N$  is the number of atoms in the unit cell. The number of possible crystal structures grows exponentially with increasing the degrees of freedom ( $d = 3N + 3$ ):  $C \approx \exp(a \cdot d)$ , where  $a$  is a constant. Clearly, it is impractical to exhaustively enumerate all possible atomic arrangements, making it necessary to design algorithms or methods for the problem of CSP. Here, we summarize the challenges faced in CSP as follows:

- High-dimensional potential energy surfaces<sup>[32,33]</sup>: A large number of atoms in the unit cell leads to very high-dimensional potential energy surfaces. The number of possible structures on the potential energy



**Figure 1.** ML-driven materials design. First, researchers use ML-based CSP methods to explore low-energy structures of target compositions in a short time. Then, the low-energy structures can be added to databases or used in quantum mechanical calculations. Finally, the potential candidates can be synthesized in experiment. ML: Machine learning; CSP: crystal structure prediction.

surface increases exponentially with the number of atoms, making it extremely difficult to search for the global minimum in high-dimensional spaces. Simple exhaustive methods are not suitable for CSP.

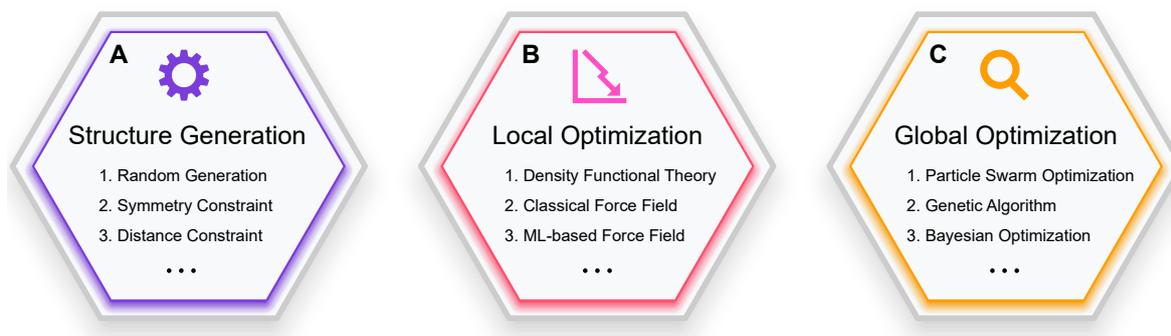
- Computational cost<sup>[13,34]</sup>: Determining the accurate energies of crystal structures typically requires first-principles calculations based on density functional theory (DFT). However, the computational complexity of DFT increases rapidly with the number of electrons, limiting the size of systems where DFT can be applied.
- Limitations of empirical force fields<sup>[35,36]</sup>: Empirical force fields can be used for energy calculations and structure optimization because they are faster. However, due to their reliance on empirical parameters, they often fail to accurately describe the entire potential energy surface.
- Local minima<sup>[37,38]</sup>: The potential energy surface contains numerous local minima corresponding to metastable structures. Without appropriate global search methods, structure searches can easily become trapped in local minima.

To overcome these challenges, various algorithms have been adopted in conventional CSP methods, including particle swarm optimization<sup>[39,40]</sup>, genetic algorithm (GA)<sup>[41,42]</sup>, Bayesian optimization<sup>[43,44]</sup>, and simulated annealing<sup>[45,46]</sup>. Nowadays, ML methods have been applied to CSP, greatly improving the efficiency of structure searches. These include the graph neural network (GNN)<sup>[47,48]</sup>, ML force field<sup>[49,50]</sup>, and generative model<sup>[51,52]</sup>.

## CONVENTIONAL CSP METHODS

Conventional CSP methods mainly refer to those that do not use ML techniques. In this section, we briefly discuss these methods to convey their basic ideas, progress, and bottlenecks.

As shown in [Figure 2](#), conventional CSP methods mainly include three parts: (i) structure generation; (ii) structure optimization; and (iii) structure search. The initial structures are always generated randomly with symmetry and distance constraints, and then DFT and global search algorithms are combined to explore low-energy structures on the potential energy surface. The main difference between different CSP methods lies in the global optimization algorithms. Therefore, we classify conventional CSP methods based on these algorithms as follows.



**Figure 2.** General steps in CSP. (A) Initial structures generated randomly with physical constraints; (B) Structure optimization by DFT or classical force fields; (C) New structures generated by global optimization algorithms. CSP: Crystal structure prediction; DFT: density functional theory.

### Random search

The random search algorithm is the most basic method in CSP<sup>[53,54]</sup>. This method searches for the lowest-energy structures through extensive random exploration. For instance, ab initio random structure searching (AIRSS)<sup>[55]</sup> is a typical implementation of the random search algorithm. AIRSS first generates a large number of structures randomly and then uses first-principles calculations to relax these structures. Using random search methods, novel structures have been discovered for defect clusters of various sizes<sup>[56]</sup>, high-pressure phases of solid hydrogen<sup>[57]</sup>, nitrogen<sup>[58]</sup>, and lithium<sup>[59]</sup>. Combining random search with a set of correlation functions as the objective, the well-known special quasirandom structures (SQS)<sup>[60,61]</sup> approach has been developed for modeling the chemically disordered state within a fixed lattice for alloys with variable compositions. By using SQS, it is also possible to investigate order-disorder phase transitions<sup>[60,61]</sup>, such as phase transitions in Fe-C alloys<sup>[62]</sup>, BeZnO<sub>2</sub> alloys<sup>[63]</sup>, and Cs<sub>2</sub>AgBiBr<sub>6</sub> perovskite<sup>[64]</sup>. Despite the accomplishments, random search algorithms face challenges due to the giant configurational space. To improve search efficiency, several strategies can be applied: using geometric constraints to reduce the search space<sup>[65,66]</sup>, adopting ML models for rapid screening and energy calculations<sup>[67]</sup>, and utilizing parallel computing to accelerate the search process<sup>[68]</sup>. These strategies have made random search algorithms reasonably practical in the field of CSP, especially in the generation of initial structures.

### Particle swarm optimization

The particle swarm optimization<sup>[69,70]</sup> is based on swarm intelligence, inspired by the collective behavior of birds or fishes in nature. In particle swarm optimization, particles move through the solution space, updating their positions and velocities based on their own experiences and the experiences of other particles in the swarm. For instance, crystal structure analysis by particle swarm optimization (CALYPSO)<sup>[69]</sup> is a CSP package based on the particle swarm optimization. The general workflow of CALYPSO includes the following steps: First, initial structures are randomly generated with physical constraints, including minimum interatomic distances and crystal symmetry. Then, structures are characterized using crystal fingerprints to eliminate duplicate or similar structures. After removing duplicate structures, local optimization is applied to candidate structures to reach the local minima. Finally, the particle swarm optimization is used for structural evolution, generating initial structures for the next iteration. These steps are repeated until the convergence conditions are met. To date, a large number of functional materials have been discovered by CALYPSO, covering wide applications in lithium batteries<sup>[71,72]</sup>, superconductors<sup>[73]</sup>, photovoltaics<sup>[74]</sup>, and electronics<sup>[75]</sup>. The particle swarm optimization is simple to implement, with relatively few parameters that are easy to adjust. However, it may get trapped in local optima, especially in complex high-dimensional spaces or non-convex optimization problems.

## GA

The GA<sup>[17,76,77]</sup> mimics the mechanism of natural selection, choosing individuals with the highest adaptability for reproduction. In CSP, each individual represents a potential crystal structure, and the fitness of the configuration is primarily determined by its energy, with lower energy indicating higher fitness. For instance, Universal Structure Predictor: Evolutionary Xtallography (USPEX)<sup>[17]</sup> is a GA-based CSP software widely used for discovering new materials, optimizing existing ones, and understanding the underlying principles of crystal formation. The core steps include: First, selecting two or more parent crystal structures from the existing population based on the fitness function. Then, a crossover operation is performed, where parts of the parent chromosomes are exchanged to generate new offspring crystal structures. Subsequently, with a certain probability, mutation is introduced in the offspring chromosomes, randomly altering some genes (e.g., unit cell parameters) to introduce genetic diversity. The new generation population includes high-fitness individuals inherited from the parents and high-fitness offspring. This process iterates until the convergence conditions are met. USPEX has been widely utilized to identify various functional materials<sup>[78–81]</sup>, such as novel electride materials  $\text{Sr}_5\text{P}_3$ <sup>[82]</sup>, hard metallic phase  $\text{TiN}_2$ <sup>[83]</sup>, high  $T_c$  superconductor  $\text{H}_3\text{S}$ <sup>[84]</sup>, and transparent high-pressure phase of sodium<sup>[85]</sup>. The GA demonstrates significant capability in handling complex, nonlinear optimization problems, effectively avoiding entrapment in local optima. However, GA-based methods for CSP sometimes require numerous evaluations of potential solutions to evolve optimal candidate structures. When combined with computationally intensive calculations such as DFT, the overall computational cost can become substantial, particularly in systems with large numbers of atoms where DFT calculations are especially time-consuming<sup>[86]</sup>. Fortunately, recent advancements, such as the integration of ML models in USPEX, have helped to alleviate some of these challenges<sup>[87]</sup>.

