Complex Engineering Systems

Perspective

Open Access

Check for updates

Dynamic reliability decision-making frameworks: trends and opportunities

Xiujie Zhao, Yi Luo

College of Management and Economics, Tianjin University, Tianjin 300072, China.

Correspondence to: Prof. Xiujie Zhao, College of Management and Economics, Tianjin University, 92 Weijin Road, Nankai District, Tianjin 300072, China. E-mail: xiujiezhao@tju.edu.cn

How to cite this article: Zhao X, Luo Y. Dynamic reliability decision-making frameworks: trends and opportunities. *Complex Eng Syst* 2024;4:22. https://dx.doi.org/10.20517/ces.2024.54

Received: 29 Aug 2024 **First Decision:** 10 Oct 2024 **Revised:** 31 Oct 2024 **Accepted:** 26 Nov 2024 **Published:** 30 Nov 2024

Academic Editor: Hamid Reza Karimi **Copy Editor:** Fangling Lan **Production Editor:** Fangling Lan

Abstract

Reliability engineering and management are becoming more important as systems evolve in functionality and complexity. Given various dynamic factors influencing reliability, static one-time decision frameworks can no longer offer optimal reliability decisions. In the paper, we discuss the recent trends in reliability decision-making methods across three stages of reliability issues: reliability testing and optimization, reliability modeling and evaluation, and post-service design. We can find a growing interest in time-dependent dynamic methods in research for all these three stages. Sequential decision modeling methods, such as the Markov decision process and its extensions, can be a resort to solve these problems, while modeling and problem-solving can be quite challenging under certain circumstances. Future research holds promising opportunities in related topics.

Keywords: Reliability, sequential decision models, maintenance optimization, reliability optimization

1. INTRODUCTION

Reliability engineering and management are crucial for product quality and market competitiveness. Demand for high performance and reliability has made decision-making a key research focus in this field.

Traditional static models assume constant conditions, failing to capture dynamic factors such as production, usage, and environment[1-3]. For instance, lithium-ion battery (LiB) reliability in electric vehicles depends on

© The Author(s) 2024. **Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License (https://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, sharing, adaptation, distribution and reproduction in any medium or format, for any purpose, even commercially, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

manufacturing, usage, and maintenance[4,5]. Therefore, dynamic reliability decision-making provides more adaptive solutions across three main stages: reliability testing and optimization, modeling and evaluation, and post-service design.

Reliability testing and optimization face challenges due to complex, long-lasting products[6]. Standard life tests under normal conditions are impractical for highly reliable consumer durables[7]. Accelerated reliability testing (ART) uses stresses such as higher temperatures or humidity to expedite failure[8]. Reliability optimization, once limited to the design phase, now requires ongoing adjustments due to dynamic usage and environments[9,10], involving complex programming and algorithm development[11].

Traditional lifetime prediction methods lack today's needed precision, driving data-driven model growth[12]. The complexity of dynamic environments and varied usage further complicates model development. Peng *et al.* highlight the role of dynamic conditions in reliability modeling[13].

Reliability engineering includes testing, modeling, optimization, and post-service design, linked to degradation and operational environments[14]. For example, Gan *et al.* propose a maintenance optimization strategy for multi-state systems in dynamic environments[15].

The shift towards dynamic decision-making frameworks is driven by the practical needs of industries such as aerospace, automotive, and energy, where systems operate under highly variable conditions. This paper explores trends in dynamic reliability decision-making, addressing challenges and highlighting future research directions for adaptive and responsive management.

2. RELIABILITY TESTING AND OPTIMIZATION

Reliability testing and optimization are typically conducted during product design. However, modern systems' complexity makes system-level testing challenging[16]. Reliability is often assessed by testing the basic units, sometimes performed by suppliers rather than manufacturers. A key challenge in reliability testing is the extrapolation needed for predictions. Long-lasting consumer durables render traditional life tests impractical, making accelerated tests (ARTs) a preferred framework[17,18].

