Li *et al*. *Intell Robot* 2023;3(2):213-21
DOI: 10.20517/ir.2023.13

Intelligence & Robotics

Research Article

Open Access

Check for updates

The cooperatability of the first-order multi-agent systems consisting of a leader and a follower with multiplicative noises under Markov switching topologies

Dianqiang Li, Tao Li

School of Mathematical Sciences, East China Normal University, Shanghai 200241, China.

Correspondence to: Dr. Dianqiang Li, School of Mathematical Sciences, East China Normal University, No.5005, South Lianhua Road, Shanghai 200241, China. E-mail: ldq2015ahdx@126.com.

How to cite this article: Li D, Li T. The cooperatability of the first-order multi-agent systems consisting of a leader and a follower with multiplicative noises under Markov switching topologies. *Intell Robot* 2023;3(2):213-21. http://dx.doi.org/10.20517/ir.2023.13

Received: 24 Mar 2023 **First Decision:** 17 Apr 2023 **Revised:** 29 May 2023 **Accepted:** 6 Jun 2023 **Published:** 28 Jun 2023

Academic Editor: Simon X. Yang, Hongtian Chen **Copy Editor:** Yanbing Bai **Production Editor:** Yanbing Bai

Abstract

We investigate the cooperatability of the first-order leader-following multi-agent systems consisting of a leader and a follower with multiplicative noises under Markov switching topologies. Each agent exhibits first-order linear dynamics, and there are multiplicative noises along with information exchange among the agents. What is more, the communication topologies are Markov switching topologies. By utilizing the stability theory of the stochastic differential equations with Markovian switching and the Markov chain theory, we establish the necessary and sufficient conditions for the cooperatability of the leader-following multi-agent systems. The conditions are outlined below: (i) The product of the system parameter and the square of multiplicative noise intensities should be less than 1/2; (ii) The transition rate from the unconnected graph to the connected graph should be twice the system parameter; (iii) The transition rate from the connected graph to the unconnected graph should be less than a constant that is related to the system parameter, the intensities of multiplicative noises, and the transition rate from the unconnected graph to the connected graph. Finally, the effectiveness of our control strategy is demonstrated by the population growth systems.

Keywords: Leader-following, Markov switching topologies, multiplicative noises, cooperatability

1. INTRODUCTION

In the past few decades, distributed cooperative control of multi-agent systems under fixed topologies has attracted much attention from the system and control community[1–3]. However, in practical systems, the communication networks connecting the agents often experience sudden interruptions and restoration. These mutations lead to the changes in the structures or parameters of the system. Here, we describe this changing topology by the Markovian switching topology. For such systems, we usually use the Markov switching systems to describe them. In recent years, the stability of linear Markov switching systems has been widely studied[4–8]. By Kronecker product and Lyapunov exponent, Mariton *et al.*[4] gave the necessary and sufficient conditions for the moment stability and the almost sure stability of the system, respectively. Feng *et al.*[5] studied the stochastic stability of the system and revealed the relationship between the moment stability and the almost sure stability. Feng *et al.*[6] studied the stabilization problem. The literature[7,8] investigated the robust stability problems and gave sufficient and necessary conditions in the form of linear matrix inequalities for the mean square stability.

In many real-world systems, it is inevitable for systems to be subjected to random noises[9]. These noises may change the trajectory of the system and even affect its stability. Therefore, an increasing number of researchers have focused on studying the stability of the Markov switching stochastic systems. The stability of linear Markov switching systems with stochastic noises was studied in previous literature[10–12]. Fragoso *et al.*[10] studied the Markov switching systems with additive noises and provided the necessary and sufficient conditions for the mean square stability of the system. On the other hand, the literature[11,12] explored the Markov switching systems with multiplicative noises. By employing the operator theory, Dragan *et al.*[11] derived the necessary and sufficient conditions in the form of linear matrix inequalities for the mean square stability. Similarly, Sheng *et al.*[12], also using the operator theory, presented a new necessary and sufficient condition for the mean square stability. Using the Lyapunov method, Mao *et al.*[13] established a sufficient condition for the pth moment exponential stability of the nonlinear Markov switching system and revealed the relationship between the pth moment exponential stability and the almost sure exponential stability of the system. In the context of nonlinear Markov switching systems, Deng *et al.*[14] addressed the problem of mean square stabilization.