## Bayesian optimization

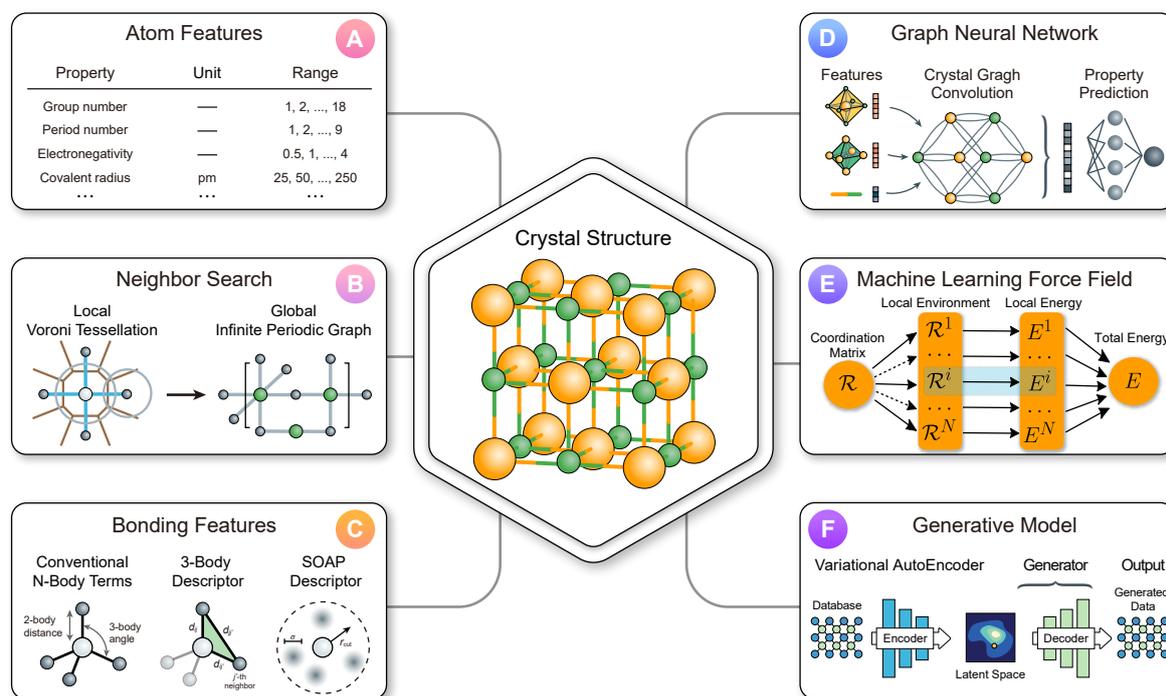
Making use of the Bayesian theory and Gaussian process regression, Bayesian optimization<sup>[88]</sup> can significantly reduce the computational time and accelerate the structure search process by constructing surrogate models. It mainly consists of two parts: the surrogate model based on Gaussian process regression, and the acquisition function, which guides the search process. Bayesian optimization has been widely applied to search for clusters, such as  $\text{Cu}_{15}$ <sup>[89]</sup>,  $\text{CuNi}$ <sup>[90]</sup>, and  $\text{C}_{24}$ <sup>[91]</sup> clusters. Bayesian optimization exhibits great potential in CSP but still faces several challenges<sup>[92–94]</sup>. First, updating the surrogate model and calculating the acquisition function can be very time-consuming. In addition, the noise and uncertainty in actual calculations can affect the accuracy of the model and the stability of the optimization process.

## Simulated annealing

The simulated annealing<sup>[95–97]</sup> is a random search method inspired by the natural phenomenon of atomic rearrangement in solid-state materials that achieve the lowest-energy state through slow cooling after heating. The core principle is to temporarily allow the system to enter higher energy states during the search process, which helps to avoid premature convergence to local optima. Simulated annealing has been successfully used to predict the crystal structures of  $\text{LiF}$ <sup>[97]</sup>,  $\text{GeF}_2$ <sup>[98]</sup> and  $\text{BN}$ <sup>[99]</sup> and to investigate the properties of  $\text{IrO}_2$  and  $\text{RuO}_2$  surfaces<sup>[100]</sup>. Simulated annealing is favored in optimization problems mainly due to its simplicity and effectiveness in avoiding trapping by local minima, thereby increasing the likelihood of finding the global minimum. However, the performance of the algorithm heavily depends on parameter settings, such as initial temperature, cooling rate, and termination temperature. Determining the optimal values for these parameters often requires extensive experience and numerous tests.

## Template-based method

Besides these ab initio methods, another widely used CSP approach is the template-based method. A well-known example of this approach is ion substitution<sup>[101]</sup>. Traditionally, this method involves replacing an ion in the crystal structure of a known compound with a chemically similar ion, guided by empirical rules such as the Goldschmidt rules<sup>[102]</sup>. This process has been further enhanced by a probabilistic model, which quantitatively



**Figure 3.** Application of machine-learning models in CSP. (A) Representation of atoms in machine-learning models. Reproduced with permission [107]. Copyright 2018, American Physical Society; (B) Neighbor search using Voronoi tessellation and construction of a global periodic graph. Reproduced from Ref. [108]. CC BY 4.0; (C) Representation of bonding between atoms in machine-learning models. Reproduced from Ref. [109]. CC BY 4.0; (D) Architecture of crystal graph convolutional neural network. Reproduced with permission [107]. Copyright 2018, American Physical Society; (E) Architecture of the neural network used in machine-learning force fields. Reproduced from Ref. [110]. CC BY-NC 4.0; (F) VAE for stable structure generation. Reproduced from Ref. [111]. CC BY-NC 4.0. CSP: Crystal structure prediction; VAE: variational autoencoder.

predicts the likelihood of successful ionic substitution by analyzing a vast database of crystal structures [103]. This data-driven model not only improves the accuracy of predicting new compounds, but also accelerates the materials discovery process by efficiently identifying novel structures with reduced computational resources. For instance, using this method, a comprehensive stability map of inorganic ternary metal nitrides has been constructed, leading to the synthesis of several new Zn- and Mg-based ternary nitrides [104].

Although conventional CSP methods have achieved remarkable accomplishments, most of them still suffer from low computational efficiencies, in addition to the limitations of each global optimization algorithm mentioned above. The main time cost lies in the optimization of structures, as there is rarely a guarantee that the structures are near the local minima on the potential energy surface, leading to tens of thousands of DFT calculations or time-consuming optimizations. Fortunately, advanced ML techniques have shed new light on tackling these challenges, opening up new possibilities in this direction.

## APPLICATIONS OF ML IN CSP

In recent years, ML has achieved a better balance between speed and accuracy by embedding physical knowledge into neural networks [105], such as energies, forces, stresses, and magnetic moments, and training on large-scale data [106]. By leveraging the advantages of ML models, they can usually be combined with CSP in the following four aspects.

- **Crystal Structure Representation:** ML-based structure representation methods can accurately capture the geometric and topological features of crystals [Figure 3A-C] [107–111], converting complex structural infor-

mation into high-dimensional crystal feature vectors<sup>[109,112]</sup>. These feature vectors not only contain sufficient structural information, but also exhibit rotational invariance, translational invariance, and index permutation invariance<sup>[107]</sup>. Most importantly, these feature vectors can reveal intrinsic connections and differences between structures, greatly enhancing the effectiveness of structure clustering during the search process.

- **Property Prediction and Rapid Screening:** ML models, especially the GNN shown in [Figure 3D](#), can quickly predict the physical properties of candidate materials<sup>[113–115]</sup>, such as energy, band gap, and performance for different applications. Moreover, by combining ML models with global optimization algorithms such as simulated annealing, GAs, and particle swarm optimization, low-energy crystal structures can be efficiently identified<sup>[116]</sup>.
- **Machine-Learning Force Field:** ML force fields [[Figure 3E](#)] can be used for structure optimization. Compared to classical force fields, ML-based force fields maintain near first-principles calculation accuracy while significantly reducing the computational cost, making the simulation of large-scale complex systems feasible<sup>[117,118]</sup>. Also, they enable high-throughput material screening and the construction of material databases<sup>[119,120]</sup>.
- **Generative Model:** Generative models [[Figure 3F](#)] learn the distribution of data and sample new data instances from this learned distribution<sup>[111]</sup>, enabling the exploration of a more diverse range of crystal structures. Some advanced generative models provide better compositional and structural diversity than substitution-based enumeration in high-throughput calculations and better structural generation efficiency<sup>[121,122]</sup> than conventional CSP techniques.

In this section, we review applications of crystal structure characterization, property prediction, and ML force fields in structure generation, global structure search, and local structure optimization, respectively. Finally, we will discuss the generative model, which differs from the typical CSP workflow.

### Structure generation

In the ML-based CSP, once the initial structures are generated, suitable descriptors are needed to capture the geometric and topological information of the crystal structure. By converting crystal structures into a readable format using ML models, we can effectively represent structures and learn the relationship between structure and properties. The descriptors used to construct ML models should meet the following three basic criteria<sup>[107,123]</sup>:

1. **Physical Consistency:** The descriptors should maintain physical invariance, meaning that their values should not change with the rotation and translation of the structure.
2. **Index Invariance:** The descriptors should be insensitive to the indexing order of the atoms. Even if the order or numbering of the atoms changes, the descriptor values should remain unchanged, ensuring model consistency and stability.
3. **Discrimination:** The descriptors should be able to distinguish different atomic environments. Similar local chemical environments should yield similar descriptors, while different local chemical environments should result in significantly different descriptors.

There are currently two main approaches to structure representation: continuous 3D voxel representation and matrix representation. In the continuous 3D voxel representation<sup>[124]</sup>, encoders and decoders are employed to prepare 2D crystal graphs and to reconstruct 3D voxel images. In the matrix representation<sup>[125–127]</sup>, crystal structure features such as lattice parameters, atomic occupation coordinates, and elemental properties are separated into different matrix rows and columns. Since widely used GNN and ML force fields mainly adopt the matrix representation, we will focus on the matrix representation, including the atom features and bonding features.

