Monisha *et al*. *Complex Eng Syst* 2024;4:23
DOI: 10.20517/ces.2024.67

Complex Engineering Systems

Research Article

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Delay-dependent observer based control for uncertain fuzzy-chaotic systems with deception attacks and faulty actuators

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How to cite this article: Monisha S, Nithya V, Sakthivel R. Delay-dependent observer based control for uncertain fuzzy-chaotic systems with deception attacks and faulty actuators. *Complex Eng Syst* 2024;4:23. http://dx.doi.org/10.20517/ces.2024.67

Received: 30 Sep 2024 **First Decision:** 18 Nov 2024 **Revised:** 22 Nov 2024 **Accepted:** 27 Nov 2024 **Published:** 30 Nov 2024

Academic Editor: Hamid Reza Karimi **Copy Editor:** Fangling Lan **Production Editor:** Fangling Lan

Abstract

In this study, the robust fault-tolerant control problem for T-S fuzzy chaotic systems is studied with the use of a proportional integral observer. Particularly, to accurately consider the real-world scenario, the chaotic system model incorporates the parameter uncertainties, input delay and external disruptions. Furthermore, a fuzzy-based observer is put forward based on analyzed system output with the intent of estimating the states of undertaken chaotic systems. Herein, the system output is susceptible to randomly occurring deception attacks and adheres to the Bernoulli distribution. Precisely, the consideration of deception attacks in the output channel allows for more secure state estimation in a networked setting. Subsequently, we develop a proportional integral observer-based fault-tolerant control that allows for the attainment of goals despite faults and delays in the input channel. Moreover, by setting up the Lyapunov-Krasovskii functional and blending it with Wirtinger's integral inequalities, we put together adequate conditions that ensure the asymptotic stability of the closed-loop systems by establishing conditions in the form of linear matrix inequalities. In the follow-up, based on the established matrix inequalities, the exact procedure for computing the gain matrices is outlined. As a last step, we put forward numerical simulation outcomes that exhibit the viability of the theoretical findings.

Keywords: Chaotic systems, T-S fuzzy technique, fault-tolerant control, proportional-integral observer approach, deception attacks, extended dissipativity

1. INTRODUCTION

Chaos is an inherent nonlinear behavior with significant avenues in various real-world uses, including encrypted communication, processing data, identifying patterns, and chemical processes[1,2]. As a result, there has been a heightened focus on chaos-associated nonlinear behavior in the fields of science and technology. Chaotic systems exhibit unexpected and inconsistent characteristics attributed to initial-value sensitivities, topological temporary nature, pseudo-randomness and the absence of periodicity which can potentially result in functionality degradation or system instability[3–5]. Consequently, chaos control has emerged as an important topic across numerous domains. Numerous intriguing control methodologies have been developed to examine chaotic systems to date, such as feedback control[6], sliding mode control[7], finite-time control[8], and so on. Given these developments and their impact on chaotic systems, this topic has gained our attention.

Additionally, various causes, including unforeseen alterations in operational circumstances, dynamic modifications in system boundaries and behavioral shifts in physical components, might render the nonlinear characteristics of chaotic systems prevalent. Furthermore, when the system is subjected to the nonlinear phenomenon, the control mechanism that was designed for chaotic systems is incapable of producing favorable dynamic outcomes for chaotic systems. For the purpose of resolving the problem discussed above, the concept of fuzzy logic[9] is utilized in this investigation in order to effectively address the nonlinear characteristics of the chaotic system. In detail, the Takagi-Sugeno (T-S) fuzzy model captured growing attention from researchers owing to its outstanding ability to represent intricate dynamical processes. In detail, the T-S fuzzy model employs a collection of fuzzy IF-THEN constraints to depict local linear input-output relationships within a nonlinear system. Upon account of this attribute, the T-S fuzzy model is advantageous for chaotic system analysis and controller design[10]. Moreover, it has been disclosed that many chaotic systems, including Rosler system, Lorenz system and Chua's system, can be precisely represented by T-S fuzzy models[11–14]. Owing to these advantageous attributes, fuzzy model-based control has been extensively utilized in chaotic systems, prompting us to undertake this research.

Besides, control systems are prone to actuator failures, which could substantially affect system operations and, in extreme cases, can cause system instability[15]. Consequently, to preserve the ideal stability and proper operation, it is crucial to take the adverse outcomes of actuator flaws into account while crafting control systems[16]. This premise lends conception to a fault-tolerant control framework for dynamical systems[17]. To put it another way, fault-tolerant control deals with the failed actuators and restores system performance for ensuring the system runs consistently. The beneficial features of fault-tolerant control have prompted substantial research efforts[18,19]. In addition to the actuator fault, if the signal transmission times between the actuator and sensor are prolonged then the control input channel could experience delay and result in unexpected degradation in performance, leading to system instability[20–22]. Consequently, while creating a control protocol, it is relevant and even necessary to consider the input delay[23,24]. Similarly, when there is a discrepancy between the real system and its mathematical model, the performance of the system is likely to suffer[25]. Because of this, embracing uncertainty in system design is important from a theoretical and practical standpoint[26,27]. Furthermore, as a result of unpredictable variables, the external disruptions are pervasive[28,29]. In order to mitigate these impacts, the extended dissipativity concept is used, which reduces the impact of the disturbance in the system model[30–32]. Thus, it is theoretically and practically significant to include parameter uncertainties, actuator faults, external disturbances and input delay in T-S fuzzy chaotic systems.

Meanwhile, estimating the system state in practice gets extremely hard. To address this pressing issue, the researchers have proposed an observer-based state estimation method[33]. It should be noted that the mathematical model of the system being studied determines the structure of the observer. The accuracy of the state estimate is therefore dependent on the mathematical model used and the characteristics of the data that are used. To that end, the proportional integral observer (PIO) has garnered a lot of interest in many real-world systems, since the application of PIO is involved in manufacturing, power circuits, and network communications[34]. The PIO improves the estimate of the system states and boosts the system resiliency which is done by including the proportional and integral feedback term with regard to the output estimation error. As a consequence, the PIO technique has gained a lot of traction among academics[35–37]. It should be noted that the PIO is set up using the system's output signal but in a networked environment, it might be impacted by cyberattacks. Consequently, the observer design should take into account aftermath impacts of cyber attacks in order to obtain a secure estimate of the system states[38,39]. As a consequence of this notion, numerous intriguing findings on secure state estimation for various dynamical systems have been reported in the literature[40–49]. Notwithstanding these many initiatives, the analysis of T-S fuzzy chaotic systems in networked conditions subject to cyber attack scenarios remains an uncharted area and needs exploration.

Driven by above-discussed factors and findings, this study delves into the topic of PIO-based fault-tolerant control for T-S fuzzy chaotic systems that are susceptible to various factors. Additionally, the following is a summary of the research achievements presented in this article:

- A PIO-based fault-tolerant control law is configured for the T-S fuzzy chaotic systems in the case of external disturbances, actuator faults, deception attacks and input delays.
- In detail, with the intent of reconstructing the states of the undertaken model, a PIO is developed with the fuzzy technique. Additionally, to strengthen the model's information security, deception attacks are considered.
- Subsequently, by making use of the estimation information procured from the designed observer along with the feedback control technique, a PIO-based control law is developed, in which actuator faults and input delays are considered to precisely mimic the actual situations.
- By configuring the suitable Lyapunov-Krasovskii functional candidate, a set of adequate constraints is established in the frame of linear matrix inequalities (LMIs) which ensures the asymptotic stability of the investigated system.

At the end of this study, numerical examples accompanied by the simulation findings are offered which substantiate the effectiveness and applicability of the offered control mechanism and the established theoretical findings.

The subsequent sections of this paper are organized as follows: Section 2 outlines the mathematical modeling of the undertaken model and control design and offers the preliminaries. Section 3 presents the adequate constraints that ensure the primary intent of this study. The potential of the proposed findings is summarized in Section 4 through a simulation example. Finally, Section 5 provides a conclusion of our study, presenting insights and emphasizing the significant findings obtained from our investigation.

2. PROBLEM FORMULATION

2.1. Chaotic system description

Consider a class of T-S fuzzy chaotic fuzzy systems subject to multiple vulnerable factors as follows:
rule for the undertaken plant i: IF $\phi_1(t)$ is Φ_1^i and ... and $\phi_h(t)$ is Φ_h^i, THEN

$$\begin{cases}\dot{x}(t) = (\mathcal{A}_i + \Delta\mathcal{A}_i(t))x(t) + \mathcal{B}_i u^f(t - \wp(t)) + \mathcal{D}_i w(t), \quad i \in \omega = \{1, 2, 3, \ldots, l\} \\ y(t) = \mathcal{C}_i x(t), \end{cases} \tag{1}$$

where $\phi_1(t), \phi_2(t), \ldots, \phi_h(t)$ are the premise variables, and Φ_1^i, Φ_2^i, Φ_3^i,...,Φ_h^i are the fuzzy sets; l is the number of fuzzy rules; $x(t) \in \mathcal{R}^n$ represents the state vector; $u^f(t) \in \mathcal{R}^m$ signifies the control input; $y(t) \in \mathcal{R}^q$

denotes the measured output vector of the system; $\wp(t)$ indicates the time-varying delay that occurs in the control channel which satisfies $0 \leq \wp(t) \leq \bar{\wp}$ and $\dot{\wp}(t) \leq \varrho$, where $\varrho < 1$; $w(t) \in \mathcal{R}^w$ indicates the external disturbance; $\mathcal{A}_i, \mathcal{B}_i, \mathcal{D}_i, C_i$ are the known constant matrices. Moreover, $\Delta\mathcal{A}_i(t)$ signifies the parameter uncertainty which has the following layout $\Delta\mathcal{A}_i(t) = \mathfrak{B}_i\mathfrak{F}_i(t)\mathfrak{N}_i$, and $\mathfrak{F}_i^T(t)\mathfrak{F}_i(t) \leq I$ is a time-varying matrix that is unknown.

