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Adaptive neural control of vehicular platoons with unknown functions and full state constraints

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Abstract

This paper investigates the adaptive neural control of vehicular platoons subject to unknown nonlinear functions and full-state constraints. To address the challenges posed by unknown functions, the neural network technology is integrated into the backstepping control framework. Additionally, the time-varying constraints on position, velocity, and acceleration are effectively managed through the application of tangent barrier Lyapunov functions. Notably, the proposed approach successfully avoids the singularity problem. Based on Lyapunov stability theory, it is rigorously shown that the closed-loop system remains bounded, with system states and error signals strictly confined within the prescribed constraint boundaries. Finally, a numerical example is presented to validate the effectiveness and feasibility of the proposed control scheme.

Keywords: Vehicular platoons, neural network (NN), backstepping, tangent barrier Lyapunov function, constraints

1. INTRODUCTION

Adaptive control is a control strategy that dynamically adjusts its parameters in response to changes in the system, with the goal of enhancing controller performance in uncertain and evolving environments. This approach finds extensive application in fields such as industrial automation [1], cruise system design [2], and traffic management [3,4]. Especially when dealing with model uncertainties and external disturbances, this approach can effectively ensure the stability and tracking performance of the closed-loop system. With the development of modern industry and intelligent transportation systems, the demand for control of complex dynamic systems is increasing, and the research on adaptive control is particularly important. By continuously optimizing and improving adaptive control algorithms, the response speed and accuracy of the system can be enhanced, which provides a theoretical basis and technical support for achieving efficient and safe automation operations. Recently, the rapid acceleration of urbanization has inevitably resulted in increased road congestion and environmental pollution. This phenomenon has adversely impacted travel efficiency, economic growth, and environmental sustainability, placing unprecedented strain on traditional transportation systems. To address these challenges, many cities are exploring intelligent transportation systems and sustainable travel solutions. Among numerous studies, connected automatic vehicles (CAVs) can achieve efficient information sharing and intelligent decision-making through vehicle-to-vehicle (V2V) communication, reduce traffic accidents, execute intelligent scheduling and dynamic route planning, and reduce energy consumption[5–8]. Note that distributed control of vehicular systems can achieve the control objective of the entire system through communication and coordination, and there are significant achievements[9–14]. Specifically, references[9,10] constructed the platoon as a dynamically decoupled system and used distributed model predictive control (DMPC) technology to solve flexible platooning scheduling problems. Reference[11] mainly investigated the distributed adaptive fixed time queue tracking problem of third-order completely non-uniform nonlinear vehicles, and solves the interference problem through robust H_∞ control theory. Distributed platoon control relies on communication between ve-hicles, such as V2V. In order to ensure the security of this highly connected network, references[12–14] discussed the vehicular system under network attacks, designed the adaptive distributed control framework, and utilized Lyapunov theory to prove the asymptotic stability. Based on these vehicular control theory results, the initial motivation of this paper is to design an adaptive control scheme for a third-order vehicle system and com-plete stability analysis. Due to model errors and parameter variations, the existence of unknown functions has brought great difficulties to the stability analysis of vehicle platoons. Usually, there are two methods for han-dling unknown functions: adaptive model predictive control (MPC)[15,16] and function approximation[17–23]. For example, the reference[15] proposed a robust adaptive MPC algorithm to achieve good tracking perfor-mance and parameter set estimation accuracy, while the tube-based adaptive MPC scheme designed by[16] ensured the asymptotic stability of the dynamic uncertainty closed-loop system. Differently[17,18], utilized the neural network (NN) technique to approximate unknown dynamics. However, references[15–18] considered linear systems or simple nonlinear systems, which might not be applicable to vehicular systems with relatively high complexity. In reference[19], the proposed new scheme combined local motion planning and adaptive MPC, guaranteeing the motion behavior of vehicles traveling at the desired speed. In addition, adaptive MPC requires high accuracy in system modeling, and additional disturbances can affect the control effect. Hence, reference[20] approximated unknown nonlinear functions through fuzzy logic systems (FLS) and proved the individual and string stability of the entire heterogeneous vehicle group using predefined time stability criteria and Lyapunov functions. By simpler single critic NN, reference[21] achieved the approximation of unknown functions and proposed an improved weight update law to ensure the stability of closed-loop systems without the need for difficult-to-find initial allowable control schemes[22]. developed a platoon tracking control strategy based on an adaptive NN algorithm, and then[23] extended this scheme to vehicular systems with limited communication resources. Therefore, this paper will investigate vehicle platoons with unknown functions and utilize the NN approxima-tion technique to solve the problem.