**Table 1. Atom features used in CGCNN**

Descriptor	Unit	Range	Number of categories
Group number	-	1,2,...,18	18
Period number	-	1,2,...,9	9
Electronegativity <sup>[129,130]</sup>	-	0.5-4.0	10
Covalent radius <sup>[131]</sup>	pm	25-250	10
Valence electrons	-	1,2,...,12	12
First ionization energy	eV	1.3-3.3	10
Electron affinity <sup>[132]</sup>	eV	-3-3.7	10
Block	-	s,p,d,f	4
Atomic volume	cm <sup>3</sup> /mol	1.5-4.3	10

These atom features are encoded using one-hot vectors. Reproduced with permission<sup>[107]</sup>. Copyright 2018, American Physical Society. CGCNN: Crystal graph convolutional neural network.

### Atom features

Atom features are used to describe different atoms in ML models<sup>[107,126,128]</sup>. As illustrated in Table 1<sup>[107,129–132]</sup>, the initial atomic feature vectors contain various elemental properties. These descriptors can uniquely determine each element and include their main physical properties. In advanced GNNs, such as message passing neural network (MPNN)<sup>[128]</sup>, crystal graph convolutional neural network (CGCNN)<sup>[107]</sup>, materials graph network (MEGNet)<sup>[126]</sup>, and atomistic line graph neural network (ALIGNN)<sup>[133]</sup>, the initial atomic features are processed through fully connected layers to construct atomic representations that are more strongly correlated with the target properties.

### Bonding features

Bonding features are used to describe the local environment of each atom. In GNNs, the bonding features are directly used as input to the ML model. In ML force fields, the input is the atom positions, and the bonding features are obtained via symmetry functions. The Behler-Parrinello and smooth overlap of atomic positions (SOAP) descriptors are two commonly used bonding features, and we introduce them as follows.

The Behler-Parrinello descriptor<sup>[32,134]</sup> uses a set of symmetry functions to characterize the local chemical environment of each atom. It consists of two types of symmetry functions: radial and angular symmetry functions, which capture distance and angle information between atoms, respectively.

SOAP is another descriptor used to characterize the local environment of atoms<sup>[135]</sup>. The SOAP descriptor represents the environment of each atom as a continuous density field, capturing the geometric properties of the atomic environment by calculating the overlap of density fields.

The calculation process of the SOAP descriptor for the local environment of atom  $i$  can be expressed by

$$\rho_i(\mathbf{r}) = \sum_{j \neq i} \exp\left(-\frac{|\mathbf{r} - \mathbf{r}_{ij}|^2}{2\sigma^2}\right), \quad (2)$$

where  $\mathbf{r}_{ij}$  is the position vector of atom  $j$  relative to atom  $i$ , and  $\sigma$  is the width of the Gaussian function.

To incorporate angular information, the atomic density  $\rho_i(\mathbf{r})$  is expanded using spherical harmonics  $Y_{lm}(\theta, \phi)$  and radial basis functions  $g_n(r)$ :

$$\rho(\mathbf{r}) = \sum_{n,l,m} c_{nlm} g_n(r) Y_{lm}(\theta, \phi). \quad (3)$$

The expansion coefficients are calculated by inner product:

$$c_{nlm} = \int \rho_i(\mathbf{r}) g_n(r) Y_{lm}(\theta, \phi) d\mathbf{r}, \quad (4)$$

where  $r = |\mathbf{r}|$ , and  $\theta$  and  $\phi$  are the polar and azimuthal angles in the spherical coordinate system, respectively.

To ensure the rotational invariance of the descriptor, the SOAP descriptor calculates the power spectrum of the expansion coefficients:

$$p_{nm'l} = \sqrt{\frac{4\pi}{2l+1}} \sum_{m=-l}^l c_{nlm} c_{n'lm}^* \quad (5)$$

The vector composed of the power spectrum  $p_{nm'l}$  constitutes the SOAP descriptor. It not only captures the local environmental characteristics of atom  $i$ , but also ensures invariance to translation and rotation. Therefore, the SOAP descriptor can accurately characterize the distances and directions between an atom and its neighbors.

When constructing ML models, atoms can be encoded by property-based one-hot vectors. The Behler-Parrinello or SOAP descriptors generate a high-dimensional bonding feature vector for each atom. These vectors serve as input for the ML models, and the output is the total energy, enabling the ML model to map the local atomic environment to energy.

### Global structure search

With the increasing size of open material databases<sup>[18,120,136–138]</sup> and the development of ML models<sup>[139–141]</sup>, it has become a common practice to screen hundreds of thousands of materials to identify potential candidates<sup>[19,142,143]</sup>. A typical workflow for applying an ML model to screen structures in CSP is shown in [Figure 4A](#)<sup>[107,126,128,133,144]</sup>. The ML model is pretrained using databases [[Figure 4B](#)] and then takes the generated structures as input, predicting their energies. In this way, the ML model can identify low-energy candidates from a vast number of initial structures, allowing the low-energy areas on the potential energy surface to be quickly located. In this section, we introduce the commonly used GNNs for property prediction in CSP and GNN-based CSP methods.

The MPNN<sup>[128]</sup> provides a general framework for GNN [[Figure 4C](#)]. It includes three stages: message generation, message passing, and message readout. Messages generated at adjacent vertices are collected by the central vertex to update its representation. By repeating this process, the GNN captures higher-order abstract features. Last, through the message readout process, the global graph features are mapped to the target properties.

Specifically, MPNN updates the representation of each vertex in the graph through the following steps:

1. Message Passing: Each vertex  $v$  collects messages from its neighbors  $w \in N(v)$  and updates its state based on these messages:

$$\mathbf{m}_v^{t+1} = \sum_{w \in N(v)} M_t(\mathbf{h}_v^t, \mathbf{h}_w^t, \mathbf{e}_{vw}), \quad (6)$$

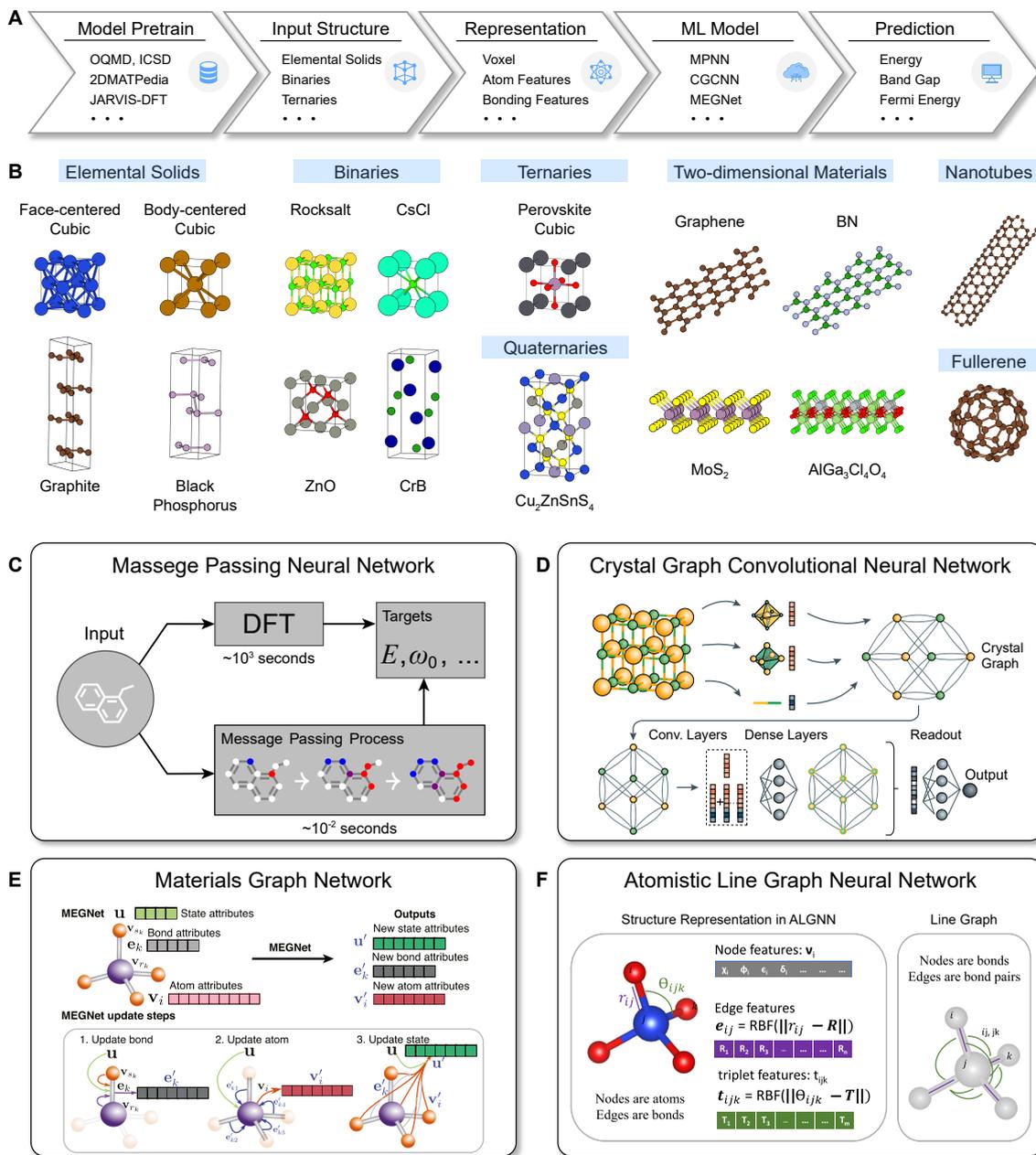
$$\mathbf{h}_v^{t+1} = U_t(\mathbf{h}_v^t, \mathbf{m}_v^{t+1}), \quad (7)$$

where  $\mathbf{h}_v^t$  is the feature vector of vertex  $v$  at the  $t$ -th iteration,  $M_t$  is used to aggregate messages from neighbors, and  $U_t$  is used to update the vertex features.