ARTs are typically categorized into accelerated life tests (ALTs) and accelerated degradation tests (ADTs), with some studies combining these methods[19]. Both methods shorten testing time and yield valuable reliability data. ALTs apply higher stress levels to accelerate failure[20], while ADTs track performance degradation over time[21,22]. In ALTs, unit failures and failure times are recorded, whereas ADTs measure periodic degradation. Planning is the first crucial step in ART for reliability assessment. Most existing studies follow a scheme that hinges on large-sample approximation to design ARTs, in which the optimality criteria are established via a function of the Fisher information and other selected variables. Figure 1 shows some key elements to be considered during the planning phase. The scheme is statistically sound, flexible, and adaptable to various scenarios.

Recent literature regarding related topics has implied several possible future directions. First, there are numerous research gaps in formulating new optimality criteria. Classical criteria such as D-, A-, and V-optimality, while statistically robust, often lack practical relevance. Bayesian optimization has been effectively applied to reliability-based design optimization, particularly in estimating design points with high accuracy[23]. Song *et al.* employ the constrained Bayesian optimization method and its variant to adaptively learn design points[24]. Emerging studies have increasingly concentrated on the connections between statistical criteria and the decision-maker's objectives[25,26]. Etemadi and Fotuhi-Firuzabad[27] propose

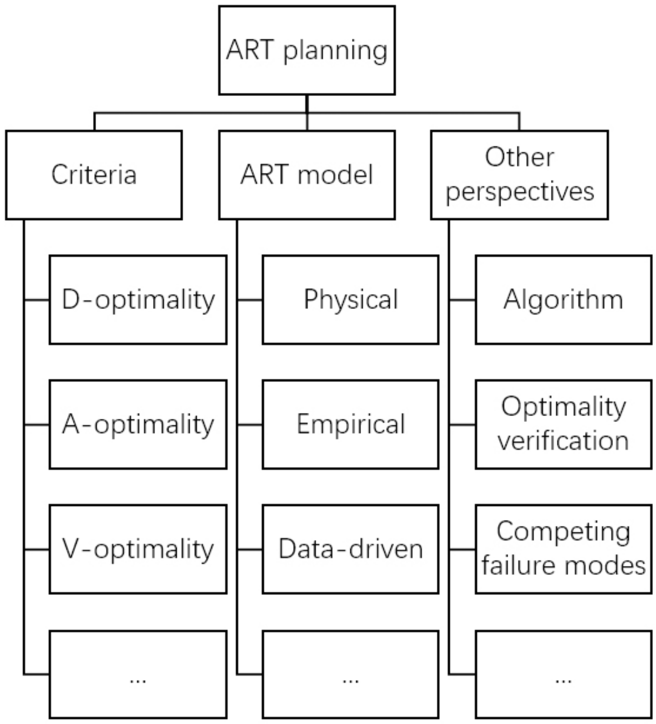

Figure 1. Important aspects in ART plan optimization.

optimizing protection system design by balancing costs and relay failure losses, integrating economic aspects into reliability. Incorporating maintenance and warranty perspectives into these criteria is promising. Second, the enrichment of the ART model and reliability model is always an outlet. Specifically, enhancing the interpretability of such models presents a significant challenge[28]. Last but not least, dynamic methods in ART planning are limited. Most extant works treat the test planning as a static design of experiment problem, yet in practice, tests can be planned and conducted in a dynamic, sequential manner[29,30]. Dynamic decision-making methods such as the Markov decision process (MDP) and reinforcement learning (RL) can be promising tools to tackle such planning problems[31].

Adding redundancy is a traditional yet effective way to boost system reliability, commonly seen in high-safety engineering systems such as airliners and automobiles. Reliability optimization covers various design problems, such as the redundancy allocation problem (RAP)[32], the reliability allocation problem and the reliability RAP (RRAP)[11]. Simply speaking, the problem is selecting the optimal components and quantities to ensure system operation. The uncertainty embedded in the reliability of units can make the problem quite onerous to solve. Risk-aware decision-makers often seek worst-case reliability solutions[33,34], leveraging distributionally robust methods. Nevertheless, the problem usually falls into the category of hard-to-solve mixed integer linear or nonlinear programs. Efficient algorithms remain a research priority, with specific system structures guiding algorithm improvements. Reihaneh *et al.* propose an exact branch-and-price (BP) algorithm for RAP with heterogeneous components under a mixed redundancy strategy[35].