The stability theory of Markov switching systems with noises has numerous practical applications[15–17]. Previous studies[18–22] have focused on the distributed control problem of multi-agent systems with random noises under Markov switching topologies. The literature[18,19] studied the distributed control problem of discrete-time multi-agent systems. By the state space decomposition method, Huang *et al.*[18] gave a sufficient condition for almost sure consensus and mean square consensus, respectively. Zhang *et al.*[19] studied the mean square consensus problem. The literature[20–22] considers the distributed control problem of continuous-time multi-agent systems. Zhang *et al.*[20] studies the distributed control problem of multi-agent systems with first-order integrator dynamics. Li *et al.*[21] studied the containment control problem. Wang *et al.*[22] studied mean square consensus and almost sure consensus of higher-order multi-agent systems.

Compared with additive noises, multiplicative noises play a stabilizing role in the almost sure stability of systems[23]. Many scholars have studied the distributed control problem of multi-agent systems with multiplicative noises[24–28]. However, as the state of the system is related to the Markov chain, we cannot write the expectation of the product of the state variable and the indicative function in the form of the expected product. This leads to the fact that the distributed control problem of multi-agent systems with multiplicative noises under the Markov switching topology has not yet been solved. As a preliminary study, we study the cooperatability of the first-order leader-following multi-agent systems consisting of a leader and a follower with multiplicative noises under Markov switching topologies. Each agent has first-order linear dynamics, and there are multiplicative noises along with information exchange among agents. What is more, the communication topologies are Markov switching topologies. Compared with existing literature[24–28], we have revealed the influence of multiplicative noises and switching rates on the cooperatability of the system. To analyze this influence, we delve into the stability theory of Markov switching systems with noises. Therefore, we introduced a new lemma to address this issue. We establish the necessary and sufficient conditions for the cooperatability of the leader-following multi-agent systems by combining the stability theory of the stochastic differential equation with Markovian switching and the Markov chain theory. These conditions are outlined below: (i) The product of the system parameter and the square of multiplicative noise intensities should be less than 1/2; (ii) The transition rate from the unconnected graph to the connected graph should be twice the value of the system parameter; (iii) The transition rate from the connected graph to the unconnected graph should be lower than a constant, which is related to the system parameter, the intensity of multiplicative noises, and the transition rate from the unconnected graph to the connected graph.

The remaining sections of this paper are structured as follows: Section 2 formulates the problem. Section 3 presents the admissible cooperative distributed control strategy. Section 4 provides the main result. Section 5 includes a numerical simulation to demonstrate the effectiveness of our control laws. Section 6 concludes the paper.

Notation: The symbols $\mathbb{R}$ and $\mathbb{R}_+$ denote real and non-negative numbers, respectively. I_n denotes the $n \times n$ dimensional identity matrix. The symbol diag$\{A_1, \ldots, A_N\}$ represents the block diagonal matrix with entries being $A_1, \ldots, A_N$. For a given vector or matrix X, X^{T} denotes its transpose, and $|X|$ represents the determinant of X. For two matrices C and D, $C \otimes D$ denotes their Kronecker product, and $C \oplus D = C \otimes I + I \otimes D$ represents the Kronecker sum. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geqslant t_0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geqslant t_0}$ that satisfies the usual conditions, namely, it is right continuous and increasing while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets; $w(t) = (w_1(t), \ldots, w_m(t))^T$ denotes a m-dimensional standard Brownian motion defined in $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geqslant t_0}, \mathbb{P}\right)$. For a given random variable X, the mathematical expectation of X is denoted by $\mathbb{E}[X]$.