Through the utilization of fuzzy blending techniques, we can attain the underneath the fuzzy chaotic system (1) as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{l} \mathfrak{b}_i(\phi(t))\big((\mathcal{A}_i + \Delta\mathcal{A}_i(t))x(t) + \mathcal{B}_i u^f(t-\wp(t)) + \mathcal{D}_i w(t)\big) \\ y(t) = \sum_{i=1}^{l} \mathfrak{b}_i(\phi(t))(C_i x(t)), \end{cases} \tag{2}$$

where $\phi(t) = [\phi_1(t), \phi_2(t), \ldots, \phi_h(t)]$, $\mathfrak{b}_i(\phi(t))$ represents the normalized membership function satisfying:

$$\mathfrak{b}_i(\phi(t)) = \frac{\prod_{j=1}^{h} \Phi_j^i(\phi_j(t))}{\sum_{i=1}^{l}\prod_{j=1}^{h} \Phi_j^i(\phi_j(t))} \geq 0, \quad \sum_{i=1}^{l} \mathfrak{b}_i(\phi(t)) = 1,$$

and $\Phi_j^i(\phi_j(t))$ indicates the grade of membership of $\phi_j(t)$ in Φ_j^i.

2.2. Deception attack model

In the current investigation, the deception attack is taken into account in the measurement output channel. The mathematical configuration of the deception attack is laid out below:

$$y_d(t) = (1-\theta(t))y(t) + \theta(t)\zeta(y(t)), \tag{3}$$

where $\theta(t) \in \{0,1\}$ indicates the occurring probability of deception attacks, and $\zeta(y(t))$ illustrates the deceptive signal sent out by the attacker. Further, $\theta(t)$ follows the Bernoulli distribution and it has ensuing probability constraints: $Pr\{\theta(t)=1\} = \mathbb{E}\{\theta(t)\} = \bar{v}$, $Pr\{\theta(t)=0\} = 1-\mathbb{E}\{\theta(t)\} = 1-\bar{v}$, $\bar{v} \in [0,1]$ is pre-defined scalar.

2.3. Description of PIO system

In practice, directly measuring the system state (2) is not an easy task. In order to address this issue, a PIO system is developed to estimate the states of the model. Moreover, the observer system is articulated as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{l} \mathfrak{b}_i(\phi(t))\big(\mathcal{A}_i\hat{x}(t) + \mathcal{B}_i u^f(t-\wp(t)) + \mathcal{L}_i(y_d(t)-\hat{y}(t)) + \mathcal{B}_i x_p(t)\big), \\ \hat{y}(t) = \sum_{i=1}^{l} \mathfrak{b}_i(\phi(t))(C_i\hat{x}(t)), \\ \dot{x_p}(t) = -x_p(t) + \mathcal{L}_{pi}(y_d(t)-\hat{y}(t)) \end{cases} \tag{4}$$

where $\hat{x}(t) \in \mathcal{R}^n$ signifies the estimated states of $x(t)$ and $\hat{y}(t) \in \mathcal{R}^q$ epitomize the estimated output of $y(t)$; L_i and $\mathcal{L}_{pi}$ indicates the proportional and integral gain matrices of the designed observer, which will be reckoned in the later portion of this investigation.

2.4. PIO-based control scheme configuration with delay

Rule for the designed control law j: IF $\phi_1(t)$ is Φ_1^j and ... and $\phi_h(t)$ is Φ_h^j, THEN

By leveraging the estimation data obtained from the established observer configuration and the feedback mechanism, a control rule is provided which includes input delay to align closer with the real-world circumstances.

The dynamic configuration of the control law is given below:

$$u(t-\wp(t)) = \sum_{j=1}^{h} \mathfrak{b}_j(\phi(t))\mathcal{K}_j\,\hat{x}(\tau-\wp(t)), \tag{5}$$

where $\mathcal{K}_j$ is the gain matrices of the controller, which will be determined later.

Apart from this, we consider the controller with faulty actuator model which is defined by $u^f(t) = \mathfrak{G}_f u(t)$. In detail, $\mathfrak{G}_f = \text{diag}\{\mathrm{g}_1, \mathrm{g}_2, \dots, \mathrm{g}_m\}$, g_s $(s = 1, 2, \dots, m)$ is the fault factor of the s^{th} actuator that is assumed to lie in the interval $[\underline{\mathrm{g}}_s, \overline{\mathrm{g}}_s]$ with $0 < \underline{\mathrm{g}_s} \le \mathrm{g}_s \le \overline{\mathrm{g}_s} \le 1$. Furthermore, the fault matrix $\mathfrak{G}_f$ is represented as $\mathfrak{G}_f = \mathfrak{G}_0 + \mathfrak{G}_1\Sigma$, where $\mathfrak{G}_0 = \frac{\underline{\mathfrak{G}}+\overline{\mathfrak{G}}}{2}$, $\mathfrak{G}_1 = \frac{\overline{\mathfrak{G}}-\underline{\mathfrak{G}}}{2}$ with $\overline{\mathfrak{G}}_f = \text{diag}\{\overline{\mathrm{g}}_1, \overline{\mathrm{g}}_2, \dots, \overline{\mathrm{g}}_m\}$, $\underline{\mathfrak{G}}_f = \text{diag}\{\underline{\mathrm{g}}_1, \underline{\mathrm{g}}_2, \dots, \underline{\mathrm{g}}_m\}$ and $0 < \Sigma = \text{diag}\{\pi_1, \pi_2, \dots, \pi_m\} \le I$. Then, in accordance with (5), the control set-up with the actuator fault model can be written as follows:

$$u^f(t-\wp(t)) = \sum_{j=1}^{h} \mathfrak{b}_j(\phi(t))\,(\mathfrak{G}_0 + \mathfrak{G}_1\Sigma)\mathcal{K}_j\,\hat{x}(\tau-\wp(t)). \tag{6}$$

Further, by defining the error system $\mathfrak{e}(t)$ as $\mathfrak{e}(t) = x(t) - \hat{x}(t)$ and making use of relations (3)-(6), the following set-up of a closed-loop system can be obtained.

$$\dot{x}(t) = \sum_{i=1}^{l}\sum_{j=1}^{h} \mathfrak{b}_j(\phi(t))\big((\mathcal{A}_i + \Delta\mathcal{A}_i(t))x(t) + \mathcal{B}_i(\mathfrak{G}_0 + \mathfrak{G}_1\Sigma)\mathcal{K}_j\,\hat{x}(\tau-\wp(t)) + \mathcal{D}_i w(t)\big), \tag{7}$$

$$\mathfrak{e}(t) = \sum_{i=1}^{j} \mathfrak{b}_i(\phi(t))\big(\mathcal{A}_i\mathfrak{e}(t) + \Delta\mathcal{A}_i(t)x(t) - \mathcal{L}_i C_i\mathfrak{e}(t) + \theta(t)\mathcal{L}_i C_i x(t) - \theta(t)\mathcal{L}_i\zeta(y(t)) + \mathcal{D}_i w(t)\big) - \mathcal{B}_i x_p(t),$$

$$\dot{x_p}(t) = \sum_{i=1}^{l} \mathfrak{b}_i(\phi(t))\big(-x_p(t) + \mathcal{L}_{pi}C_i\mathfrak{e}(t) - \theta(t)\mathcal{L}_{pi}C_i x(t) + \theta(t)\mathcal{L}_{pi} f(y(t))\big). \tag{8}$$

Assumption 1 *For the given real symmetric matrices κ_1, κ_2, κ_3 and κ_4, the following constraints are satisfied:*
1)$\kappa_1 \le 0,\ \kappa_3 > 0\ \ and\ \ \kappa_4 \ge 0;$
2)$(\|\kappa_1\| + \|\kappa_2\|)\kappa_4 = 0.$

Assumption 2 *The nonlinear function $f(y(t))$ satisfies the following constraint*

$$||f(y(t))||^2 \le |\mathcal{H}y(t)|^2$$

where $\mathcal{H}$ is the constant matrix that denotes the upper bound of nonlinear function.

Definition 1 [28] *For the given matrices κ_1, κ_2, κ_3 and κ_4 with Assumption 1, the undertaken system (1) with the controller (6) is extended dissipative if the following inequality holds for $T_f \ge 0$:*

$$\mathbb{E}\left\{\int_0^{T_f} \zeta(t)dt\right\} - \sup_{0\le t\le T_f} \mathbb{E}\left\{y^T(t)\kappa_4 y(t)\right\} \ge \gamma, \tag{9}$$

where $\zeta(t) = y^T(t)\kappa_1 y(t) + y^T(t)\kappa_2 w(t) + w^T(t)\kappa_3 w(t)$.