Traditionally, reliability optimization was viewed as a static design-phase problem. However, with modern systems experiencing diverse usage and dynamic environments, system design now spans the entire lifecycle. Peiravi *et al.* introduced a novel Universal Redundancy Strategy (URS) aimed at improving system reliability by dynamically adjusting the configuration of redundant components during system operation[36].

Planning for redundancy upgrades and maintenance interactions is a time-dependent problem that classic methods may not handle effectively. Sequential decision models present a promising approach for tackling such dynamic challenges.

3. RELIABILITY MODELING AND EVALUATION

Data-driven reliability modeling methods have significantly evolved[37], but conventional lifetime-based methods, such as Weibull and Poisson models, remain popular in both research and applications[38]. Meanwhile, degradation modeling that utilizes the health characteristics of products for reliability analysis plays an increasingly important part in related fields[39]. This approach has been used to study reliability through performance loss in batteries, wear in machine tools, and crack growth in turbine blades. Reliability modeling and evaluation is not inherently a decision-making problem, but it provides crucial input for reliability-focused decision-making frameworks.

Dynamic factors can significantly affect system reliability during operation, as most systems function in time-varying environments. Changes in external conditions, such as higher temperatures, can negatively impact electronic products such as batteries and chipsets. Singpurwalla[40] and Eryilmaz and Rıza Bozbulut[41] have explored these effects on reliability modeling. Luo *et al.* propose a statistical model that accounts for both correlated component lifetimes and lifetime ordering constraints under dynamic conditions to show direct impacts on reliability[42]. Dynamic environments are generally modeled as time-varying covariates linked to reliability parameters[1,43]. Zhang *et al.* use a Wiener process for system degradation, treating the dynamic environment as a covariate[44].

Hybrid models have emerged as a transformative approach by integrating the strengths of physics-based models and data-driven techniques to address key challenges in reliability modeling. One such challenge is managing the heterogeneous usage patterns of products, which introduce complex operational variances and significantly affect system reliability. Accurately managing high-dimensional, often correlated, covariates is crucial. Machine learning-based methods can be a resort, yet incorporating the physical principles of reliability into these models can be complex. For a comprehensive review, refer to the study by Xu and Saleh[45]. Physics-informed machine learning methods need more in-depth investigation.

Dependent and Competing Failure Processes (DCFP) are increasingly important in reliability modeling, especially for systems facing multiple, interacting failure mechanisms such as wear, corrosion, and fatigue[46]. In DCFP, one failure process can influence the likelihood of others. This is particularly relevant for industrial applications such as LiB systems[47] and micro-electro-mechanical systems (MEMS)[48]. Additionally, Wu and Ding[49] explore how Markov environments influence systems with DCFP, highlighting dynamic environmental factors. Zhou and Li[50] study interactions between minor and major failures. Future research could integrate DCFP into dynamic decision-making frameworks for a deeper understanding of system reliability in changing conditions.

Reliability modeling for complex multi-component systems is another challenging task especially with component dependencies. Complex systems can be onerous to characterize using classical block diagrams. Network or graph-based characterizations are far more flexible and informative[51]. Combining stochastic models with regression-based methods can enhance the efficiency and accuracy of reliability models.

4. POST-SALE SERVICE DESIGN VIA RELIABILITY MODELS

Maintenance modeling and optimization are crucial post-sale strategies for manufacturers and consumers, with dynamic methods widely studied. MDP-based maintenance optimization dates back to the 1950s when reliability engineering first emerged. Early research discussed system deterioration without explicitly mentioning "condition-based maintenance", favoring the term "preventive maintenance". In recent decades, condition-based maintenance (CBM) has garnered huge attention[52]. As shown in Figure 2, maintenance schemes often rely on different key state thresholds, which may lead to over-maintenance or unexpected failures. CBM dynamically adjusts maintenance decisions by continuously monitoring the system's degradation state.