2. Message Readout: After  $T$  rounds of message passing, the global representation of the graph can be obtained by aggregating the feature vectors of all vertices:

$$\hat{y} = R(\{\mathbf{h}_v^T \mid v \in \mathcal{V}\}), \quad (8)$$

where  $R$  is the readout function, which can be a simple summation or a more complex pooling operation.



**Figure 4.** GNNs applied in CSP. (A) Prediction pipeline; (B) Examples of structures stored in database. Reproduced from Ref. [144]. CC BY 4.0; (C) MPNN predicts the quantum properties of an organic molecule. Reproduced from Ref. [128]. CC BY-NC 4.0; (D) Illustration of the CGCNN, including construction of the crystal graph and then building the structure of the convolutional neural network on top of the crystal graph. Reproduced with permission [107]. Copyright 2018, American Physical Society; (E) Overview of MEGNet. The initial graph is represented by the set of atomic attributes  $\mathbf{v} = \{\mathbf{v}_i\}_{i=1}^{N_c}$ , bond attributes  $\mathbf{E} = \{(e_k, r_k, s_k)\}_{k=1}^{N_c}$ , and global state attributes  $\mathbf{u}$ . Reproduced with permission [126]. Copyright 2019, American Chemical Society; (F) ALIGNN convolution layer alternates between message passing on the bond graph and its line graph. Reproduced from Ref. [133]. CC BY 4.0. GNNs: graph neural networks; CSP: crystal structure prediction; MPNN: message passing neural network; CGCNN: crystal graph convolutional neural network; MEGNet: materials graph network; ALIGNN: atomistic line graph neural network.

Most GNNs can be represented using the MPNN framework, including the Molecular Fingerprint Convolution Network [145], the Gated Graph Neural Network [146], Interaction Networks [147], Molecular Graph Convolutional Networks [148], Deep Tensor Networks [127], and Graph Laplacian Matrix Networks [149]. Here, we concentrate on three GNNs suitable for crystal property prediction: CGCNN [107], MEGNet [126], and ALIGNN [133], which can also be built using the MPNN framework.

CGCNN is a well-known GNN model designed for predicting crystal properties [Figure 4D]. It effectively transforms periodic crystal structures into undirected multigraph representations and introduces a crystal graph convolution operator as a message aggregation function. Specifically, the graph convolution operator is defined as

$$\mathbf{v}_i^{(t+1)} = \mathbf{v}_i^{(t)} + \sum_{j,k} \sigma(\mathbf{z}_{(i,j)k}^{(t)} \mathbf{W}_f^{(t)} + \mathbf{b}_f^{(t)}) \odot g(\mathbf{z}_{(i,j)k}^{(t)} \mathbf{W}_s^{(t)} + \mathbf{b}_s^{(t)}), \quad (9)$$

where  $\sigma$  is the Sigmoid function,  $\mathbf{W}_f^{(t)}$  and  $\mathbf{W}_s^{(t)}$  are the shared weights for the  $t$ -th aggregation, and  $\mathbf{b}_f^{(t)}$  and  $\mathbf{b}_s^{(t)}$  are the shared biases for the  $t$ -th aggregation. The interaction feature is defined as  $\mathbf{z}_{(i,j)k}^{(t)} = \mathbf{v}_i^{(t)} \oplus \mathbf{v}_j^{(t)} \oplus \mathbf{u}_{(i,j)k}$ .

Thus, the graph convolution operator effectively represents atomic interactions using  $\sigma(\cdot)$  for weights and  $g(\cdot)$  for bonding features. By repeatedly aggregating the atomic feature vectors  $\mathbf{v}_i^{(t)}$  using the graph convolution operator, we obtain the feature representation of atoms in the crystal  $\mathbf{v}_N^{(R)}$ . The crystal feature vector is obtained by a pooling layer  $\mathbf{v}_c = \text{Pool}(\mathbf{v}_0^{(0)}, \mathbf{v}_1^{(0)}, \dots, \mathbf{v}_N^{(0)}, \dots, \mathbf{v}_N^{(R)})$ . Finally, a fully connected network associates the crystal feature vector with material properties to predict properties.

MEGNet is a universal property prediction model for molecules and crystals. Compared to CGCNN, MEGNet encodes the macroscopic properties of the system (such as temperature, pressure, entropy, *etc.*) into feature vectors, enhancing the ability to predict material properties. As shown in Figure 4E, the feature encoding and aggregation process in MEGNet includes the atomic feature vector  $\mathbf{v}_i$ , the bond feature vector  $\mathbf{e}_k$ , and the system feature vector  $\mathbf{u}$ , which contains information about the macroscopic properties of the system.

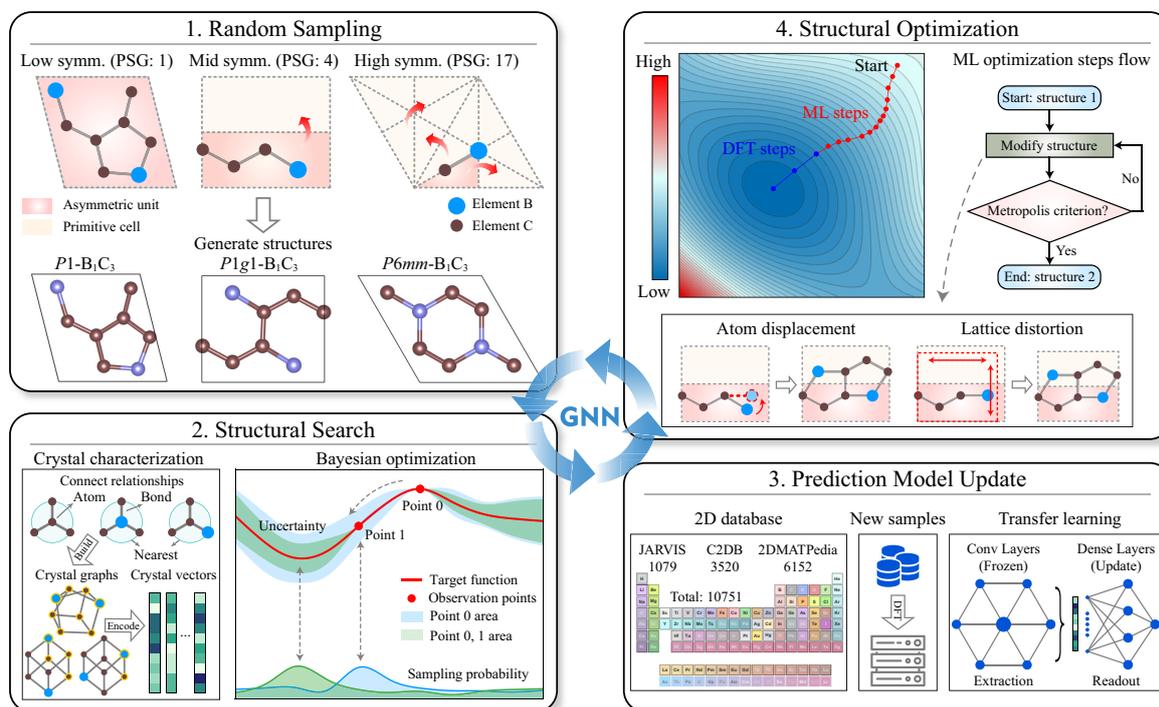
ALIGNN is a GNN model designed for predicting crystal properties [Figure 4F]. Its key innovation is the construction of a line graph that includes angular information, allowing messages to be passed between the bond graph and its corresponding line graph. Compared to CGCNN, ALIGNN explicitly incorporates angular information, enhancing the ability to distinguish between different structures.

Combining GNN and CSP, Cheng *et al.* have developed an accelerated CSP framework<sup>[67]</sup>. It mainly includes three parts: (i) pre-training of ML models; (ii) structure generation with physical constraints; and (iii) structure search and optimization based on ML. In the framework, GNNs such as CGCNN, MEGNet, ALIGNN, and CHGNet can be potentially used as prediction models, while algorithms such as random search, simulated annealing, GAs, or particle swarm optimization can be employed.

Recently, the symmetry-based combinatorial crystal optimization program (SCCOP)<sup>[150,151]</sup> has been developed for 2D materials. The workflow of SCCOP is shown in Figure 5. SCCOP first converts the structures generated from 17 plane space groups to crystal vectors using a direct asymmetry space-based GNN and predicts their energies. Then, Bayesian optimization is performed to explore the structure located at the minimum of the potential energy surface.

For the desired structures, SCCOP optimizes them with ML-accelerated simulated annealing, in conjunction with a limited number of DFT calculations, to obtain the lowest-energy structure.