2. PROBLEM FORMULATIONS

Consider a leader-following multi-agent system consisting of a leader and a follower, where the leader and the follower are indexed by 0 and 1, respectively. The dynamics of the leader is given by

$$\dot{x}_0(t) = a x_0(t), \tag{1}$$

where $x_0(t) \in \mathbb{R}$ is the state, and $a \in \mathbb{R}^+$ is a known constant.

The dynamics of the follower is given by

$$\dot{x}_1(t) = a x_1(t) + b u(t), \tag{2}$$

where $x_1(t) \in \mathbb{R}$ is the state, $u(t) \in \mathbb{R}$ is the input, and $a \in \mathbb{R}^+$ and $b \in \mathbb{R}/0$ are known constants.

In this section, we assume that the topology graph is a Markovian switching topology. Let the switching signal $\theta(t)$ be defined in the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geqslant 0}, \mathbb{P})$. The signal $\theta(t)$ is a right continuous homogeneous Markov chain and has a finite state space $\mathbb{S} = \{1, 2\}$. The matrix $Q = [q_{ij}]_{1 \leqslant i,j \leqslant 2}$ is the transfer rate matrix of the Markov chain $\theta(t)$ and satisfies

$$P(\theta(t+\triangle) = j | \theta(t) = i) = \begin{cases} q_{ij} \triangle + o(\triangle), & i \neq j, \\ 1 + q_{ij} \triangle + o(\triangle), & i = j, \end{cases}$$

where if $i \neq j$, q_{ij} is the transition rate of the Markov chain from the state i to the state j with $q_{ij} \geqslant 0$; if $i = j$, $q_{ii} = -\sum_{j \neq i}^{2} q_{ij}$; $\triangle > 0$ and $\lim_{t \to \infty} \frac{o(\triangle)}{\triangle} = 0$. We use $\mathcal{G}_{(\theta(t))} = (\mathcal{V}, \mathcal{E}(\theta(t)), \mathcal{A}(\theta(t)))$ to represent a weighted graph formed by the leader and the follower, where the set of nodes $\mathcal{V} = \{0, 1\}$ and the set of edges $\mathcal{E}(\theta(t)) \subseteq \mathcal{V}\times\mathcal{V}$. Denote the neighbors of the ith agent by $\mathcal{N}_i(\theta(t))$. The adjacency matrix $\mathcal{A}(\theta(t)) = \begin{bmatrix} 0 & 0 \\ a_{10}(\theta(t)) & 0 \end{bmatrix} \in \mathbb{R}^{2\times 2}$, where if $0 \in \mathcal{N}_1(\theta(t))$, then $a_{10}(\theta(t)) = 1$, otherwise $a_{10}(\theta(t)) = 0$. The Laplacian matrix of $\mathcal{G}_{(\theta(t))}$ is given by $\mathcal{L}(\theta(t)) = \mathcal{D}(\theta(t)) - \mathcal{A}(\theta(t))$, where $\mathcal{D}(\theta(t)) = \mathrm{diag}(0, a_{10}(\theta(t)))$. Without losing generality, we assume that the transition rate matrix of the Markov chain $\theta(t)$ is the matrix $Q = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix}$, where α represents the transition rate from the unconnected graph to the connected graph; β represents the transition rate from the connected graph to the unconnected graph.

3. ADMISSIBLE DISTRIBUTED COOPERATIVE CONTROL STRATEGY

In the real network, the relative state measurement information obtained by the follower from the leader is often affected by noises. Therefore, for the leader–following multi-agent system (1)–(2), we assume that the relative state measurement information has the following form

$$y_{10}(t) = x_1(t) - x_0(t) + \sigma_{10}\left(x_1(t) - x_0(t)\right)\xi_{10}(t), \tag{3}$$

where $\xi_{10}(t)$ represents the multiplicative measurement noise, and σ_{10} represents the intensity of multiplicative measurement noise.

We consider the following set of admissible distributed cooperative control strategies based on (3) and the randomness of the communication topology

$$\mathcal{U} = \{U = \{u(t) = k a_{10}(\theta(t)) y_{10}(t), t > 0\}, k \in \mathbb{R}\}. \tag{4}$$

This paper primarily focuses on investigating the necessary and sufficient conditions for the cooperatability of the first-order leader-following multi-agent systems. These systems are composed of a leader and a follower and are subjected to multiplicative noises under Markov switching topologies.