Maintenance optimization via MDP and its variants has been studied intensely[53-55]. MDP is a framework for decision-making under uncertainty, representing a system where an agent chooses actions that lead to different states with specific rewards[56,57].

Unlike traditional CBM methods that set a constant threshold, MDP-based approaches use dynamic criteria linked to long-term utility for optimal policies. Semi-MDP (SMDP) and partially observed MDP (POMDP) are among the most commonly used variants of MDP in such problems[58,59]. The decisions are planned as illustrated in Figure 3. Under the MDP framework, system states (S_1 to S_5) are fully observable, while the POMDP framework addresses situations where system states are not directly observable. Instead, the decision-maker relies on observations to update belief states, which serve as probabilistic representations of the system's underlying condition. Mahmoodi *et al.* explore SMDP-based maintenance for parallel unit systems in dynamic environments[60], while Arcieri *et al.* propose a framework for inferring POMDP parameters using Markov Chain Monte Carlo (MCMC) to and address model uncertainty through domain randomization in RL training[61].

Warranty management stands at the interfaces of various engineering and management perspectives, including reliability, quality, marketing, design, *etc.*[62]. Maintenance and warranty policies for sold systems can be optimized jointly using MDP. Developing adaptive, tailored strategies in this context shows great potential.

Maintenance modeling can be expanded by incorporating system dynamics and varied maintenance actions to suit different scenarios. One intriguing direction is to incorporate time-varying dynamic covariates into degradation models, enabling decisions based on both system state and external factors. For example, Zheng *et al.* use an SMDP integrated with lot sizing and maintenance scheduling[63], while Luo *et al.* model the dynamic environment as a time-varying covariate affecting the degradation drift via a Markov process[64]. Joint policies combining maintenance with other decisions, such as production planning and inventory management[65,66], offer significant benefits. Paraschos *et al.* study a stochastic production and inventory system that accounts for multiple deterioration failures and product quality[67]. Zheng *et al.* consider the quality of spare parts supplied by vendors as a critical factor in their model[68]. MDP can facilitate the joint optimization of maintenance and warranty policies, particularly by exploring adaptive and customized strategies, incorporating adaptive approaches and risk-aware criteria, as shown by Xu *et al.*[69].

To tackle sequential decision problems, especially in larger-scale scenarios, algorithms can leverage system reliability characteristics. For example, modern electricity distribution systems consist of intricate networks with interdependent subsystems, presenting significant maintenance challenges due to their scale. RL-driven CBM policies are increasingly popular for their adaptability to dynamic, uncertain systems[70]. However, the dependencies among these subsystems can be limited, allowing for factored MDP modeling to simplify the problem and make the computation of optimal policies more feasible. Furthermore, current RL methodologies applied in CBM optimization primarily focus on determining optimal policies through

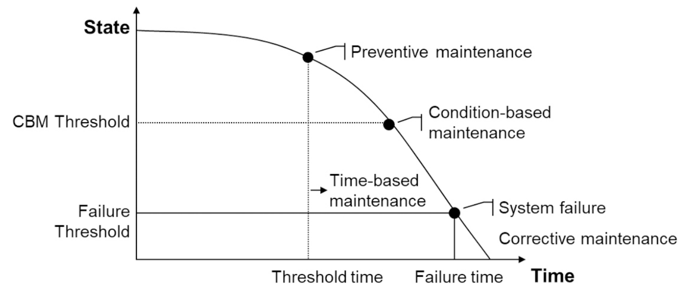

Figure 2. Different maintenance planning schemes.