Lemma 1 [50] *For any positive definite matrix $\mathcal{Z}$, given scalars u and v satisfying $u < v$, the condition given below is satisfied for all continuously differentiable function $\xi : [u, v] \to \mathcal{R}^\alpha$:*

$$(u-v)\int_u^v \xi^T(\mathrm{s})\mathcal{Z}\xi(\mathrm{s})d\mathrm{s} \ge \begin{bmatrix} \int_u^v \xi(k)dk \\ \int_u^v dk\int_u^s \xi(s)ds \\ \int_u^v dk\int_u^s ds\int_u^r \xi(r)dr \end{bmatrix}^T \begin{bmatrix} 9\mathcal{Z} & -\frac{36}{u-v}\mathcal{Z} & \frac{60}{(u-v)^2}\mathcal{Z} \\ * & \frac{192}{(u-v)^2}\mathcal{Z} & -\frac{360}{(u-v)^3}\mathcal{Z} \\ * & * & \frac{720}{(u-v)^4}\mathcal{Z} \end{bmatrix} \begin{bmatrix} \int_u^v \xi(k)dk \\ \int_u^v dk\int_u^s \xi(s)ds \\ \int_u^v dk\int_u^s ds\int_u^r \xi(r)dr \end{bmatrix}.$$

3. MAIN RESULTS

In this section, we focus on deriving sufficient conditions for ascertaining the asymptotic stability of the considered system by configuring Lyapunov-Krasovskii functional and blending it with the extended Wirtinger's integral inequality.

Theorem 1 *For given positive scalars $\bar{\wp}$, $\bar{v}$, ϱ, actuator fault matrix $\mathfrak{G}_f$ and controller gain matrices $\mathcal{K}_j$, $\mathcal{L}_i$, $\mathcal{L}_{pi}$, system (7) is asymptotically stable, if there exist apt dimension matrices $\mathcal{U}_i > 0$, $(i = 1, 2, 3)$, $\mathcal{W}_g > 0$ $(g = 1, 2, 3, 4)$, $\check{\mathcal{W}}_1$ and $\check{\mathcal{W}}_2 > 0$, such that the following condition is satisfied:*

$$\sum_{i=1}^{l}\sum_{j=1}^{h}\left[\Lambda_{ij}\right]_{15\times 15} < 0, \tag{10}$$

where $\Lambda_{1,1} = sym(\mathcal{U}_{1i}\mathcal{A}_i) + \mathcal{W}_1 + \mathcal{W}_2 + \bar{\wp}^2\check{\mathcal{W}}_1 + C^T\mathcal{H}^T\mathcal{H}C$, $\Lambda_{1,2} = (\bar{v}\mathcal{U}_{2i}\mathcal{L}_iC_i)^T$, $\Lambda_{1,3} = \bar{v}C_i^T\mathcal{L}_{pi}^T\mathcal{U}_{3i}^T$, $\Lambda_{1,5} = \mathcal{U}_{1i}\mathcal{B}_i\mathfrak{G}_f\mathcal{K}_j$, $\Lambda_{1,7} = -\mathcal{U}_{1i}\mathcal{B}_i\mathfrak{G}_f\mathcal{K}_j$, $\Lambda_{1,9} = \mathcal{U}_{1i}D_i$, $\Lambda_{2,2} = sym(\mathcal{U}_{2i}\mathcal{A}_i - \mathcal{L}_iC_i) + \mathcal{W}_3 + \mathcal{W}_4 + \bar{\wp}^2\check{\mathcal{W}}_2$, $\Lambda_{2,3} = -\mathcal{U}_{2i}\mathcal{B}_i + C_i^T\mathcal{L}_{pi}^T\mathcal{U}_{3i}^T$, $\Lambda_{2,8} = -\bar{v}\mathcal{U}_{2i}\mathcal{L}_i$, $\Lambda_{2,9} = \mathcal{U}_{2i}D_i$, $\Lambda_{3,3} = -sym(\mathcal{U}_{3i})$, $\Lambda_{3,8} = \bar{v}\mathcal{U}_{3i}\mathcal{L}_{pi}$, $\Lambda_{4,4} = -\mathcal{W}_1$, $\Lambda_{5,5} = -(1-\varrho)\mathcal{W}_2$, $\Lambda_{6,6} = -\mathcal{W}_3$, $\Lambda_{7,7} = -(1-\varrho)\mathcal{W}_4$, $\Lambda_{8,8} = -I$, $\Lambda_{9,9} = -I$, $\Lambda_{10,10} = -9\check{\mathcal{W}}_1$, $\Lambda_{10,11} = \frac{36}{\bar{\wp}}\check{\mathcal{W}}_1$, $\Lambda_{10,12} = -\frac{60}{\bar{\wp}^2}\check{\mathcal{W}}_1$, $\Lambda_{11,11} - \frac{192}{\bar{\wp}^2}\check{\mathcal{W}}_1$, $\Lambda_{11,12} = \frac{360}{\bar{\wp}^3}\check{\mathcal{W}}_1$, $\Lambda_{12,12} = -\frac{720}{\bar{\wp}^3}\check{\mathcal{W}}_1$, $\Lambda_{13,13} = -9\check{\mathcal{W}}_2$, $\Lambda_{13,14} = \frac{36}{\bar{\wp}}\check{\mathcal{W}}_2$, $\Lambda_{13,15} = -\frac{60}{\bar{\wp}^2}\check{\mathcal{W}}_2$, $\Lambda_{14,14} - \frac{192}{\bar{\wp}^2}\check{\mathcal{W}}_2$, $\Lambda_{14,15} = \frac{360}{\bar{\wp}^3}\check{\mathcal{W}}_2$ *and* $\Lambda_{15,15} = -\frac{720}{\bar{\wp}^3}\check{\mathcal{W}}_2$.

Proof 1 *Let us construct the Lyapunov-Krasovskii functional for systems (7)-(8) in the following structure:*

$$\mathcal{V}(t) = \sum_{a=1}^{3}\mathcal{V}_a(t), \tag{11}$$

where

$$\begin{aligned}
\mathcal{V}_1(t) =& x^T(t)\mathcal{U}_{1i}x(t) + \mathtt{e}^T(t)\mathcal{U}_{2i}\mathtt{e}(t) + x_p^T(t)\mathcal{U}_{3i}x_p(t),\\
\mathcal{V}_2(t) =& \int_{\tau-\bar{\wp}}^{\tau} x^T(\mathtt{m})\mathcal{W}_1x(\mathtt{m})d\mathtt{m} + \int_{\tau-\wp(t)}^{\tau} x^T(\mathtt{s})\mathcal{W}_2x(\mathtt{s})d\mathtt{s} + \int_{\tau-\bar{\wp}}^{\tau} \mathtt{e}^T(\mathtt{m})\mathcal{W}_3\mathtt{e}(\mathtt{m})d\mathtt{m} + \int_{\tau-\wp(t)}^{\tau} \mathtt{e}^T(\mathtt{s})\mathcal{W}_4\mathtt{e}(\mathtt{s})d\mathtt{s},\\
\mathcal{V}_3(t) =& \bar{\wp}\int_{\tau-\bar{\wp}}^{\tau}\int_{s}^{\tau} x^T(\mathtt{v})\check{\mathcal{W}}_1x(\mathtt{v})d\mathtt{v}d\mathtt{s} + \bar{\wp}\int_{\tau-\bar{\wp}}^{\tau}\int_{s}^{\tau} \mathtt{e}^T(\mathtt{v})\check{\mathcal{W}}_2\mathtt{e}(\mathtt{v})d\mathtt{v}d\mathtt{s}.
\end{aligned}$$

Then, by taking the derivative of (11) and applying mathematical expectation along with the solutions from

(7)-(8), we get

$$\begin{aligned}\mathbb{E}\{\dot{\mathcal{V}}_1(t)\} =& x^T(t)\mathcal{U}_{1i}\dot{x}(t) + \dot{x}^T(t)\mathcal{U}_{1i}x(t) + \mathfrak{e}^T(t)\mathcal{U}_{2i}\dot{\mathfrak{e}}(t) + \dot{\mathfrak{e}}^T(t)\mathcal{U}_{2i}\mathfrak{e}(t) + x_p^T(t)\mathcal{U}_{3i}\dot{x}_p(t) + \dot{x}_p^T(t)\mathcal{U}_{3i}x_p(t)\\ =& 2x^T(t)\mathcal{U}_i\left[\sum_{i=1}^{l}\sum_{j=1}^{h}\mathfrak{b}_j(\phi(t))\big(\mathcal{A}_i x(t) + \mathcal{B}_i\mathfrak{G}_f\mathcal{K}_j\,\hat{x}(\tau-\wp(t)) + \mathcal{D}_i w(t)\big)\right]\\ &+ 2\mathfrak{e}^T(t)\mathcal{U}_{2i}\left[\sum_{i=1}^{j}\mathfrak{b}_i(\phi(t))\big(\mathcal{A}_i\mathfrak{e}(t) - \mathcal{L}_iC_i\mathfrak{e}(t) + \theta(t)\mathcal{L}_iC_ix(t) - \theta(t)\mathcal{L}_i\zeta(y(t)) + \mathcal{D}_iw(t)\big) - \mathcal{B}_ix_p(t)\right]\\ &+ 2x_p^T\mathcal{U}_{3i}\left[\sum_{i=1}^{l}\mathfrak{b}_i(\phi(t))\big(-x_p(t) + \mathcal{L}_{pi}C_i\mathfrak{e}(t) - \theta(t)\mathcal{L}_{pi}C_ix(t) + \theta(t)\mathcal{L}_{pi}\zeta(y(t))\big)\right] \end{aligned} \tag{12}$$