The assumption and lemma required in this section are given below.

Assumption 1 The noise process $\xi_{10}(t)$ satisfies $\int_0^t \xi_{10}(s)\mathrm{d}s = w_{10}(t), t \geqslant 0$, where $w_{10}(t)$ is a one-dimensional standard Brownian motion.

Lemma 1 [12] The solution of the Markov switching stochastic differential equations

$$\mathrm{d}x(t) = A(\theta(t))x(t)\mathrm{d}t + C(\theta(t))x(t)\mathrm{d}w(t) \tag{5}$$

is mean square stable if and only if $F = \mathrm{diag}(A(1) \oplus A(1), \ldots, A(S) \oplus A(S)) + \mathrm{diag}(C(1) \oplus C(1), \ldots, C(S) \oplus C(S)) + Q^{\mathrm{T}} \otimes I_{n^2}$ is a Hurwitz matrix, where $Q = [q_{ij}]_{1\leqslant i,j\leqslant S}$ is the transition rate matrix of the Markov chain $\theta(t)$. If $\theta(t) = i$, we denote $A(\theta(t)) = A(i)$, $C(\theta(t)) = C(i)$, and $i = 1, \ldots, S$.

4. MAIN RESULTS

By leveraging the stability theory of stochastic differential equations with Markovian switching and the Markov chain theory, we provide the necessary and sufficient conditions for the cooperatability of the leader-following multi-agent systems.

Theorem 1 Suppose Assumption 1 is satisfied. In that case, there exists an admissible cooperative control strategy denoted by $U \in \mathcal{U}$, which ensures that the follower can track the leader for any initial value under the distributed control law U. This holds if and only if the following conditions are met: $2a\sigma_{10}^2 < 1$, $\alpha > 2a$, $0 \leqslant \beta < \frac{(\alpha-2a)(1-2a\sigma_{10}^2)}{2a\sigma_{10}^2}$.

Proof: Denote $\delta(t) = x_1(t) - x_0(t)$. By Assumptions 1 and (1)–(4), we get

$$d\delta(t) = [a + bka_{10}(\theta(t))]\delta_1(t)dt + bka_{10}(\theta(t))\sigma_{10}\delta_1(t)dw_{10}(t), \tag{6}$$

where if $a_{10}(\theta(t)) = 0$, then we denote $A(1) = a$ and $C(1) = 0$; if $a_{10}(\theta(t)) = 1$, then we denote $A(2) = a + bk$ and $C(2) = bk\sigma_{10}$.

Denote $F = \mathrm{diag}(A(1) \oplus A(1), A(2) \oplus A(2)) + \mathrm{diag}(C(1) \oplus C(1), C(2) \oplus C(2)) + Q^{\mathrm{T}} \otimes I_1$. By the definition of F, we have

$$\begin{aligned} F &= \begin{bmatrix} 2a & 0 \\ 0 & 2(a+bk) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & b^2k^2\sigma_{10}^2 \end{bmatrix} + \begin{bmatrix} -\alpha & \beta \\ \alpha & -\beta \end{bmatrix} \\ &= \begin{bmatrix} 2a - \alpha & \beta \\ \alpha & 2(a+bk) + b^2k^2\sigma_{10}^2 - \beta \end{bmatrix}. \end{aligned} \tag{7}$$

Necessity: If there exists an admissible cooperative control strategy denoted by $U \in \mathcal{U}$, such that for any initial value, the follower can track the leader under the distributed control law U, it implies that the system (6) is mean square stable. According to Lemma 1, it can be inferred that all eigenvalues of F have negative real parts.

Noting that $|\lambda I - F| = \begin{vmatrix} \lambda - 2a + \alpha & -\beta \\ -\alpha & \lambda - 2(a+bk) - b^2k^2\sigma_{10}^2 + \beta \end{vmatrix}$, we have $|\lambda I - F| = (\lambda - 2a)^2 + (\beta + \alpha - 2bk - b^2k^2\sigma_{10}^2)(\lambda - 2a) - 2bk\alpha - \alpha b^2k^2\sigma_{10}^2$.