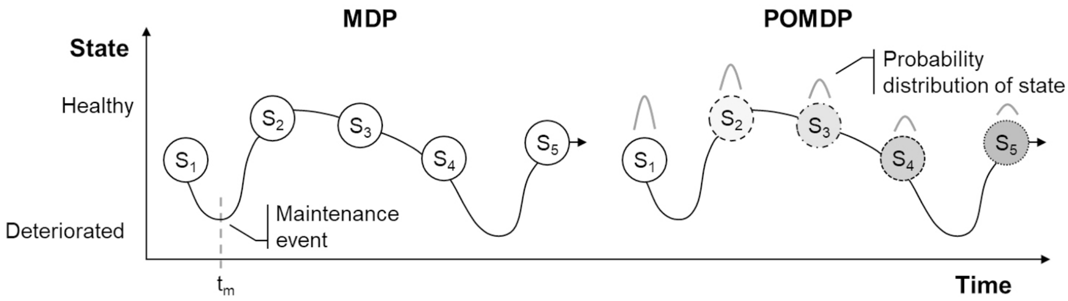

Figure 3. MDP and POMDP in maintenance planning.

numerical approaches. There is a notable gap in research regarding the convergence of regrets, a promising area for exploring CBM policy structures within the RL framework.

5. CONCLUSIONS

The paper highlights progress and future directions in dynamic decision-making for reliability. Advancements in operations research, computer science, and statistics offer valuable tools for modeling and solving dynamic optimization problems. Integrating domain expertise from mechanical and systems engineering enhances applicability. Future research should prioritize physics-informed machine learning to merge physical principles with data-driven reliability models. Transitioning to dynamic ART can yield more realistic simulations, while ensuring model interpretability and computational efficiency is critical as complexity rises. Tailored algorithms for specific systems will enable real-time applications. In CBM, RL can improve policy decisions, addressing challenges such as regret convergence. Developing adaptive maintenance and warranty strategies and new optimality criteria that balance statistical and practical aspects will be essential for practical deployment. The focus on these dynamic approaches signals a promising path for future research across system life cycles.

DECLARATIONS

Authors' contributions

Conceptualization and main drafting of the manuscript: Zhao X
Writing and revision: Luo Y

Availability of data and materials

Not applicable.

Financial support and sponsorship

This work is supported by the National Natural Science Foundation of China [grant numbers 72371182 and 72002149] and the National Social Science Fund of China [grant number 23CGL018].

Conflicts of interest

Zhao X is a Junior Editorial Board member of the journal *Complex Engineering Systems*, while the other author have declared that they have no conflicts of interest.

Ethical approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

Copyright

© The Author(s) 2024.