To evaluate the effectiveness of SCCOP, it was applied to a total of 35 representative 2D materials. Figure 6A provides a comparison between the lowest-energy structure in the 2D material database and the structures discovered by SCCOP. The results demonstrated that SCCOP successfully reaches the lowest energy level for 30 compounds within only a few minutes. Additionally, Figure 6B shows the three lowest-energy structures for eight compounds. For example, in the case of AgI, the lowest-energy structure in the database corresponds to a honeycomb structure (-2.308 eV/atom), while SCCOP identifies an energetically more favorable puckered



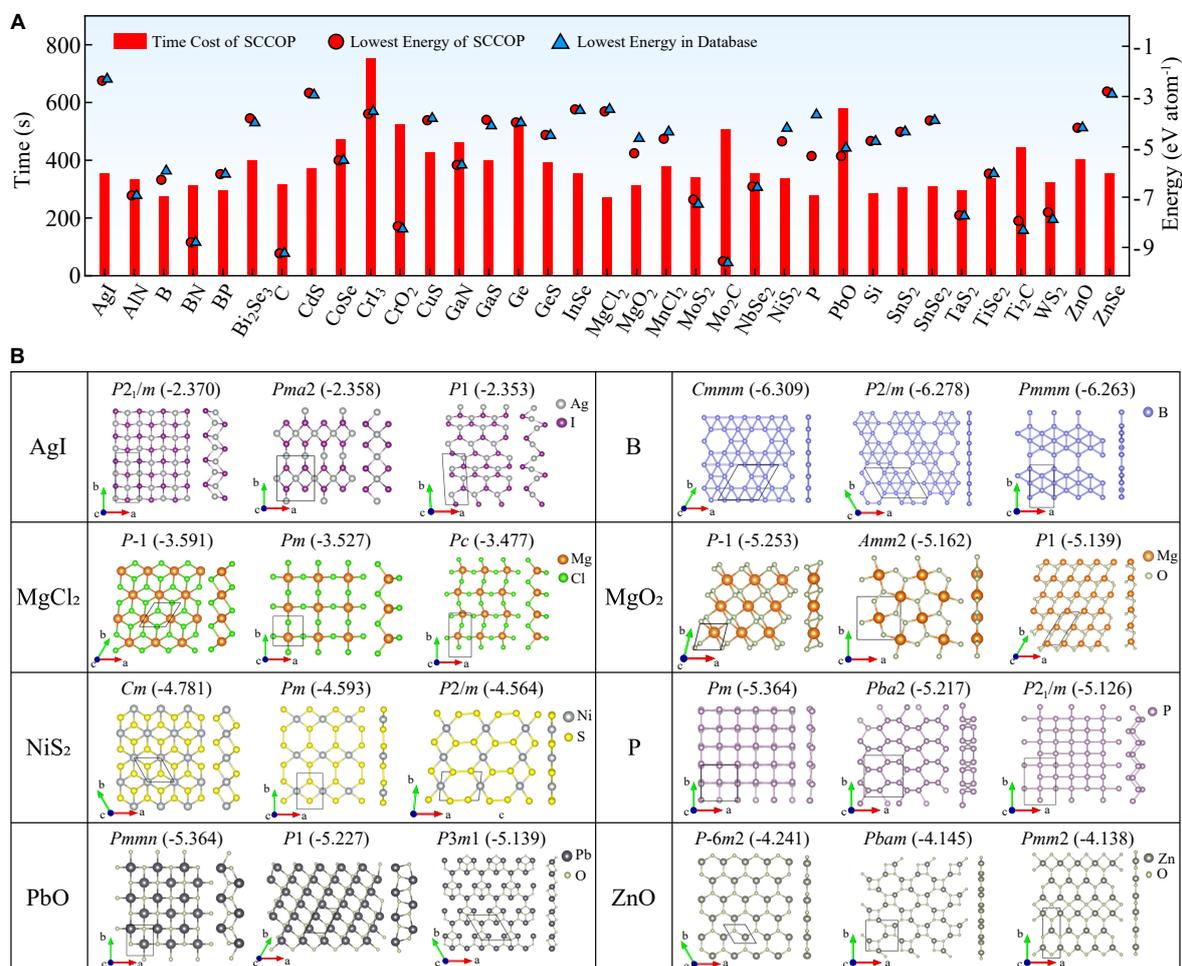
**Figure 5.** Workflow of SCCOP for the search of two-dimensional materials. Step 1: generating structures by symmetry. Step 2: characterizing structures into crystal vectors and exploring the potential energy surface by Bayesian optimization. Step 3: updating the energy prediction model. Step 4: optimizing structures to obtain the lowest-energy configuration by ML and DFT. The whole program runs in a closed loop. Reproduced from Ref. [150]. CC BY 4.0. SCCOP: symmetry-based combinatorial crystal optimization program; ML: machine learning; DFT: density functional theory.

structure with space group  $P2_1/m$  (-2.37 eV/atom). Furthermore,  $MgCl_2$  is recorded as having a four-fold coordination (-3.509 eV/atom) in the database, but SCCOP discovers that a structure with six-fold coordination exhibits lower energy (-3.591 eV/atom). SCCOP has been further applied to validate the stability of Cu- and Ag-based ternary compounds in the chalcopyrite structure prototype [152]. It has also been successfully utilized to investigate the mixed-coordination structures of IB-VA-VIA2 compounds, which have the lowest free energy at low temperatures compared to the octahedrally coordinated structure in experiments [153]. These applications highlight the wide-range applicability of SCCOP and demonstrate the feasibility of using GNNs to accelerate CSP.

### Local structure optimization

Although ML models can identify potential candidates in a short time, high-accuracy structure optimization is still needed to fully relax structures to their local minima on the potential energy surface. In conventional CSP methods, structures are optimized by DFT or classical force fields. While DFT has high accuracy, it is time-consuming. Classical force fields are much faster than DFT, but often lack sufficient precision when dealing with complex systems, such as metal-organic frameworks and biological macromolecules, especially in scenarios involving intricate electronic effects and chemical reactions [154,155].

Currently, ML force fields exhibit the potential to speed up the structure optimization. ML force fields can maintain the speed advantage of classical force fields while significantly improving prediction accuracy for complex systems, particularly in cases where classical force fields perform poorly. Through collective efforts of the field, many ML force fields have been developed, e.g., MEGNet [126], CHGNet [105], NequIP [156], and MACE [157]. ML force fields are commonly applied in studying the properties of new materials [158], the mechanisms of drug molecules [159], and the protein folding process [160]. Thus, using ML force fields to replace



**Figure 6.** Performance of SCCOP on 35 representative compounds. (A) Time cost and lowest energy for each compound, with all energy calculations evaluated with DFT; (B) Three lowest-energy structures identified by SCCOP. Each compound has been explored five times by SCCOP, with up to ten atoms in the unit cell. Reproduced from Ref. [150]. CC BY 4.0. SCCOP: symmetry-based combinatorial crystal optimization program; DFT: density functional theory.

time-consuming structural relaxation is a feasible way to speed up conventional CSP. In this section, we will discuss ML force fields and their applications in CSP.

While constructing ML force fields [Figure 7], the total energy is usually expressed as follows [32]:

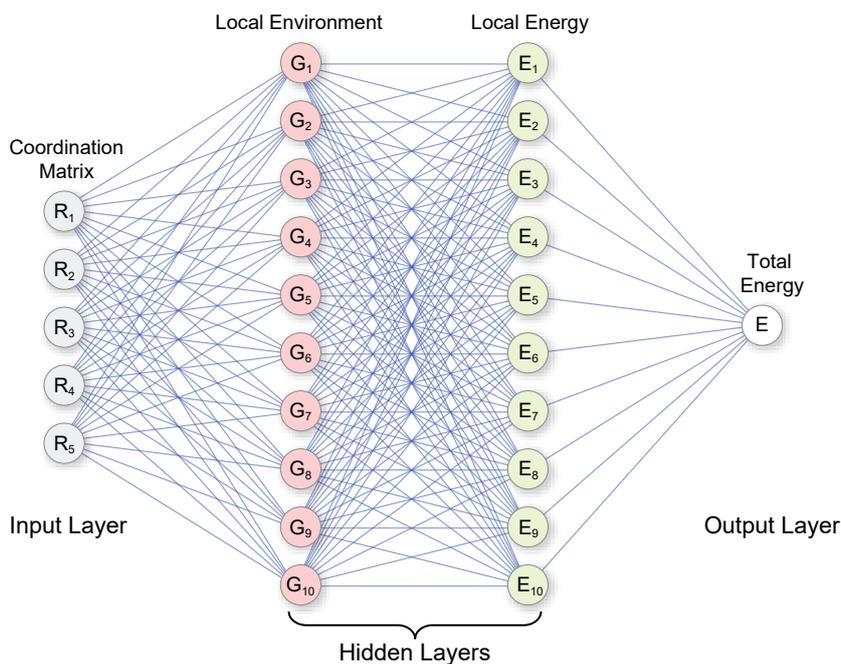
$$E_{\text{tot}} = \sum_i^N E_i(\mathbf{G}_i), \quad (10)$$

where  $N$  is the number of atoms in the unit cell, and  $\mathbf{G}_i$  is the feature vector of the  $i$ -th atom, representing the local chemical environment.  $E_i$  is the energy contribution of the  $i$ -th atom.

To train the ML force field, the simplest loss function only fits the energy:

$$\mathcal{L} = \sum_{i=1}^M (E_i^{\text{ref}} - E_i^{\text{pred}})^2, \quad (11)$$

where  $M$  represents the number of samples in the training set. More generally, by calculating the gradient of the energy with respect to the coordinates (i.e., atomic forces), we can derive a loss function commonly used



**Figure 7.** Neural network of ML force fields. This feedforward neural network consists of an input layer, two hidden layers, and an output layer. The input is a coordination matrix. The hidden layers transform the inputs to the local environment of each atom, mapping it to local energy, and finally summing them to get the total energy. Reproduced from Ref. [110]. CC BY-NC 4.0. ML: Machine learning.