Denote $m = \lambda - 2a$ and $f(m) = m^2 + (\beta + \alpha - 2bk - b^2k^2\sigma_{10}^2)m - 2bk\alpha - \alpha b^2k^2\sigma_{10}^2$. As all eigenvalues of F have negative real parts, we know that the real parts of the zero point of $f(m)$ are less than $-2a$. As the real parts of the zero point of $f(m)$ are less than $-2a$, by considering the image of the function $f(m)$, the following two conditions can be inferred.

Condition $(\mathbf{C_1})$: $f(-2a) = 4a^2 - 2a(\beta + \alpha - 2bk - b^2k^2\sigma_{10}^2) - 2bk\alpha - \alpha b^2k^2\sigma_{10}^2 > 0$.

Condition $(\mathbf{C_2})$: $\frac{2bk-\alpha-\beta+b^2k^2\sigma_{10}^2}{2} + 2a < 0$.
By Condition $(\mathbf{C_1})$, we obtain

$$(2a - \alpha)(2a + 2bk + b^2k^2\sigma_{10}^2) > 2a\beta. \tag{8}$$

In the following, we discuss the Conditions $(\mathbf{C_1})$ and $(\mathbf{C_2})$.

(1) If $2a + 2bk + b^2k^2\sigma_{10}^2 = 0$ holds, then we have $(2a - \alpha)(2a + 2bk + b^2k^2\sigma_{10}^2) = 0$. This contradicts the inequality (8). Therefore, this situation does not hold.

(2) If $2a + 2bk + b^2k^2\sigma_{10}^2 > 0$ holds, by (8), we get $0 \leq \alpha < 2a$.
By $\frac{2bk-\alpha-\beta+b^2k^2\sigma_{10}^2}{2} + 2a < 0$, we have $\beta + \alpha - 2bk - b^2k^2\sigma_{10}^2 > 4a$. By $\beta + \alpha - 2bk - b^2k^2\sigma_{10}^2 > 4a$, $2a + 2bk + b^2k^2\sigma_{10}^2 > 0$ and $0 \leq \alpha < 2a$, we obtain $f(-2a) = 4a^2 - 2a(\beta + \alpha - 2bk - b^2k^2\sigma_{10}^2) - 2bk\alpha - \alpha b^2k^2\sigma_{10}^2 < -4a^2 - 2bk\alpha - \alpha b^2k^2\sigma_{10}^2 = -4a^2 - \alpha(2bk + b^2k^2\sigma_{10}^2) < -4a^2 + 2\alpha a = 2a(\alpha - 2a) < 0$. This contradicts Condition $(\mathbf{C_1})$. Therefore, this situation also is not valid.

(3) If $2a + 2bk + b^2k^2\sigma_{10}^2 < 0$, then by $\beta \geqslant 0$, $a > 0$ and (8), we have $\alpha > 2a$.

By $\alpha > 2a$ and (8), we get

$$b^2k^2\sigma_{10}^2 + 2bk < \frac{2a\beta}{2a-\alpha} - 2a. \tag{9}$$

By Condition ($\mathbf{C_2}$), we obtain

$$b^2k^2\sigma_{10}^2 + 2bk < \alpha + \beta - 4a. \tag{10}$$

By $\beta \geqslant 0$, $a > 0$ and $\alpha > 2a$, we get

$$\alpha + \beta - 4a - (\frac{2a\beta}{2a-\alpha} - 2a) = \alpha - 2a + \beta + \frac{2a\beta}{\alpha - 2a} > 0, \tag{11}$$

which implies $\alpha + \beta - 4a > \frac{2a\beta}{2a-\alpha} - 2a$.