REFERENCES

1. Hong Y, Duan Y, Meeker WQ, Stanley DL, Gu X. Statistical methods for degradation data with dynamic covariates information and an application to outdoor weathering data. *Technometrics* 2015;57:180-93. DOI
2. Diallo C, Venkatadri U, Khatab A, Bhakthavatchalam S. State of the art review of quality, reliability and maintenance issues in closed-loop supply chains with remanufacturing. *Int J Prod Res* 2017;55:1277-96. DOI
3. Zhang D, Zhang Y. Dynamic decision-making for reliability and maintenance analysis of manufacturing systems based on failure effects. *Enterp Inf Syst* 2017;11:1228-42. DOI
4. Gandoman FH, Jaguemont J, Goutam S, et al. Concept of reliability and safety assessment of lithium-ion batteries in electric vehicles: basics, progress, and challenges. *Appl Energy* 2019;251:113343. DOI
5. Zhao X, Gaudoin O, Doyen L, Xie M. Optimal inspection and replacement policy based on experimental degradation data with covariates. *IISE Trans* 2019;51:322-36. DOI
6. Lobato FS, da Silva MA, Cavalini Jr AA, Steffen Jr V. Reliability-based robust multi-objective optimization applied to engineering system design. *Eng Optim* 2020;52:1-21. DOI
7. Liu L, Li X, Jiang T, Sun F. Utilizing accelerated degradation and field data for life prediction of highly reliable products. *Qual Reliab Eng Int* 2016;32:2281-97. DOI
8. Collins DH, Freels JK, Huzurbazar AV, Warr RL, Weaver BP. Accelerated test methods for reliability prediction. *J Qual Technol* 2013;45:244-59. DOI
9. Baladeh AE, Zio E. A two-stage stochastic programming model of component test plan and redundancy allocation for system reliability optimization. *IEEE Trans Reliab* 2021;70:99-109. DOI
10. Mellal MA, Al-Dahidi S, Williams EJ. System reliability optimization with heterogeneous components using hosted cuckoo optimization algorithm. *Reliab Eng Syst Saf* 2020;203:107110. DOI
11. Coit DW, Zio E. The evolution of system reliability optimization. *Reliab Eng Syst Saf* 2019;192:106259. DOI
12. Li X, Zhang W, Ding Q. Deep learning-based remaining useful life estimation of bearings using multi-scale feature extraction. *Reliab Eng Syst Saf* 2019;182:208-18. DOI
13. Peng W, Hong L, Ye Z. Degradation-based reliability modeling of complex systems in dynamic environments. Singapore: Springer; 2017; pp. 81-103. DOI
14. Li C, Wang X, Li L, Xie M, Wang X. On dynamically monitoring aggregate warranty claims for early detection of reliability problems. *IISE Trans* 2020;52:568-87. DOI
15. Gan S, Hu H, Coit DW. Maintenance optimization considering the mutual dependence of the environment and system with decreasing effects of imperfect maintenance. *Reliab Eng Syst Saf* 2023;235:109202. DOI
16. Friederich J, Lazarova-Molnar S. Reliability assessment of manufacturing systems: a comprehensive overview, challenges and opportunities. *J Manuf Syst* 2024;72:38-58. DOI
17. Meeker WQ, Escobar LA. A review of recent research and current issues in accelerated testing. *Int Stat Rev* 1993;61:147. DOI
18. Limon S, Yadav OP, Liao H. A literature review on planning and analysis of accelerated testing for reliability assessment. *Qual Reliab Eng Int* 2017;33:2361-83. DOI
19. Zhao X, Xu J, Liu B. Accelerated degradation tests planning with competing failure modes. *IEEE Trans Reliab* 2018;67:142-55. DOI
20. Meeker WQ, Escobar LA, Hong Y. Using accelerated life tests results to predict product field reliability. *Technometrics* 2009;51:146-61. DOI