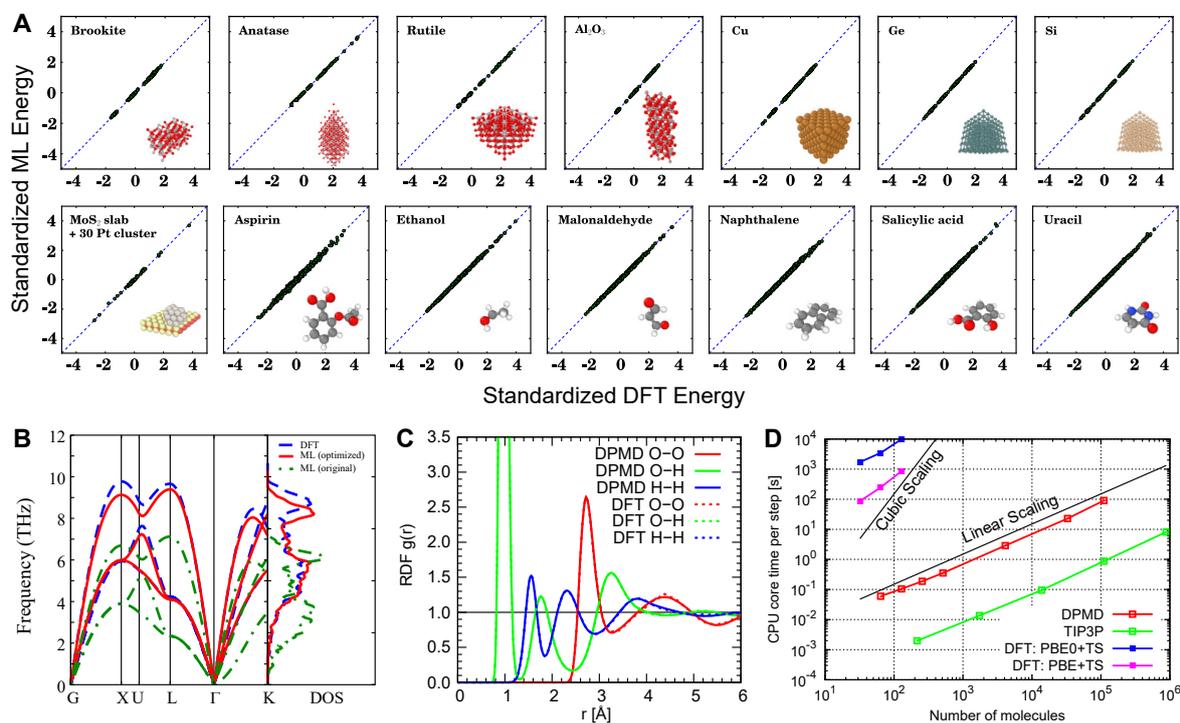
for model training<sup>[161]</sup>:

$$\mathcal{L} = \sum_{k=1}^M \left[ \alpha \left( \frac{E_k^{\text{ref}}}{N_k} - \frac{E_k^{\text{pred}}}{N_k} \right)^2 + \frac{\beta}{3N_k} \sum_{l=1}^3 \sum_{i=1}^{N_j} (\mathbf{F}_{il}^{\text{ref}} - \mathbf{F}_{il}^{\text{pred}})^2 \right], \quad (12)$$

where  $N_j$  is the number of atoms in sample  $k$ ,  $l$  denotes the  $x$ ,  $y$ , or  $z$  direction in the Cartesian coordinate system, and  $\alpha$  and  $\beta$  are the weighting coefficients for energy and force, respectively. Some ML force fields add stress to the loss function during training<sup>[162,163]</sup>, thereby improving the efficiency of data utilization.

As shown in Figure 8A, ML force fields have been tested on the TiO<sub>2</sub> system<sup>[110]</sup>, which contains three different polymorphs: Anatase, Brookite, and Rutile. They have been demonstrated for other crystal systems such as Al<sub>2</sub>O<sub>3</sub>, Cu, Ge, and Si, as well as on MoS<sub>2</sub> slabs and small molecular systems. These results demonstrate that the trained ML force fields can adapt to various types of systems, and the energy is fitted with high precision, indicating its strong capability in force calculations.

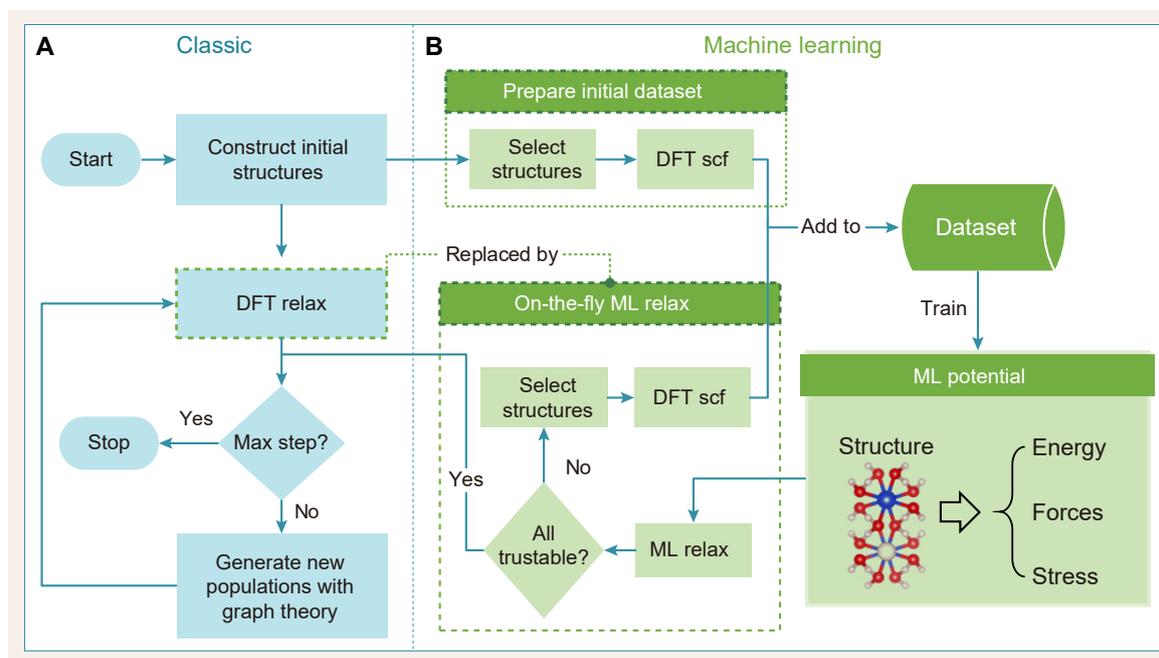
In addition to energy prediction, ML force fields can be used for more complex tasks, including phonon spectra and solid-liquid phase transitions. Figure 8B shows the phonon spectrum of fcc Al calculated using both DFT and ML force fields. The ML results are consistent with the DFT ones, demonstrating that ML force fields can accurately describe the vibrational behavior of materials<sup>[162]</sup>. In the case of water and ice, ML force fields and DFT were used to simulate different thermodynamic conditions<sup>[164]</sup>. The average energy, density, radial distribution functions [Figure 8C], and a representative angular distribution function (i.e., a three-body correlation function) have been reproduced with high accuracy. These results indicated that ML force fields can maintain the accuracy of DFT by model training. As shown in Figure 8D, the computational cost of ML force fields scales linearly with the number of atoms. Since all the physical quantities in an ML force field are sums of local contributions, this also means that after training on a relatively small system, the ML force field can be directly applied to much larger systems.



**Figure 8.** Validity and time cost of machine-learning force fields. (A) Comparison of the DFT energies and the DeepPot-SE predicted energies on the testing snapshots. Reproduced from Ref. [110]. Copyright 2018, Curran Associates Inc.; (B) Phonon band structure and DOS of fcc Al using DFT (blue dashed lines), and optimized (red solid lines) and original (green dashed lines) ML interatomic potentials. Reproduced with permission [162]. Copyright 2019, AIP Publishing; (C) Correlation functions of liquid water from DPMD and PI-AIMD. Reproduced with permission [164]. Copyright 2018, Elsevier; (D) Computational cost of MD steps versus system size with DPMD, TIP3P, PBE + TS, and PBE0 + TS. Reproduced with permission [164]. Copyright 2018, Elsevier. DFT: Density functional theory; DOS: density of states; ML: machine learning; DPMD: deep potential molecular dynamics; PI-AIMD: path-integral Ab initio molecular dynamics; MD: molecular dynamics; TIP3P: transferable intermolecular potential with 3 points; PBE: Perdew-Burke-Ernzerhof functional; TS: Tkatchenko-Scheffler functional.

To alleviate the bottleneck in CSP, ML force fields have been employed to replace the time-consuming DFT optimization. For example, a ML and graph theory assisted universal structure searcher (MAGUS) combines ML force fields with global optimization algorithms for structure search [Figure 9A] [165]. Specifically, the initial population is first generated by seeding and random generation. In each generation, the structures in the population are optimized using DFT or other force fields. Next, duplicate structures are removed from the population to maintain diversity. The remaining structures are then selected for crossover and mutation to create offspring. Generally, structures with higher fitness are more likely to be chosen as parents for crossover and mutation. The selection process can also incorporate the confidence level of the fitness with Bayesian optimization methods. As illustrated in Figure 9B, MAGUS trains an on-the-fly ML model during structure search, and uses this model to select and relax candidate structures to accelerate global searches. Using MAGUS, a stable superhard tungsten nitride (WN<sub>6</sub>) has been discovered, which can be quenched to ambient pressure after high-pressure synthesis [166]. Two different stable stoichiometries for helium-water compounds have also been predicted [167], both of which exhibit a superionic state at high pressures and temperatures.