By (9), (10), and (11), we have

$$b^2k^2\sigma_{10}^2 + 2bk < \frac{2a\beta}{2a-\alpha} - 2a. \tag{12}$$

Denote $g(t) = \sigma_{10}^2 t^2 + 2t - \frac{2a\beta}{2a-\alpha} + 2a$ and $t = bk$. By (12), we know that $g(t) < 0$ has a solution for variable t. By $g(t) < 0$, we have $\Delta = 4 - 4\sigma_{10}^2(-\frac{2a\beta}{2a-\alpha} + 2a) > 0$. By $\Delta > 0$, we get $\beta < \frac{(\alpha-2a)(1-2a\sigma_{10}^2)}{2a\sigma_{10}^2}$. Combining $0 \leqslant \beta < \frac{(\alpha-2a)(1-2a\sigma_{10}^2)}{2a\sigma_{10}^2}$ and $\alpha > 2a$, we have $2a\sigma_{10}^2 < 1$. In summary, we obtain $2a\sigma_{10}^2 < 1$, $\alpha > 2a$, $0 \leqslant \beta < \frac{(\alpha-2a)(1-2a\sigma_{10}^2)}{2a\sigma_{10}^2}$.

Sufficiency: By $2a\sigma_{10}^2 < 1$, $\alpha > 2a$ and $0 \leqslant \beta < \frac{(\alpha-2a)(1-2a\sigma_{10}^2)}{2a\sigma_{10}^2}$, we get (12). By (12), we have $bk \in \left(\frac{-2(\alpha-2a)-\sqrt{\rho}}{2(\alpha-2a)\sigma_{10}^2}, \frac{-2(\alpha-2a)+\sqrt{\rho}}{2(\alpha-2a)\sigma_{10}^2}\right)$, where $\rho = 4(\alpha-2a)^2 - 4(\alpha-2a)\sigma_{10}^2[2a(\alpha-2a)+2a\beta]$. From the value range of bk, it can be seen that Condition ($\mathbf{C_1}$) and Condition ($\mathbf{C_2}$) hold. Therefore, since the real parts of the zero point of $f(m)$ are less than $-2a$, it can be concluded that all eigenvalues of F have negative real parts. Lemma 1 implies that the system (6) is mean square stable. Therefore, there exists an admissible cooperative control strategy $U \in \mathcal{U}$, such that for any initial value, the follower can track the leader under the distributed control law U.

Remark 1 The conditions $\alpha > 2a$, $2a\sigma_{10}^2 < 1$ and $0 \leqslant \beta < \frac{(\alpha-2a)(1-2a\sigma_{10}^2)}{2a\sigma_{10}^2}$ stated in Theorem 1 highlight the influence of multiplicative noises and both the transition rates α and β on the cooperatability of the system. It is shown that smaller multiplicative noises, lower transition rate β, and higher transition rate α are all favorable for the cooperatability of the system. Moreover, the transition rates α and β have lower and upper bounds, respectively. What is more, the noises and the system parameters satisfy the corresponding inequality.

We have the following corollary for the case without measurement noises.

Corollary 1 Suppose Assumption 1 and $\sigma_{10} = 0$ hold. In that case, there exists an admissible cooperative control strategy denoted by $U \in \mathcal{U}$, such that for any initial value, the follower can track the leader under the distributed control law U, if and only if $\alpha > 2a$.

5. NUMERICAL SIMULATION

In this section, we will use a numerical example to demonstrate the effectiveness of our control laws.

Figure 1. The communication topology graphs.

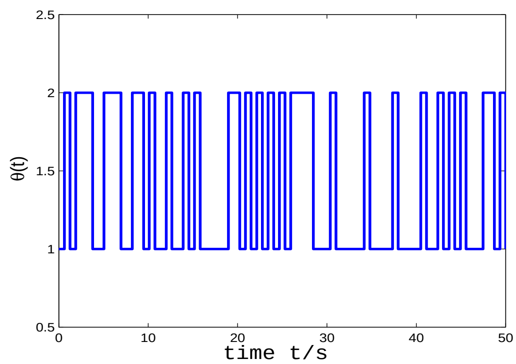

Figure 2. Markov chain $\theta(t)$.