$$\begin{aligned}\mathbb{E}\{\dot{\mathcal{V}}_2(t)\} =& x^T(t)\mathcal{W}_1x(t) - x^T(\tau-\bar{\wp})\mathcal{W}_1x(\tau-\bar{\wp}) + x^T(t)\mathcal{W}_2x(t) - (1-\varrho)x^T(\tau-\wp(t))\mathcal{W}_2x(\tau-\wp(t))\\ &+ \mathfrak{e}^T(t)\mathcal{W}_3\mathfrak{e}(t) - \mathfrak{e}^T(\tau-\bar{\wp})\mathcal{W}_3\mathfrak{e}(\tau-\bar{\wp}) + \mathfrak{e}^T(t)\mathcal{W}_4\mathfrak{e}(t) - (1-\varrho)\mathfrak{e}^T(\tau-\wp(t))\mathcal{W}_4\mathfrak{e}(\tau-\wp(t)),\end{aligned} \tag{13}$$

$$\mathbb{E}\{\dot{\mathcal{V}}_3(t)\} = \bar{\wp}^2x^T(t)\check{\mathcal{W}}_1x(t) + \bar{\wp}^2\mathfrak{e}^T(t)\check{\mathcal{W}}_2\mathfrak{e}(t) - \bar{\wp}\int_{\tau-\bar{\wp}}^{\tau}x^T(\mathfrak{s})\check{\mathcal{W}}_1x(\mathfrak{s})d\mathfrak{s} - \bar{\wp}\int_{\tau-\bar{\wp}}^{\tau}\mathfrak{e}^T(\mathfrak{s})\check{\mathcal{W}}_2\mathfrak{e}(\mathfrak{s})d\mathfrak{s}. \tag{14}$$

Now, redrafting the integral term in inequality (14) by making use of Lemma 1, we get

$$-\bar{\wp}\int_{\tau-\bar{\wp}}^{\tau}x^T(t)\check{\mathcal{W}}_1x(t)d(t) \le \Psi^T(t)\begin{bmatrix}-9\check{\mathcal{W}}_1 & \frac{36}{\bar{\wp}}\check{\mathcal{W}}_1 & -\frac{60}{\bar{\wp}^2}\check{\mathcal{W}}_1\\ * & -\frac{192}{\bar{\wp}^2}\check{\mathcal{W}}_1 & \frac{360}{\bar{\wp}^3}\check{\mathcal{W}}_1\\ * & * & -\frac{720}{\bar{\wp}^3}\check{\mathcal{W}}_1\end{bmatrix}\Psi(t), \tag{15}$$

$$-\bar{\wp}\int_{\tau-\bar{\wp}}^{\tau}\mathfrak{e}^T(t)\check{\mathcal{W}}_2\mathfrak{e}(t)d(t) \le \hat{\Psi}^T(t)\begin{bmatrix}-9\check{\mathcal{W}}_2 & \frac{36}{\bar{\wp}}\check{\mathcal{W}}_2 & -\frac{60}{\bar{\wp}^2}\check{\mathcal{W}}_2\\ * & -\frac{192}{\bar{\wp}^2}\check{\mathcal{W}}_2 & \frac{360}{\bar{\wp}^3}\check{\mathcal{W}}_2\\ * & * & -\frac{720}{\bar{\wp}^3}\check{\mathcal{W}}_2\end{bmatrix}\hat{\Psi}(t), \tag{16}$$

$\Psi^T(t) = \left[\int_{\tau-\bar{\wp}}^{\tau}x^T(s)ds \quad \int_{\tau-\bar{\wp}}^{\tau}ds\int_{\tau-\bar{\wp}}^{s}x^T(v)dv \quad \int_{\tau-\bar{\wp}}^{\tau}ds\int_{\tau-\bar{\wp}}^{s}dv\int_{\tau-\bar{\wp}}^{v}x^T(u)du\right]$ and

$\hat{\Psi}^T(t) = \left[\int_{\tau-\bar{\wp}}^{\tau}\mathfrak{e}^T(s)ds \quad \int_{\tau-\bar{\wp}}^{\tau}ds\int_{\tau-\bar{\wp}}^{s}\mathfrak{e}^T(v)dv \quad \int_{\tau-\bar{\wp}}^{\tau}ds\int_{\tau-\bar{\wp}}^{s}dv\int_{\tau-\bar{\wp}}^{v}\mathfrak{e}^T(u)du\right]$.

Moreover, in light of the presumption on deception attack function $\zeta(y(t))$ on (3), the ensuing inequality is attained:

$$x^T(t)C^T\mathcal{H}^T\mathcal{H}Cx(t) - \zeta^T(y(t))\zeta(y(t)) \ge 0. \tag{17}$$

Further, by combining (12)-(17), we end up with the hereunder relation:

$$\mathbb{E}\{\dot{\mathcal{V}}(t) - w^T(t)w(t)\} \le \pi^T(t)\left[\sum_{i=1}^{l}\sum_{j=1}^{h}\mathfrak{b}_j(\phi(t))\Lambda_{i,j}\right]\pi(t) \tag{18}$$

where $\pi^T(t) = \left[x^T(t)\ \mathfrak{e}^T(t)\ x_p^T(t)\ x^T(\tau-\bar{\wp})\ x^T(\tau-\wp(t))\ \mathfrak{e}^T(\tau-\bar{\wp})\ \mathfrak{e}^T(\tau-\wp(t))\ \zeta^T(y(t))\ w^T(t)\ \Psi^T(t)\ \hat{\Psi}^T(t)\right]$ and $\Lambda_{i,j}$ is defined in statement of theorem. Thereafter, if the constraint (10) holds, then in light of the theory of Lyapunov's stability, it is obvious that the closed-loop system (7)-(8) is asymptotically stable. This completes the proof of the theorem.

Next, in the following theorem, the delay-dependent conditions are derived for the uncertain chaotic system given in (1) under the known actuator fault (6) and unknown gain. Added to this, adequate conditions are configured in the form of matrix inequalities with the extended dissipativity theory.

Theorem 2 *Let $\bar{\wp}$, v, ϱ, ε, γ be positive scalars and system (1) with known actuator fault (6) is asymptotically stable with extended dissipativity, if there exist scalars μ, ϑ_1, ϑ_2 and positive definite matrices $\mathfrak{X}_{mi} > 0$, $(m = 1, 2, 3)$, $\hat{\mathcal{W}}_g > 0$ $(g = 1, 2, 3, 4)$, $\hat{\hat{\mathcal{W}}}_1$ and $\hat{\hat{\mathcal{W}}}_2 > 0$ such that the following conditions are met:*

$$\begin{bmatrix} -\mu I & C_i\mathfrak{X}_{2i} - \bar{\mathfrak{X}}_{2i}C_i \\ * & -I \end{bmatrix} < 0, \tag{19}$$

$$\Theta_{ij} = \begin{bmatrix} \Pi_{ij_{15\times15}} & \daleth & \Xi_1 & \varepsilon\Xi_2^T & \Xi_3 & \varepsilon\Xi_4^T & \Xi_5^T \\ * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & 0 & -I \end{bmatrix} < 0, \tag{20}$$