Many conventional CSP methods have adopted ML force fields to accelerate the optimization process. To integrate ML force fields with CSP, a sampling strategy using disordered structures to train ML models has been developed [168]. By combining ML force fields and CALYPSO, the putative global minimum structure for the B<sub>84</sub> cluster has been uncovered, and the computational cost was substantially reduced by 1-2 orders of magnitude compared to full DFT-based structure searches [169,170]. In the ML-based USPEX, the methodology was first tested on the prediction of crystal structures of carbon, high-pressure phases of sodium, and boron



**Figure 9.** Workflow of MAGUS. (A) Classical evolutionary algorithm; (B) Machine-learning CSP. Reproduced from Ref. [165]. Copyright 2023, Oxford University Press. MAGUS: Machine learning and graph theory assisted universal structure searcher; CSP: crystal structure prediction.

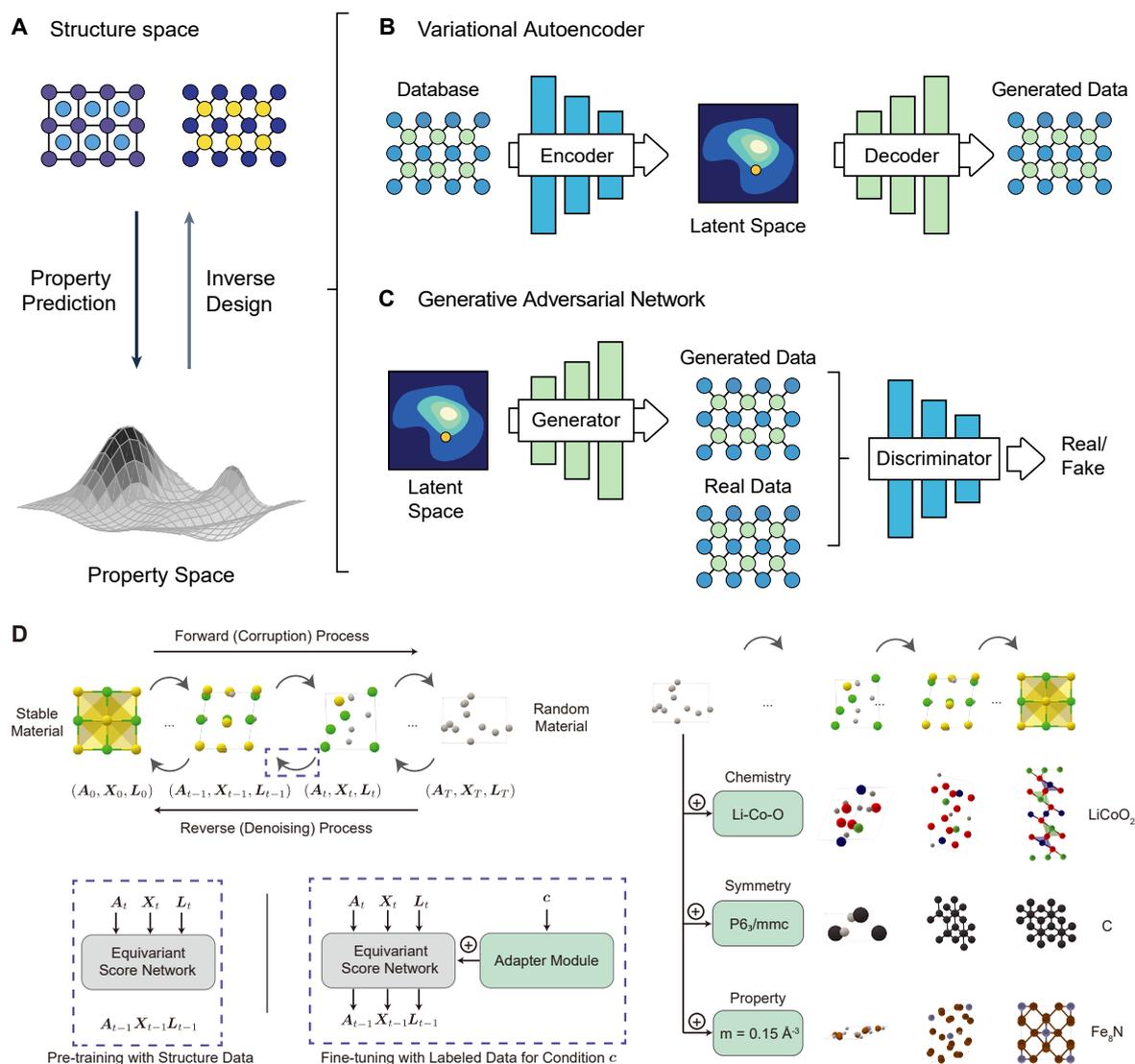
allotropes. For the test cases, the main allotropes have been reproduced, and a previously unknown 54-atom structure of boron has been predicted with very moderate computational effort [87]. Additionally, by integrating ML force fields with GAs, the structure prediction of inorganic crystals using neural network potentials with evolutionary and random searches (SPINNER) method has been presented, which identified experimentally known or theoretically more stable phases with a success rate of 80% for 60 ternary compounds [171], and high-throughput discovery of oxide materials using SPINNER has been conducted [172]. Furthermore, the  $\beta$ -rhombohedral boron structure has been studied [173] by ML-based AIRSS.

Despite these achievements, the implementation of ML force fields for structure optimization faces several challenges, including data requirements, model complexity, transferability, and computational efficiency. Solutions to these challenges include using data augmentation and transfer learning to enlarge datasets [174,175], applying explainable tools for better model interpretability [176], developing domain-specific and hybrid models to improve generalization [32], and employing model compression and efficient algorithms to enhance computational efficiency [177,178]. These strategies assist researchers in effectively utilizing ML force fields for accurate and efficient structure optimization.

### Generative model

Combining ML models with the general CSP steps has achieved significant progress in CSP, but it still struggles with the vast search space of feasible materials. Nowadays, thanks to breakthroughs in image generation [179,180], video generation [181,182], and realistic text generation [183], generative models in materials science show an unprecedented ability to learn the mapping between the structure and property spaces [Figure 10A]. Thus, generative models such as variational autoencoders (VAEs), generative adversarial networks (GANs), and diffusion models offer a powerful approach for predicting material properties and discovering new materials, significantly reducing the computational cost and enabling rapid screening of target systems.

Among the generative models, VAEs, composed of an encoder and a decoder, minimize the reconstruct-



**Figure 10.** Material property prediction and inverse design by generative models. (A) Schematic showing material property prediction from the structure space to the property space (downward arrow), and inverse material design from the property space back to the structure space (upward arrow). Reproduced from Ref. [111]. CC BY-NC 4.0; (B) VAE. The VAE consists of an encoder that transforms the input sample feature vector to a latent distribution space, and a decoder that reconstructs the sample given the hidden distribution. The VAE also models the latent space vector  $z$  from a normal distribution  $N(\mu, \sigma)$  with a mean  $\mu$  and a standard deviation  $\sigma$ . Reproduced from Ref. [111]. CC BY-NC 4.0; (C) GAN. GAN uses a generator to transform a random noise variable into the generated sample, and a discriminator to distinguish whether a sample is real or generated. Reproduced from Ref. [111]. CC BY-NC 4.0; (D) Inorganic materials design with MatterGen. It generates stable materials by reversing a corruption process by iteratively denoising an initially random structure. Reproduced from Ref. [121]. CC BY-NC 4.0. VAE: Variational autoencoder; GAN: generative adversarial network.

tion error between the decoded and input data [Figure 10B]. Representative VAE structure predictors include image-based materials generators (iMatGen) [184] and the Fourier-transformed crystal properties (FTCP) framework [185]. Specifically, iMatGen uses an invertible image-based representation to encode solid-state materials, leading to the generation of synthesizable V-O compounds. FTCP adds a target-learning branch to map latent points to target properties, resulting in the generation of 142 new crystals with desired ground- and excited-state properties. VAEs are relatively easy to train and provide more diversified structures that better cover the distribution compared to other generative models. These models generate diversified structures, but may have a lower output validity rate.

GANs use a minimax game theory approach, with a generator transforming a random latent variable into a sample and a discriminator distinguishing real from generated samples [Figure 10C]. Many GAN-based CSP methods have been developed, such as the composition-conditioned crystal GAN<sup>[186]</sup>, crystalGAN<sup>[187]</sup>, the zeolite GAN (ZeoGAN)<sup>[188]</sup>, and the constrained crystals deep convolutional generative adversarial network (CCDCGAN)<sup>[189]</sup>. For instance, the composition-conditioned crystal GAN allows the extension of the latent variable  $z$  with desired conditions, such as user-defined composition, leading to the discovery of 23 novel Mg-Mn-O potential photoanode materials. CrystalGAN utilizes a cross-domain GAN to generate complex ternary Pd-H-Ni structures from simpler binary Pd-H and Ni-H structures. CCDCGAN employs a VAE to learn a reverse map from a latent 2D crystal representation back to crystal structures, which is then used to train a GAN to generate new crystal structures. Fung *et al.* applied CCDCGAN to explore the binary Bi-Se system, revealing distinct crystal structures that cover the entire composition range<sup>[189]</sup>. While GANs produce realistic structures, they are more challenging to train, requiring a balance between the generator and discriminator to prevent issues such as non-convergence and mode collapse.

The diffusion model generates samples by learning a score network to reverse a fixed destruction process<sup>[190]</sup>. In image generation, the diffusion process typically adds Gaussian noises. However, crystals have unique periodic structures and symmetries that require a customized diffusion process. In MatterGen, Zeni *et al.* introduced a novel diffusion process tailored for crystal structures<sup>[121]</sup>. As shown in Figure 10D, they defined a destruction process for each component that fits its geometry and has a physically meaningful noise distribution. Specifically, the coordinate diffusion adopts a wrapped normal distribution to obey periodic boundaries, approaching a uniform distribution at the noise limit. The lattice diffusion uses a symmetric form, approaching a cubic lattice distribution with an average value corresponding to the average atomic density in the training data. Atom diffusion is defined in a categorical space, where individual atoms are damaged to a masked state. Based on the destroyed structure, a score network is learned, which outputs equivariant scores for atom types, coordinates, and lattice, respectively, eliminating the need to learn symmetry from the data. Compared to prior generative models, e.g., crystal diffusion variational autoencoders (CDVAE), structures produced by MatterGen were more than twice as likely to be novel and stable, and more than 15 times closer to the local energy minimum.