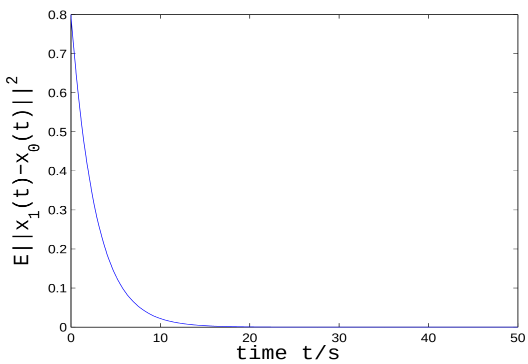

Figure 3. Mean square tracking errors.

Referring to the literature[29], the population growth system is given by (1). Consider the leader-following population growth systems (1)-(2), where $a = 0.01$ and $b = 0.2$, we will verify that the population of the follower can track the population of the leader under the distributed control law U.

The communication topology graphs are shown in Figure 1, and the trajectory of the Markov chain $\theta(t)$ is shown in Figure 2. The intensity of multiplicative measurement noise in (3) is given by $\sigma_{10} = 0.4$. The transition rate matrix is given by $Q = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$. The initial states of agents are given by $x_0(0) = 0.2$ and $x_1(0) = 0.4$.

If we choose $k = -2$, then under the control law U, the mean square error of the population between the follower and the leader is shown in Figure 3. From Figure 3, we can see that the mean square error of the population tends to zero, which implies that the follower can achieve mean square tracking under the control law of U.

6. CONCLUSION

In this paper, we have studied the cooperatability of the first-order leader-following multi-agent systems that consist of a leader and a follower. The systems are subjected to multiplicative noises under Markov switching topologies. Each agent in this system follows first-order linear dynamics, and there are multiplicative noises along with information exchange among agents. Additionally, the communication topologies are characterized by Markov switching. By employing the stability theory of the stochastic differential equation with Markovian switching and the Markov chain theory, we have established the necessary and sufficient conditions for achieving the cooperatability in the leader-following multi-agent systems. Furthermore, there are several other interesting topics that can be explored in future research. For instance, it would be valuable to investigate the cooperatability of the leader-following multi-agent systems with both multiplicative noises and delays under Markov switching topologies

DECLARATIONS

Authors' contributions

Made substantial contributions to the research and investigation process, reviewed and summarized the literature, and wrote and edited the original draft: Li D

Performed oversight and leadership responsibility for the research activity planning and execution and performed critical review, commentary, and revision: Li T

Availability of data and materials

Not applicable.

Financial support and sponsorship

Not applicable.

Conflicts of interest

All authors declared that there are no conflicts of interest.

Ethical approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

REFERENCES

1. Mao J, Huang S, Xiang Z, Wang Y, Zheng D. Practical finite-time sampled-data output consensus for a class of nonlinear multiagent systems via output feedback. *Int J Robust Nonlinear Control* 2021;31:920-49. DOI
2. Mao J, Yan T, Huang S, Li S, Jiao J. Sampled-data output feedback leader-following consensus for a class of nonlinear multi-agent systems with input unmodeled dynamics. *Int J Robust Nonlinear Control* 2021;31:4203-26. DOI
3. Shang Y, Ye Y. Leader-follower fixed-time group consensus control of multiagent systems under directed topology. *Complexity* 2017;2017:1-9. DOI
4. Mariton M. Almost sure and moments stability of jump linear systems. *Syst Control Lett* 1988;11:393-7. DOI
5. Feng X, Loparo K, Ji Y, Chizeck H. Stochastic stability properties of jump linear systems. *IEEE Trans Automat Contr* 1992;37:38-53. DOI
6. Fang Y, Loparo K. Stabilization of continuous-time jump linear systems. *IEEE Trans Automat Contr* 2002;47:1590-603. DOI
7. El Ghaoui L, Rami MA. Robust state feedback stabilization of jump linear systems via LMIs. *Int J Robust Nonlin Contr* 1996;6:1015-22. DOI
8. Costa OLV, Boukas EK. Necessary and sufficient condition for robust stability and stabilizability of continuous-time linear systems with markovian jumps. *J Optim Theory Appl* 1998;99:359-79. DOI
9. Somarakis C, Motee N. Aggregate fluctuations in networks with drift-diffusion models driven by stable non-gaussian disturbances. *IEEE Trans Control Netw Syst* 2020;7:1248-58. DOI
10. Fragoso MD, Costa OL. A unified approach for mean square stability of continuous-time markovian jumping linear systems with additive disturbances. In: Proceedings of the 39th IEEE Conference on Decision and Control, Sydney, Australia, 12–15 December, 2000, pp. 2361–2366. DOI
11. Dragan V, Morozan T, Stoica AM. Mathematical methods in robust control of linear stochastic systems. 2006, New York, USA: Springer.