where $\Pi_{1,1} = sym(\mathcal{A}_i\mathcal{X}_{1i}) + \hat{\mathcal{W}}_1 + \hat{\mathcal{W}}_2 + \bar{\wp}^2\hat{\hat{\mathcal{W}}}_1$, $\Pi_{1,2} = \bar{v}C_i^T\mathcal{L}_{oi}^T$, $\Pi_{1,3} = \bar{v}C_i^T\mathcal{L}_{pi}^T$, $\Pi_{1,4} = \mathcal{B}_i\mathfrak{G}_f\mathcal{Y}_j$, $\Pi_{1,6} = -\delta_1\mathcal{B}_i\mathfrak{G}_f\mathcal{Y}_j$, $\Pi_{1,9} = D_i - \kappa_2C_i^T$, $\Pi_{2,2} = \delta_1 sym(\mathcal{A}_i\mathcal{X}_{2i}) - \delta_1\mathcal{L}_{oi}C_i + \hat{\mathcal{W}}_3 + \hat{\mathcal{W}}_4 + \bar{\wp}^2\hat{\hat{\mathcal{W}}}_2$, $\Pi_{2,3} = -\delta_2\mathcal{B}_i\mathcal{X}_{2i} + \delta_1C_i^T\mathcal{L}_{si}^T$, $\Pi_{2,8} = \bar{v}I$, $\Pi_{2,9} = \mathcal{D}_i$, $\Pi_{3,3} = \delta_2 sym(\mathcal{M}_p\mathcal{X}_{3i})$, $\Pi_{3,8} = \bar{v}I$, $\Pi_{4,4} = -\hat{\mathcal{W}}_1$, $\Pi_{5,5} = -(1-\varrho)\hat{\mathcal{W}}_2$, $\Pi_{6,6} = -\hat{\mathcal{W}}_3$, $\Pi_{7,7} = -(1-\varrho)\hat{\mathcal{W}}_4$, $\Pi_{8,8} = -I$, $\Pi_{9,9} = -\kappa_3 I$, $\Pi_{10,10} = -9\hat{\hat{\mathcal{W}}}_1$, $\Pi_{10,11} = \frac{36}{\bar{\wp}}\hat{\hat{\mathcal{W}}}_1$, $\Pi_{10,12} = -\frac{60}{\bar{\wp}^2}\hat{\hat{\mathcal{W}}}_1$, $\Pi_{11,11} - \frac{192}{\bar{\wp}^2}\hat{\hat{\mathcal{W}}}_1$, $\Pi_{11,12} = \frac{360}{\bar{\wp}^3}\hat{\hat{\mathcal{W}}}_1$, $\Pi_{12,12} = -\frac{720}{\bar{\wp}^3}\hat{\hat{\mathcal{W}}}_1$, $\Pi_{13,13} = -9\hat{\hat{\mathcal{W}}}_2$, $\Pi_{13,14} = \frac{36}{\bar{\wp}}\hat{\hat{\mathcal{W}}}_2$, $\Pi_{13,15} = -\frac{60}{\bar{\wp}^2}\hat{\hat{\mathcal{W}}}_2$, $\Pi_{14,14} - \frac{192}{\bar{\wp}^2}\hat{\hat{\mathcal{W}}}_2$, $\Pi_{14,15} = \frac{360}{\bar{\wp}^3}\hat{\hat{\mathcal{W}}}_2$ *and* $\Pi_{15,15} = -\frac{720}{\bar{\wp}^3}\hat{\hat{\mathcal{W}}}_2$. $\daleth = \begin{bmatrix} \mathcal{X}_{1i}\mathcal{H}_i^TC_i^T & \underbrace{0\cdots0}_{14\ times} \end{bmatrix}^T$, $\Xi_1 = \begin{bmatrix} \mathfrak{B}_i^T & \mathfrak{B}_i^T & \underbrace{0\cdots0}_{13\ times} \end{bmatrix}^T$, $\Xi_2 = \begin{bmatrix} \mathfrak{N}_i\mathcal{X}_{1i} & \underbrace{0\cdots0}_{14\ times} \end{bmatrix}$, $\Xi_3 = \begin{bmatrix} \mathcal{X}_{1i}\mathfrak{B}_i^T & \underbrace{0\cdots0}_{14\ times} \end{bmatrix}^T$, $\Xi_4 = \begin{bmatrix} \mathfrak{N}_i^T\mathcal{X}_{1i} & \underbrace{0\cdots0}_{14\ times} \end{bmatrix}$, $\Xi_5 = \begin{bmatrix} (\mathcal{X}_{1i}C_i^T\tilde{\kappa}_1)^T & \underbrace{0\cdots0}_{14\ times} \end{bmatrix}^T$. *In addition, if the above-offered constraints have a workable solution, then the gain matrices can be computed by using the ensuing connection:* $\mathcal{L}_i = \mathcal{L}_{oi}\bar{\mathfrak{X}}_{2i}^{-1}$, $\mathcal{L}_{pi} = \mathcal{L}_{si}\bar{\mathfrak{X}}_{2i}^{-1}$ *and* $\mathcal{K}_j = \mathcal{Y}_j\mathfrak{X}_{1i}^{-1}$.

Proof 2 *The proof of this theorem follows from the results (10) obtained in Theorem 1 along with system uncertainty and extended dissipative theory. To proceed ahead, let us impose the conditions* $\mathcal{U}_{1i}^{-1} = \mathfrak{X}_{1i}$, $\mathcal{U}_{2i}^{-1} = \mathfrak{X}_{2i}$, $\mathcal{U}_{3i}^{-1} = \mathfrak{X}_{3i}$, $\mathfrak{X}_{1i}\mathcal{W}_1\mathfrak{X}_{1i} = \hat{\mathcal{W}}_1$, $\mathfrak{X}_{1i}\mathcal{W}_2\mathfrak{X}_{1i} = \hat{\mathcal{W}}_2$, $\mathfrak{X}_{2i}\mathcal{W}_3\mathfrak{X}_{2i} = \hat{\mathcal{W}}_3$, $\mathfrak{X}_{2i}\mathcal{W}_4\mathfrak{X}_{2i} = \hat{\mathcal{W}}_4$, $C_i\mathfrak{X}_2 = \bar{\mathfrak{X}}_2C_i$, $\mathfrak{X}_{1i}\breve{\mathcal{W}}_1\mathfrak{X}_{1i} = \hat{\hat{\mathcal{W}}}_1$, $\mathfrak{X}_{2i}\breve{\mathcal{W}}_2\mathfrak{X}_{1i} = \hat{\hat{\mathcal{W}}}_2$, *along with pre- and post- multiplication by* $\{\mathcal{U}_{1i}^{-1}, \mathcal{U}_{2i}^{-1}, \mathcal{U}_{3i}^{-1}, \mathcal{U}_{1i}^{-1}, \mathcal{U}_{1i}^{-1}, \mathcal{U}_{2i}^{-1}, \mathcal{U}_{2i}^{-1}, \mathcal{U}_{2i}^{-1}, I, \mathcal{U}_{1i}^{-1}, \mathcal{U}_{1i}^{-1}, \mathcal{U}_{1i}^{-1}, \mathcal{U}_{2i}^{-1}, \mathcal{U}_{2i}^{-1}, \mathcal{U}_{2i}^{-1}\}$ *of the matrix* $\Lambda_{i,j}$ *in the previous theorem. As a result of this presumption and by making use of Schur complement, we arrive at the matrix* Θ_{ij} *defined in the theorem statement.*

Further by tracing the similar fashion of Theorem 1, we can attain $\mathbb{E}\{\dot{\mathcal{V}}(t)\} < 0$. *Then, with the extended dissipative criterion* $\zeta(t)$, *we can obtain the following relation:*

$$\mathbb{E}\{\dot{\mathcal{V}}(t)\} - \zeta(t) \le 0.$$

Moreover, by integrating the above inequality from 0 *to* τ, *we obtain*

$$\mathbb{E}\{\mathcal{V}(t)\} \le \mathbb{E}\{\int_0^\tau \zeta(s)ds\}. \tag{21}$$

On the basis of Definition 1, we have

$$\mathbb{E}\left\{\int_0^{T_f} \zeta(t)dt\right\} - \sup_{0\le t\le T_f} \mathbb{E}\left\{y^T(t)\kappa_4 y(t)\right\} \ge \gamma. \tag{22}$$

Subsequently, we need to establish that the above constraint obeys for any matrices $\kappa_1, \kappa_2, \kappa_3$ *and* κ_4 *under the setting of Assumption 1. In this regard, we can characterize the parameter* κ_4 *into two types.*

First, if $\|\kappa_4\| = 0$*, then the inequality (22) implies for any* $T_f \geq 0$ *that*

$$\int_0^{T_f} \zeta(s)ds \geq \gamma. \tag{23}$$

If $\|\kappa_4\| \neq 0$*, we can have from Assumption 1 that the associated matrices are* $\kappa_1 = 0$*,* $\kappa_2 = 0$*,* $\kappa_3 > 0$*, thus for any* $T_f \geq t \geq 0$*, we have*

$$\int_0^{T_f} \zeta(s)ds \geq \int_0^{t} \zeta(s)ds \geq \gamma. \tag{24}$$

Suppose there exists a scalar $0 < \eta < 1$ *such that*

$$\int_0^{T_f} \zeta(s)ds \geq \eta x^T(t)\mathcal{U}_{1i}x(t) + \gamma. \tag{25}$$

By letting $C_i^T \kappa_4 C_i < \eta \mathcal{U}_{1i}$*, then we have*

$$y^T(t)\kappa_4 y(t) = x^T(t)C_i^T \kappa_4 C_i x(t) < \eta x^T(t)x(t). \tag{26}$$

It is clear from (25) and (26) that

$$\int_0^{T_f} \zeta(s)ds \geq y^T(t)\kappa_4 y(t) + \gamma. \tag{27}$$

Thus, the inequality (9) holds for $T_f \geq 0$*. Further, if conditions (19)-(20) hold, then we conclude that addressed system (1) is asymptotically stable with the extended dissipative which concludes the proof of this theorem.*

At the end of this portion, we prove that the fuzzy chaotic system (1) is asymptotically stable with extended dissipative thorough the designed fault-tolerant controller (6) under the unknown actuator faults.

Theorem 3 *Let* $\bar{\wp}$*,* v*,* ϱ*,* ε*,* γ *be given positive scalars. Then, system (1) with the unknown fault controller (6) is asymptotically stable with the extended dissipative, if there exist positive scalars* μ*,* ϑ_1*,* ϑ_2 *and matrices* $\mathfrak{X}_{mi} > 0$*,* $(m = 1, 2, 3)$*,* $\hat{\mathcal{W}}_g > 0$ $(g = 1, 2, 3, 4)$*,* $\hat{\hat{\mathcal{W}}}_1$ *and* $\hat{\hat{\mathcal{W}}}_2 > 0$ *such that the ensuing LMIs hold:*

$$\begin{bmatrix} -\mu I & C_i \mathfrak{X}_{2i} - \bar{\mathfrak{X}}_{2i} C_i \\ * & -I \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} \Theta_{ij} & \varepsilon \mathfrak{S}_1^T & \mathfrak{S}_2 & \mathfrak{S}_3^T \\ * & -\varepsilon I & 0 & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{29}$$

where $\mathfrak{S}_1 = \begin{bmatrix} \mathcal{B}_i^T \mathfrak{G}_1^T & \underbrace{0 \cdots 0}_{20\ times} \end{bmatrix}$, $\mathfrak{S}_2 = \begin{bmatrix} 0 & 0 & 0 & \mathcal{Y}_j^T & 0 & \mathcal{Y}_j^T & \underbrace{0 \cdots 0}_{15\ times} \end{bmatrix}^T$, $\mathfrak{S}_3 = \begin{bmatrix} \underbrace{0 \cdots 0}_{15\ times} & \mathcal{B}_i^T \mathfrak{G}_1 & \underbrace{0 \cdots 0}_{5\ times} \end{bmatrix}$.