From the introduction of advanced generative models applied in CSP, we can see that the biggest difference between generative models and the applications of ML in general CSP steps is that generative models, such as VAE, GAN, and diffusion models, are end-to-end systems. This means that the structure generation, structure search, and structure optimization are all done by neural networks, making it difficult to control each step. Interestingly, this is also the biggest advantage of current advanced ML models: they reduce human intervention. Parameters are determined by algorithms and training data, giving ML models the potential to extract better features and design better workflows than humans. However, there is still a long way to go, and more efforts are needed to fully control these advanced ML models.

At the end of this section, to help the readers quickly learn about the progress of CSP method development or to apply CSP codes in their research, we summarize the conventional CSP and ML-based CSP methods in Tables 2 and 3.

When selecting CSP methods, it is important to consider the system's complexity and specific needs. GAs, such as those in USPEX, are effective for exploring large search spaces, making them ideal for complex, multi-modal problems. Random search methods in AIRSS provide a straightforward, computationally inexpensive option for initial explorations. Particle swarm optimization, as used in CALYPSO, is suitable for systems requiring quick convergence. For versatile applications, evolutionary algorithms in genetic algorithm for structure and phase predictions (GASP) and module for ab initio structure evolution (MAISE) are recommended. Bayesian optimization in global optimization with first-principles energy expressions (GOFEE) and BEACON excels

**Table 2. Summary of CSP algorithm categories with their advantages and disadvantages**

Category	Advantages	Disadvantages/limitations
Conventional Methods	Effective for complex search spaces Fast convergence	Computationally expensive Can get trapped in local minima
ML-based Methods	Efficient with large datasets Captures structural properties well	Requires extensive training data Complexity in integration
Generative Models	Good for exploring novel structures Generates diverse structures	Computationally intensive Complex model training

CSP: Crystal structure prediction.

**Table 3. Some conventional and ML-based CSP codes, along with their applications**

Software	Methods	Part of applications
USPEX (2006) [66]	Evolutionary algorithm	NaCl (2013) [191], W-B (2018) [192]
XtalOPT (2010) [193]	Evolutionary algorithm	NaH <sub>n</sub> (2011) [194], H <sub>2</sub> O (2012) [195]
AIRSS (2011) [53,196]	Random search	SiH <sub>4</sub> (2006) [196], NH <sub>3,x</sub> (2008) [197]
CALYPSO (2012) [69,70]	Particle swarm optimization	Li (2011) [198], LaH <sub>10</sub> (2017) [199], P (2024) [200]
GASP (2013) [201]	Evolutionary algorithm	Li-Be (2008) [202], Li-Si (2013) [203]
AGA (2013) [86]	Adaptive GA	Zr-Co (2014) [204], MgO-SiO <sub>2</sub> (2017) [205]
MUSE (2014) [206]	Evolutionary algorithm	IrB <sub>4</sub> (2016) [207], NbSe <sub>2</sub> (2017) [208]
IM <sup>2</sup> ODE (2015) [209]	Differential evolution	TiO <sub>2</sub> (2014) [210], 2D SiS (2016) [211]
SYDSS (2018) [54]	Random search	H <sub>2</sub> O-NaCl (2018) [54], Cl-F (2020) [212]
MAISE (2021) [213]	Evolutionary algorithm	Fe-B (2010) [214], NaSn <sub>2</sub> (2016) [215]
GOFEE (2020) [216]	Bayesian optimization & GA	C <sub>24</sub> (2022) [91], Carbon clusters (2022) [91]
BEACON (2021) [89,90]	Bayesian optimization	Cu <sub>15</sub> (2021) [89], CuNi clusters (2021) [90]
CrySPY (2021) [217]	Bayesian optimization & GA	Y <sub>2</sub> Co <sub>17</sub> (2018) [218], Al <sub>2</sub> O <sub>3</sub> (2018) [218]
FTCP (2022) [185]	VAE	Au <sub>2</sub> Sc <sub>2</sub> O <sub>3</sub> (2022) [185], Y <sub>2</sub> Zn <sub>2</sub> As <sub>2</sub> O <sub>3</sub> (2022) [185]
GN-OA (2022) [67]	GNN & Optimization algorithms	Tested on typical compounds (2022) [67]
MAGUS (2023) [165,219]	GA & Bayesian optimization	WN <sub>6</sub> (2018) [166], HeH <sub>2</sub> O (2019) [167]
SCCOP (2023) [150]	GNN & Simulated annealing	B-C-N (2023) [150], AgBiS <sub>2</sub> (2024) [152,153]
iMatGen (2019) [184]	VAE	V-O (2019) [184]
CrystalGAN (2019) [187]	GAN	Pd-Ni-H (2019) [187], Mg-Ti-H (2019) [187]
CCDCGAN (2021) [189]	GAN	MoSe <sub>2</sub> (2021) [189]
MatterGen (2024) [121]	Diffusion model	V-Sr-O (2024) [121]
UniMat (2024) [220]	Diffusion model	Tested on typical compounds (2024) [220]
DiffCSP (2024) [221]	Diffusion model	Tested on typical compounds (2024) [221]
LLaMA-2 (2024) [222]	Large language-based model	Tested on typical compounds (2024) [222]

in optimizing expensive functions with fewer evaluations, which is ideal for computationally intensive problems. Generative models, such as those in iMatGen and CrystalGAN, are excellent for innovative materials design and exploring unknown structures by learning complex distributions. For systems requiring relevant property modeling, GNNs in SCCOP and graph network-optimization algorithm (GN-OA) are powerful tools. Finally, to leverage large datasets, consider language-based models such as LLaMA-2. For beginners, starting with USPEX or AIRSS is recommended, while CALYPSO and MAGUS are better suited for complex systems. MatterGen and iMatGen are ideal for innovative designs, while IM<sup>2</sup>ODE is great for constrained problems. SCCOP can greatly shorten the time while maintaining the DFT accuracy.

In general, conventional CSP methods remain successful due to their proven reliability and ability to handle complex systems [14,73,191,192,198]. These methods are grounded in fundamental physical and chemical principles, making them robust and trustworthy for a wide range of materials. They also benefit from incorporating geometric constraints and prior knowledge. Despite being computationally intensive, ongoing improvements and the integration of ML techniques have further solidified their status in modern materials science. For the ML-based CSP methods, they can significantly reduce computational time compared to conventional methods such as DFT. Traditional CSP approaches can take days to weeks to predict a structure on a small server containing dozens to hundreds of CPU cores, while ML models, once trained, can predict structures in seconds to minutes using the same computational resources [87,150,170]. This efficiency is achieved because ML approaches learn from existing data, facilitating effective feature extraction, rapid structure screening, and optimization,

thereby offering a more cost-effective alternative to conventional methods.

## SUMMARY

In this review, we discussed the current progress in CSP, particularly focusing on the applications of ML in CSP. To help the readers understand the basic concepts, progress, and challenges in this field, we first introduced the basics of conventional CSP methods. Next, we reviewed ML models combined with general CSP steps, including descriptors in structure generation, GNNs in structure search, and ML force fields in structure optimization. The application of ML models has significantly reduced the time required for CSP, and ML-based CSP methods have helped to find more low-energy structures for desired compositions<sup>[113–115]</sup>. We further discussed generative models, which differ greatly from ML models combined with general CSP steps. Generative models for CSP are entirely based on neural networks without DFT calculations; thus, they can be applied to very large systems.

Although ML models have made significant progress in solving CSP, they still face several challenges: (i) Overfitting and data collapse: ML models may overfit the database, preventing them from identifying low-energy structures in CSP, or may cause data collapse in generative models. To mitigate overfitting, techniques such as data augmentation<sup>[223]</sup>, dropout regularization<sup>[224]</sup>, and ensemble learning can be employed<sup>[225]</sup>. Additionally, employing early stopping and cross-validation methods can help prevent overfitting by ensuring the model is generalizing well on unseen data; (ii) Limited training data: ML models are often trained on stable or metastable structures stored in databases, which represent only a small part of the complex potential energy surface; thus, the generalization of ML models cannot be guaranteed. To address this, transfer learning<sup>[175]</sup> and active learning can be used to enhance model performance by incrementally expanding the training dataset with more diverse structures; (iii) Mismatch between local fitting models and global optimization algorithms: in CSP, ML models lack a theoretical guarantee of global generalization, which may cause global optimization algorithms to fail while converging to the correct results. This issue can be tackled by techniques such as multi-fidelity modeling<sup>[226]</sup>, which combine high-fidelity simulations with ML predictions to improve the reliability of global optimization. Despite these challenges, we remain optimistic that ML models will ultimately solve the challenging task of CSP, similar to the advancements seen in protein structure prediction<sup>[227]</sup>, thereby boosting materials science research and the discovery and design of new materials.

## DECLARATIONS

### Authors' contributions

Writing-original draft preparation: Li CN

Proposed the conception and design: Li CN, Liang HP, Zhang X

References collection: Li CN, Liang HP, Zhao BQ

Writing-review and editing: Zhang X, Wei SH

Supervision: Zhang X, Wei SH

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Not applicable.

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### Conflicts of interest

All authors declared that there are no conflicts of interest.

## Ethical approval and consent to participate

Not applicable.

## Consent for publication

Not applicable.

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