12. Sheng L, Gao M, Zhang W. Spectral characterisation for stability and stabilisation of linear stochastic systems with Markovian switching and its applications. *IET Control* 2013;7:730-7. DOI
13. Mao X. Stability of stochastic differential equations with Markovian switching. *Stoch* 1999;79:45-67. DOI
14. Deng F, Luo Q, Mao X. Stochastic stabilization of hybrid differential equations. *Automatica* 2012;48:2321-8. DOI
15. Shang Y. Consensus seeking over Markovian switching networks with time-varying delays and uncertain topologies. *Appl Math Comput* 2016;273:1234-45. DOI
16. Shang Y. Couple-group consensus of continuous-time multi-agent systems under Markovian switching topologies. *J Franklin Inst* 2015;352:4826-44. DOI
17. Shang Y. Consensus of noisy multiagent systems with markovian switching topologies and time-varying delays. *Math Probl Eng* 2015;2015:1-13. DOI
18. Huang M, Dey S, Nair GN, Manton JH. Stochastic consensus over noisy networks with Markovian and arbitrary switches. *Automatica* 2010;46:1571-83. DOI
19. Zhang Y, Tian Y. Consentability and protocol design of multi-agent systems with stochastic switching topology.*Automatica* 2009;45:1195-201. DOI
20. Zhang Q, Zhang JF. Zhang Q, Zhang JF. Distributed consensus of continuous-time multi-agent systems with Markovian switching topologies and stochastic communication noises. *Int J Math Syst Sci* 2011;31:1097-110. DOI
21. Li W, Xie L, Zhang J. Containment control of leader-following multi-agent systems with Markovian switching network topologies and measurement noises. *Automatica* 2015;51:263-7. DOI
22. Wang Y, Cheng L, Ren W, Hou Z, Tan M. Seeking consensus in networks of linear agents: communication noises and markovian switching topologies. *IEEE Trans Automat Contr* 2015;60:1374-9. DOI
23. Li T, Wu F, Zhang J. Multi-agent consensus with relative-state-dependent measurement noises. *IEEE Trans Automat Contr* 2014;59:2463-8. DOI
24. Long Y, Liu S, Xie L. Distributed consensus of discrete-time multi-agent systems with multiplicative noises: consensus of discrete-time multi-agent systems. *Int J Robust Nonlin Contr* 2015;25:3113-31. DOI
25. Zong X, Li T, Zhang J. Consensus control of discrete-time multi-agent systems with time-delays and multiplicative measurement noises. *Scientia Sinica Mathematica* 2016;46:1617–36. DOI
26. Ni Y, Li X. Consensus seeking in multi-agent systems with multiplicative measurement noises. *Syst Control Lett* 2013;62:430-7. DOI
27. Djaidja S, Wu Q. Leader-following consensus for single-integrator multi-agent systems with multiplicative noises in directed topologies. *Int J Syst Sci* 2015;46:2788-98. DOI
28. Zong X, Li T, Zhang J. Consensus conditions of continuous-time multi-agent systems with time-delays and measurement noises. *Automatica* 2019;99:412-9. DOI
29. Malthus TR. An Essay on the Principle of Population. Fifth edition, Volume III, 1817, London: John Murray.