Furthermore, the gain matrices of offered controller and observer are obtained by solving the ensuing connection $\mathcal{L}_i = \mathcal{L}_{oi}\bar{\mathfrak{X}}_{2i}^{-1}$, $\mathcal{L}_{pi} = \mathcal{L}_{si}\bar{\mathfrak{X}}_{2i}^{-1}$ *and* $\mathcal{K}_j = \mathcal{Y}_j \mathfrak{X}_{1i}^{-1}$.

Proof 3 *The proof of this theorem can be straightforwardly obtained by replacing the actuator fault matrix* $\mathfrak{G}_f$ *with* $\mathfrak{G}_0 + \mathfrak{G}_1\Sigma$ *in the previous theorem. Thus, the proof is omitted here.*

Remark 1 *It should be mentioned that the Lyapunov-Krasovskii functional is specifically designed to handle systems with delays. It accounts for the past states of the system, making it suitable for systems where delay affects stability, which traditional Lyapunov functionals may not address adequately. Further, it allows for a more comprehensive analysis of stability, as it includes integral terms that capture the system's history over a time interval. This inclusion can result in less conservative stability criteria compared to conventional Lyapunov functionals. On the other hand, extended Wirtinger inequality plays a significant role in maximizing the feasible region of stability criteria and obtaining maximum delay bounds of time delays for guaranteeing system stability in a given constraint. Thus, based on these considerations, in our work, Lyapunov-Krasovskii functional with double-integral term is considered. Subsequently, the LMI conditions are derived based on the constructed Lyapunov-Krasovskii functional by properly applying the extended Wirtinger inequality.*

4. SIMULATION VERIFICATION

This section includes an illustrative example demonstrating the effectiveness and superiority of the proposed criterion. Notably, we leverage MATLAB software to aid in the acquisition of simulation results. For this purpose, we consider a chaotic Rossler's system[51] and its dynamics are laid out below:

$$\begin{cases} \dot{x}_1(t) = -x_2(t) - x_3(t), \\ \dot{x}_2(t) = x_1(t) + \mathfrak{a}x_2(t), \\ \dot{x}_3(t) = \mathfrak{b}x_1(t) - (\mathfrak{c} - x_1(t))x_3(t) + u(t), \end{cases} \tag{30}$$

where $x_1(t) \in [\mathfrak{c} - \mathfrak{d} \quad \mathfrak{c} + \mathfrak{d}]$, by using fuzzy modeling method as in[51], we get the system matrices as $\mathcal{A}_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \mathfrak{a} & 0 \\ \mathfrak{b} & 0 & -\mathfrak{d} \end{bmatrix}$, $\mathcal{A}_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \mathfrak{a} & 0 \\ \mathfrak{b} & 0 & \mathfrak{d} \end{bmatrix}$, $\mathcal{B}_1 = \mathcal{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\mathcal{D}_1 = \mathcal{D}_2 = \begin{bmatrix} 0.2 & 0.3 & 0.1 \\ 0.1 & 0.2 & 0.3 \\ 0.1 & 0.7 & 0.4 \end{bmatrix}$,
$C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathfrak{B}_1 = \begin{bmatrix} 0.12 & 0.15 & 0 \\ 0.22 & 0 & 0.3 \\ 0.31 & 0.2 & 0.2 \end{bmatrix}$, $\mathfrak{B}_2 = \begin{bmatrix} 0.12 & 0.1 & 0 \\ 0.32 & 0 & 0.4 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}$, $\mathfrak{N}_1 = \mathfrak{N}_2 = \begin{bmatrix} 0.1 & 0.4 & 0 \\ 0.3 & 0 & 0.5 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$.
Further, we have taken the parameters involved in the simulation as $\mathfrak{a} = 0.3$, $\mathfrak{b} = 0.5$, $\mathfrak{c} = 5$, $\mathfrak{d} = 10$, $\bar{\nu} = 0.3$, $\bar{\wp} = 0.5$, $\alpha = 0.6$, $\delta_1 = 1.2$, $\delta_2 = 1.5$, $\gamma = 0.01$, $\kappa_1 = 5$, $\kappa_2 = 0.2$, $\kappa_3 = 0.7$, $\varepsilon = 0.9$, and $\varrho = 1$. The fuzzy membership function to this network transmission is taken as $\mathfrak{b}_1(\phi(t)) = \frac{\mathfrak{c}+\mathfrak{d}-x_1(t)}{2\mathfrak{d}}$ and $\mathfrak{b}_2(\phi(t)) = 1 - \mathfrak{b}_1(\phi(t))$. Moreover, the actuator fault $\mathfrak{G}_f$ is assumed within the interval (0.2, 0.8) and the time-varying delay is selected as $\wp(t) = 0.3 + 0.6(0.9 + 0.5sin(0.3\tau))$. Specifically, the chaotic behavior of the considered system model is supplied in Figure 1.

Moreover, by inputting the aforementioned parameters in Theorem 3 and solving it via standard software, the controller and observer gains are obtained as follows:

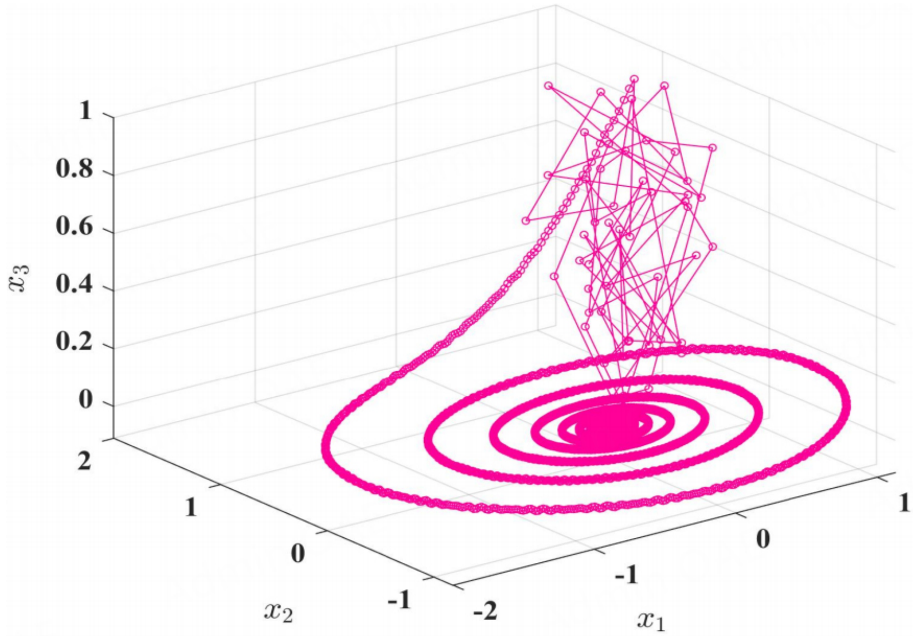

Figure 1. Chaotic behavior of considered system under these parameters.

$$\mathcal{K}_1 = \begin{bmatrix} 0.3703 & 0.0422 & -0.0448 \end{bmatrix}, \qquad \mathcal{K}_2 = \begin{bmatrix} 0.3674 & 0.1812 & 0.0209 \end{bmatrix},$$

$$\mathcal{L}_1 = \begin{bmatrix} 8.3546 & -1.0633 & -1.6445 \\ -1.0633 & 12.0220 & -0.1654 \\ -1.6445 & -0.1654 & 4.5937 \end{bmatrix}, \ \mathcal{L}_2 = \begin{bmatrix} 24.5503 & -7.0704 & -2.3350 \\ -7.0704 & 51.0683 & 0.7260 \\ -2.3350 & 0.7260 & 33.8401 \end{bmatrix},$$

$$\mathcal{L}_{p_1} = \begin{bmatrix} 1.7967 & -0.1956 & 0.0187 \\ -0.1956 & 1.3038 & 0.0149 \\ 0.0187 & 0.0149 & 0.9877 \end{bmatrix}, \ \mathcal{L}_{p_2} = \begin{bmatrix} 3.9617 & 0.1771 & -0.2038 \\ 0.1771 & 2.5764 & -0.0264 \\ -0.2038 & -0.0264 & 0.0819 \end{bmatrix}.$$

For simulation purposes, the external disturbance signal and nonlinear deception attack function are considered as

$$w(t) = \begin{bmatrix} 0.1\sin(0.4\pi\tau) & 0.2\sin(0.4\pi\tau) & 0.3\sin(0.4\pi\tau) \end{bmatrix}^T$$

and

$$f(y(t)) = \begin{bmatrix} 0.01\tanh(0.2x_1(t)) & 0.03\tanh(0.5x_2(t))) & 0.04\tanh(0.2x_3(t)) \end{bmatrix}.^T$$

In the follow-up, by utilizing the gain matrices and the parameters put forward above, the simulation task is carried out under random initial conditions. Specifically, in Figure 2, the response of the state trajectories in the presence and absence of the control is presented. It can be seen that the system trajectories diverge when the controller is absent, whereas, under the developed controller, the intended performance of the system is accomplished. In other words, the system trajectories converge to the equilibrium point, even in the presence of several deteriorating factors. From these figures, the inherent potential of the developed control is straightforwardly exhibited. Furthermore, the significance of the developed observer is demonstrated in Figure 3. Herein, the observer trajectories precisely estimate the system trajectories, showcasing the viability of the observer.