21. Meeker WQ, Escobar LA, Lu CJ. Accelerated degradation tests: modeling and analysis. *Technometrics* 1998;40:89. DOI
22. Ye Z, Chen L, Tang LC, Xie M. Accelerated degradation test planning using the inverse gaussian process. *IEEE Trans Reliab* 2014;63:750-63. DOI
23. Ghaderi A, Hassani H, Khodaygan S. A Bayesian-reliability based multi-objective optimization for tolerance design of mechanical assemblies. *Reliab Eng Syst Saf* 2021;213:107748. DOI
24. Song J, Cui Y, Wei P, Valdebenito MA, Zhang W. Constrained Bayesian optimization algorithms for estimating design points in structural reliability analysis. *Reliab Eng Syst Saf* 2024;241:109613. DOI
25. Fang G, Pan R, Stufken J. Optimal setting of test conditions and allocation of test units for accelerated degradation tests with two stress variables. *IEEE Trans Reliab* 2021;70:1096-111. DOI
26. Zhao X, Chen P, Lv S, He Z. Reliability testing for product return prediction. *Eur J Oper Res* 2023;304:1349-63. DOI
27. Etemadi AH, Fotuhi-Firuzabad M. Design and routine test optimization of modern protection systems with reliability and economic constraints. *IEEE Trans Power Deliv* 2012;27:271-8. DOI
28. Wang R, Xu J, Zhang W, Gao J, Li Y, Chen F. Reliability analysis of complex electromechanical systems: state of the art, challenges, and prospects. *Qual Reliab Eng Int* 2022;38:3935-69. DOI
29. He K, Sun Q, Xie M, Kuo W. Sequential Bayesian planning for accelerated degradation tests considering sensor degradation. *IEEE Trans Reliab* 2023;72:964-74. DOI
30. Lee I, Hong Y, Tseng S, Dasgupta T. Sequential Bayesian design for accelerated life tests. *Technometrics* 2018;60:472-83. DOI
31. Sutton RS, Barto AG. Reinforcement learning: an introduction. MIT Press; 2018. Available from: https://books.google.com/books?hl=zh-CN&lr=&id=uWV0DwAAQBAJ&oi=fnd&pg=PR7&dq=Sutton,+R.+S.+%26+Barto,+A.+G.+Reinforcement+Learning:+An+Introduction+(MIT+Press,+2018).&ots=mjqHm-YYo0&sig=qXB9jA_4jYcn0-4JysHj08ubr1I#v=onepage&q=Sutton%2C%20R.%20S.%20%26%20Barto%2C%20A.%20G.%20Reinforcement%20Learning%3A%20An%20Introduction%20(MIT%20Press%2C%202018).&f=false [Last accessed on 30 Nov 2024]
32. Zhang Z, Yang L, Xu Y, Zhu R, Cao Y. A novel reliability redundancy allocation problem formulation for complex systems. *Reliab Eng Syst Saf* 2023;239:109471. DOI
33. Li J, Huang Y, Li Y, Wang S. Redundancy allocation under state-dependent distributional uncertainty of component lifetimes. *Prod Oper Manag* 2023;32:930-50. DOI
34. Wang S, Li YF. Distributionally robust design for redundancy allocation. *INFORMS J Comput* 2020;32:620-40. DOI
35. Reihaneh M, Abouei Ardakan M, Eskandarpour M. An exact algorithm for the redundancy allocation problem with heterogeneous components under the mixed redundancy strategy. *Eur J Oper Res* 2022;297:1112-25. DOI
36. Peiravi A, Nourelfath M, Zanjani MK. Universal redundancy strategy for system reliability optimization. *Reliab Eng Syst Saf* 2022;225:108576. DOI
37. Meng Z, Zhang Z, Zhou H. A novel experimental data-driven exponential convex model for reliability assessment with uncertain-but-bounded parameters. *Appl Math Model* 2020;77:773-87. DOI
38. Murthy DNP, Xie M, Jiang R. Weibull models. Wiley; 2004. DOI
39. Hajiha M, Liu X, Hong Y. Degradation under dynamic operating conditions: modeling, competing processes and applications. *J Qual Technol* 2021;53:347-68. DOI
40. Singpurwalla ND. Survival in dynamic environments. *Stat Sci* 1995;10:86-103. DOI
41. Eryilmaz S, Rıza Bozbulut A. An algorithmic approach for the dynamic reliability analysis of non-repairable multi-state weighted k-out-of-n:G system. *Reliab Eng Syst Saf* 2014;131:61-5. DOI
42. Luo C, Shen L, Xu A. Modelling and estimation of system reliability under dynamic operating environments and lifetime ordering constraints. *Reliab Eng Syst Saf* 2022;218:108136. DOI
43. Ye Z, Xie M. Stochastic modelling and analysis of degradation for highly reliable products. *Appl Stoch Models Bus & Ind* 2015;31:16-32. DOI
44. Zhang S, Zhai Q, Shi X, Liu X. A wiener process model with dynamic covariate for degradation modeling and remaining useful life prediction. *IEEE Trans Reliab* 2023;72:214-23. DOI
45. Xu Z, Saleh JH. Machine learning for reliability engineering and safety applications: review of current status and future opportunities. *Reliab Eng Syst Saf* 2021;211:107530. DOI
46. Song S, Coit DW, Feng Q, Peng H. Reliability analysis for multi-component systems subject to multiple dependent competing failure processes. *IEEE Trans Reliab* 2014;63:331-45. DOI
47. Han X, Ouyang M, Lu L, Li J, Zheng Y, Li Z. A comparative study of commercial lithium ion battery cycle life in electrical vehicle: aging mechanism identification. *J Power Sources* 2014;251:38-54. DOI
48. Fan M, Zeng Z, Zio E, Kang R. Modeling dependent competing failure processes with degradation-shock dependence. *Reliab Eng Syst Saf* 2017;165:422-30. DOI
49. Wu B, Ding D. A gamma process based model for systems subject to multiple dependent competing failure processes under Markovian environments. *Reliab Eng Syst Saf* 2022;217:108112. DOI
50. Zhou H, Li Y. Optimal replacement in a proportional hazards model with cumulative and dependent risks. *Comput Ind Eng* 2023;176:108930. DOI
51. Zhai Q, Ye Z, Li C, Revie M, Dunson DB. Modeling recurrent failures on large directed networks. *J Am Stat Assoc* 2024:1-15. DOI
52. de Jonge B, Scarf PA. A review on maintenance optimization. *Eur J Oper Res* 2020;285:805-24. DOI