Subsequently, to showcase the impact of actuator faults, Figures 4-6 are provided. Specifically, the trajectories of the system output with and without fault are presented in Figure 4. Likewise, the response of state and

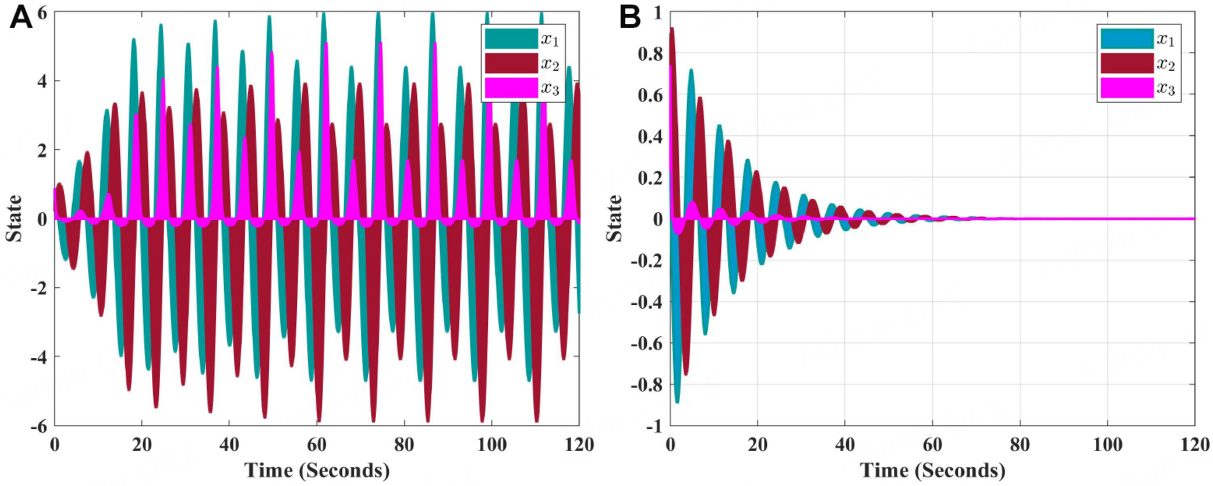

Figure 2. Time evolution of states of the undertaken system. (A) Open loop system; (B) Closed loop system.

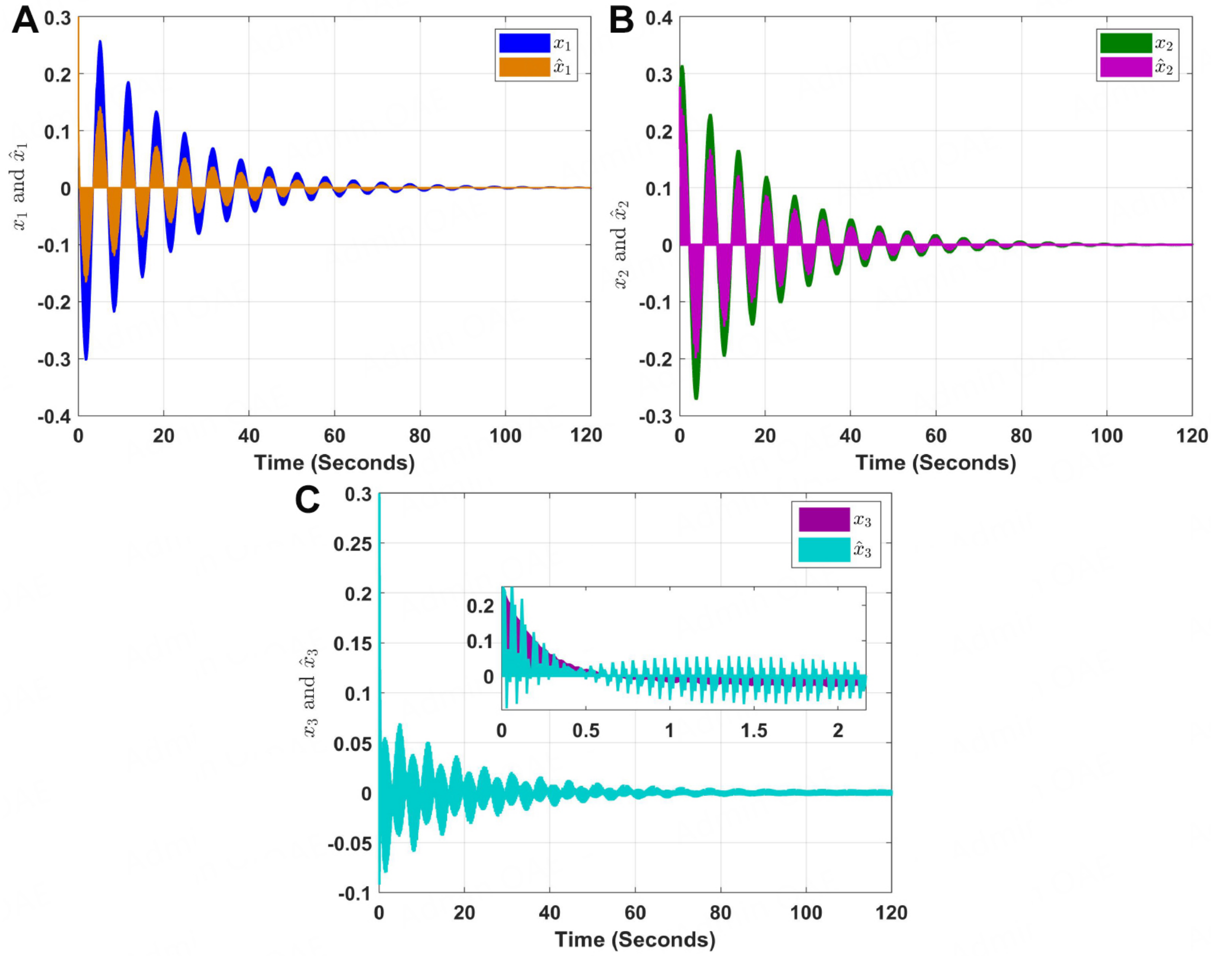

Figure 3. Response of system and its corresponding observer trajectories. (A) $x_1(t)$ *and* $\hat{x}_1(t)$; (B) $x_2(t)$ *and* $\hat{x}_2(t)$; (C) $x_3(t)$ *and* $\hat{x}_3(t)$.

its respective observer response in the situation where the actuator is functioning normally without the fault are presented in Figure 5. It is evidently visible that the estimation performance is better when compared

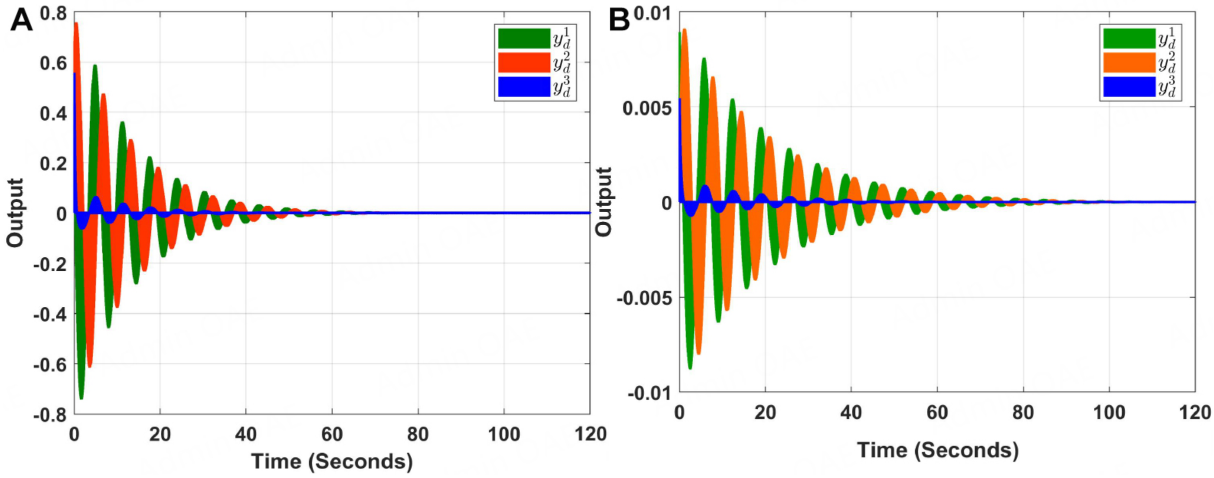

Figure 4. Progression of output trajectories. (A) With actuator faults; (B) Without actuator faults.