53. Hu J, Huang Y, Shen L. Maintenance optimization of a two-component series system considering masked causes of failure. *Qual Reliab Eng Int* 2024;40:388-405. DOI
54. Sun Q, Chen P, Wang X, Ye Z. Robust condition-based production and maintenance planning for degradation management. *Prod Oper Manag* 2023;32:3951-67. DOI
55. Uit Het Broek MA, Teunter RH, de Jonge B, Veldman J. Joint condition-based maintenance and load-sharing optimization for two-unit systems with economic dependency. *Eur J Oper Res* 2021;295:1119-31. DOI
56. Puterman ML. Chapter 8 Markov decision processes. Elsevier; 1990; pp. 331-434. DOI
57. Wiesemann W, Kuhn D, Rustem B. Robust Markov decision processes. *Math Oper Res* 2013;38:153-83. DOI
58. Deep A, Zhou S, Veeramani D, Chen Y. Partially observable Markov decision process-based optimal maintenance planning with time-dependent observations. *Eur J Oper Res* 2023;311:533-44. DOI
59. Drent C, Drent M, Arts J, Kapodistria S. Real-time integrated learning and decision making for cumulative shock degradation. *Manuf Serv Oper Manag* 2023;25:235-53. DOI
60. Mahmoodi S, Hamed Ranjkesh S, Zhao YQ. Condition-based maintenance policies for a multi-unit deteriorating system subject to shocks in a semi-Markov operating environment. *Qual Eng* 2020;32:286-97. DOI
61. Arcieri G, Hoelzl C, Schwery O, Straub D, Papakonstantinou KG, Chatzi E. POMDP inference and robust solution via deep reinforcement learning: an application to railway optimal maintenance. *Mach Learn* 2024;113:7967-95. DOI
62. González-Prida V, Crespo Márquez A. A framework for warranty management in industrial assets. *Comput Ind* 2012;63:960-71. DOI
63. Zheng R, Zhou Y, Gu L, Zhang Z. Joint optimization of lot sizing and condition-based maintenance for a production system using the proportional hazards model. *Comput Ind Eng* 2021;154:107157. DOI
64. Luo Y, Zhao X, Liu B, He S. Condition-based maintenance policy for systems under dynamic environment. *Reliab Eng Syst Saf* 2024;246:110072. DOI
65. Farahani A, Tohidi H. Integrated optimization of quality and maintenance: a literature review. *Comput Ind Eng* 2021;151:106924. DOI
66. Van Horenbeek A, Buré J, Cattrysse D, Pintelon L, Vansteenwegen P. Joint maintenance and inventory optimization systems: A review. *Int J Prod Econ* 2013;143:499-508. DOI
67. Paraschos PD, Koulinas GK, Koulouriotis DE. Reinforcement learning for combined production-maintenance and quality control of a manufacturing system with deterioration failures. *J Manuf Syst* 2020;56:470-83. DOI
68. Zheng M, Ye H, Wang D, Pan E. Joint decisions of components replacement and spare parts ordering considering different supplied product quality. *IEEE Trans Automat Sci Eng* 2024;21:1952-64. DOI
69. Xu J, Zhao X, Liu B. A risk-aware maintenance model based on a constrained Markov decision process. *IISE Trans* 2022;54:1072-83. DOI
70. Ogunfowora O, Najjaran H. Reinforcement and deep reinforcement learning-based solutions for machine maintenance planning, scheduling policies, and optimization. *J Manuf Syst* 2023;70:244-63. DOI