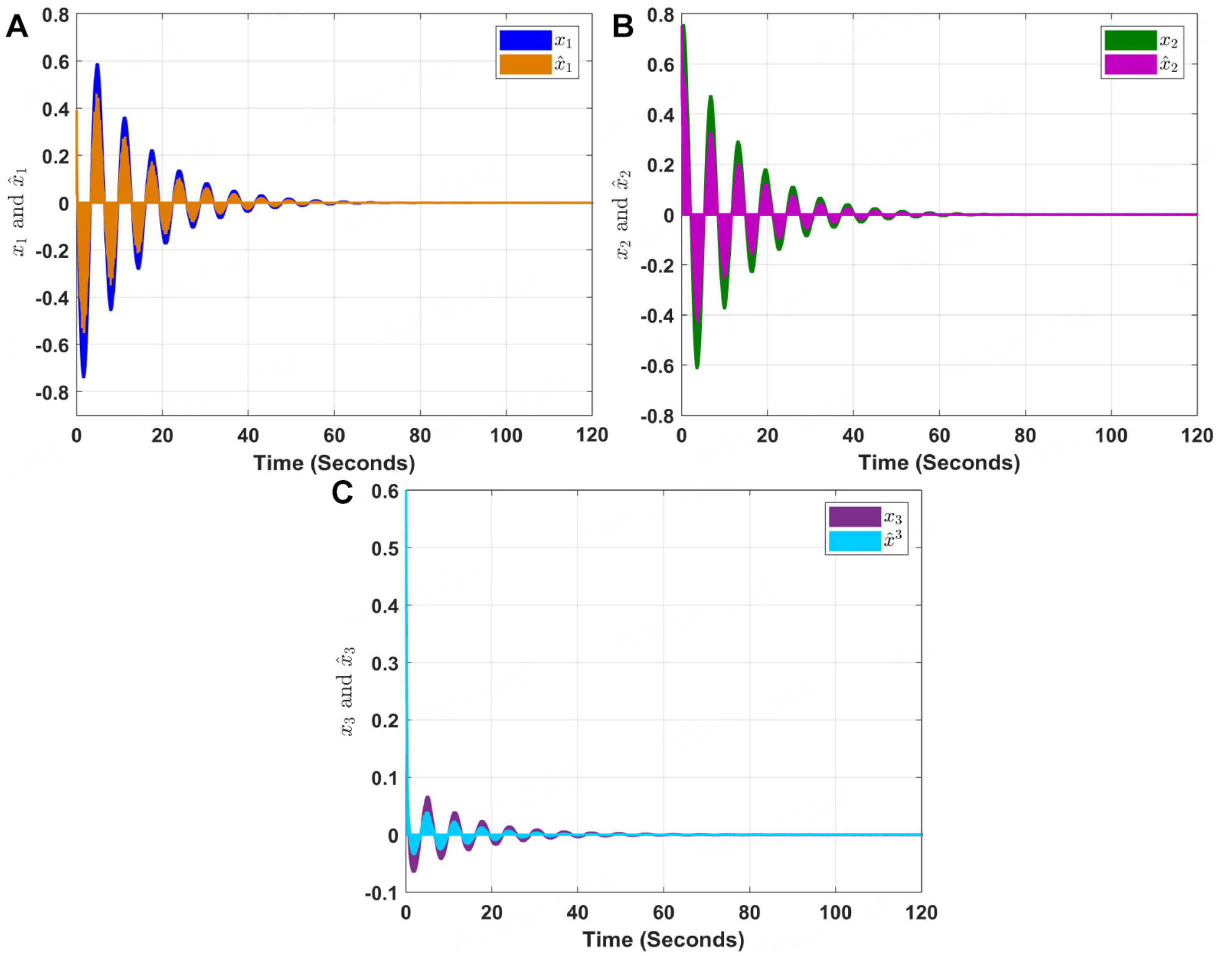

Figure 5. Time profile of system and its corresponding observer without actuator faults. (A) $x_1(t)$ *and* $\hat{x}_1(t)$; (B) $x_2(t)$ *and* $\hat{x}_2(t)$; (C)$x_3(t)$ *and* $\hat{x}_3(t)$.

with Figure 3. Eventually, the pictorial representation of the developed control protocol under the same two scenarios is laid out in Figure 6. Overall, from these figures, the impact of actuator faults of the considered

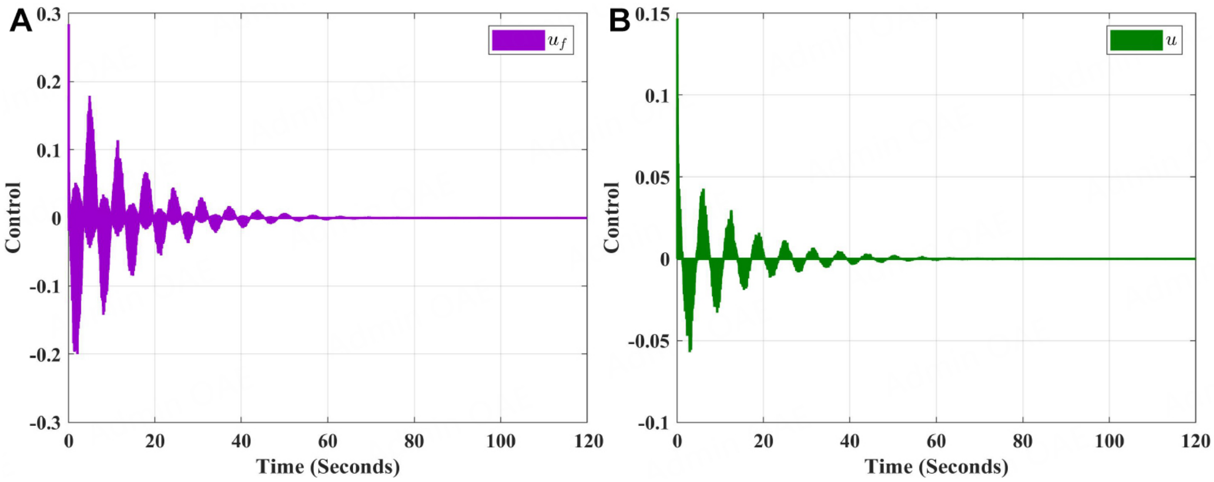

Figure 6. Response of control trajectories. (A) With actuator faults; (B) Without actuator faults.

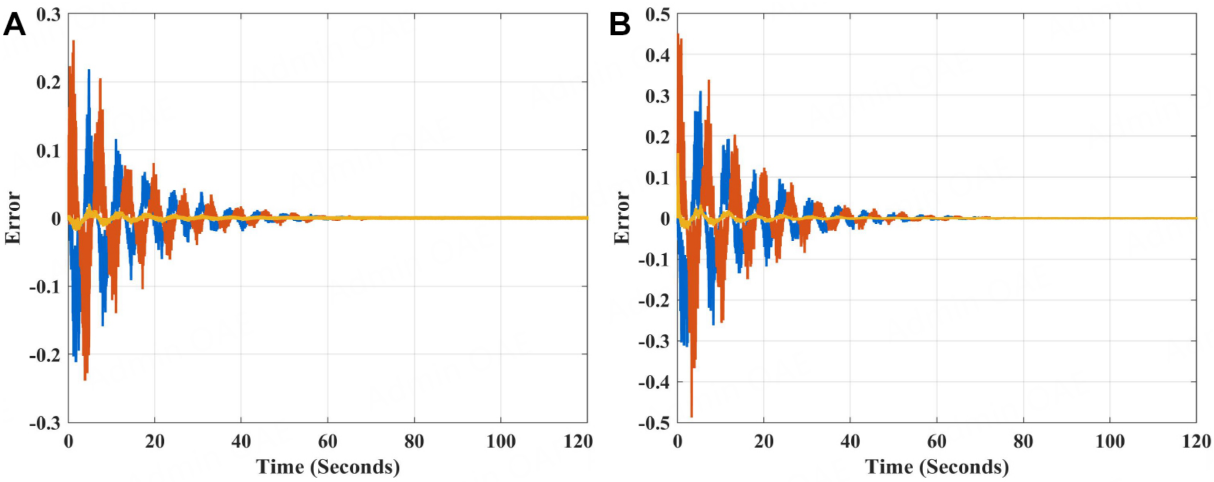

Figure 7. Response of state estimation error. (A) Under PIO; (B) Under Luenberger observer.

system and the observer is clearly exhibited.

Ultimately, to highlight the potential of the configured PIO over the conventional Luenberger observer, Figure 7 is depicted. Specifically, in this figure, the state estimation trajectories for these two observers are offered. It is seen that the estimation error is reduced significantly. In sum, the results of this simulation study indicate that the suggested control mechanism assists the assayed systems in attaining asymptotic stability despite parameter uncertainties, input delay, deception attacks, and external disturbances.

5. CONCLUSION

Over this research endeavor, we have tackled the robust fault-tolerant control issue for the T-S fuzzy chaotic system in the midst of parameter uncertainties, input delay, deception assaults, and external disturbances. To begin, the system states have been estimated by means of configuring fuzzy-based observer and therein, the output measurement of the system model considers deception attacks in the output channel to ensure secure estimation. Therein, it is presupposed that the deception attacks occur at random which considers the Bernoulli distribution. Secondly, the robust fault-tolerant control has been developed by incorporating the actuator faults and input delays. Here, the system is driven to achieve its goals by substantially mitigating the actuator faults. Thirdly, adequate stability criteria are formed with the assistance of Lyapunov stability theory and the LMI approach and the gain matrices have been emitted based on the derived conditions. In the end, the numerical results are provided to show off the envisaged use of the set-up control technique.

DECLARATIONS

Acknowledgments
The authors would like to thank the reviewers for their valuable comments.

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Conceptualization, methodology, validation, writing-original draft, writing - review & editing: Monisha S
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Availability of data and materials
Not applicable.

Financial support and sponsorship
None.

Conflicts of interest
All authors declared that there are no conflicts of interest.

Ethical approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

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