On the other hand, the safety issue also needs to be taken seriously, which was not discussed in the above articles[24]. proposed an intermittent privacy protection mechanism to ensure the security of communication data between subsystems. References[25,26] aimed to achieve specified performance queue control and strictly maintain tracking errors within the specified region[27]. Constrains the safe distance between vehicles, and[28] simulta-neously established constraints on the position and speed of the vehicle platoon. However, references[25–28] ignored the constraint condition on acceleration, which may affect the control performance of the vehicle during rapid braking. Besides, imposing constraints is certainly beneficial for system safety, but at the same time, it can also generate redundant terms, which can cause trouble for control design. Notably, using barrier Lyapunov functions (BLFs) in the process of backstepping design is an effective method to overcome the shortcomings of state constraints. For instance, reference[29] introduced time-varying integral BLFs (IBLFs) into adaptive control de-sign, which ensured the stability of the closed-loop system without violating the constraints of the states[30]. obtained the result of asymptotic convergence of tracking error to zero through logarithm BLFs (log-BLFs) the-ory analysis. In[31–33], tangent BLFs (tan-BLFs) were used to design an adaptive controller. Subsequently, reference[34] applied BLFs to vehicle platoons and completed stability analysis.

Motivated by the above discussion, this paper is dedicated to researching adaptive neural control of vehicular platoons with unknown functions and full state constraints. Main contributions are summarized as follows.

(1) Different from[15,16], we establish an effective adaptive NN control framework for third-order vehicle platoons with external disturbances, which avoids excessive assumptions about unknown functions and simplifies the design process.
(2) Unlike traditional vehicle control[9,10], we additionally consider safety factors during the driving process. By applying time-varying constraints on the position, velocity, and acceleration of the vehicle, it can ensure that the system states don't fluctuate significantly and remain within predefined boundaries, thereby improving predictability to a certain extent.
(3) Compared to the constant constraints investigated by[30], time-varying state boundaries are more difficult to handle. We adopt tan-BLFs, which allow the controller to quickly adjust the control input when system state approaches constraints, bringing the system state back within a safe range. This helps to improve the response speed and stability of the system and avoid singularity phenomena.

The composition of the paper is summarized as follows. Section 2 introduces the third-order vehicular platoon and key technologies. Section 3 provides the design process of the adaptive NN controller. Section 4 presents the stability analysis. Section 5 shows an example to validate the effectiveness of the control approach. Section 6 concludes this paper.

2. PROBLEM FORMULATION

As shown in Figure 1, we consider the platoon of CAVs with a bidirectional communication topology of the vehicle ahead and the leading reference signal. $\beta_{1,i}$, $\beta_{2,i}$ and $\beta_{3,i}$ denote the position, velocity, and acceleration of vehicle $i, i \in \{1, \cdots, N\}$, respectively. d_i is the distance between two CAVs, and $d_{i,des}$ is a desired spacing between two CAVs. y_{c0} is the reference signal. The considered CAV system is equipped with road maps, GPS, and onboard sensors. In addition, each vehicle is capable of V2V communication and receiving its own controller signals and communication between the two vehicles. During the driving process, the spacing can be kept within a safe range, that is $0 < d_{i,\min} < d_i < d_{i,\max}$, where $d_{i,\min}$ and $d_{i,\max}$ are the minimum safety spacing and maximum distance, respectively.

2.1. Vehicle dynamics

Based on[28,34], consider the dynamics of the vehicle i as follows

$$\begin{cases} \dot{\beta}_{1,i}(t) = \beta_{2,i}(t) + \eta_{1,i}(\bar{\beta}_{1,i}(t)), \\ \dot{\beta}_{2,i}(t) = \beta_{3,i}(t) + \eta_{2,i}(\bar{\beta}_{2,i}(t)), \\ \dot{\beta}_{3,i}(t) = G_{3,i}(\beta_{2,i}(t), \beta_{3,i}(t)) + \Psi_{3,i}(\beta_{2,i}(t))\bar{a}_i(t) + k_i^{-1}\eta_{3,i}(\bar{\beta}_{3,i}(t)) + k_i^{-1}\tau_i(t), \end{cases} \tag{1}$$

where $\bar{\beta}_{j,i}(t) = [\beta_{1,i}(t), \cdots, \beta_{j,i}(t)]^T$, $j = 1, 2, 3$. $\eta_{1i}(\cdot)$, $\eta_{2i}(\cdot)$ and $\eta_{3i}(\cdot)$ are unknown functions. $\tau_i(t)$ is the disturbance. $k_i > 0$ denotes the engine time constant. $G_{3,i}(\beta_{2,i}(t), \beta_{3,i}(t))$ and $\Psi_{3,i}(\beta_{2,i}(t))$ are given by

$$G_{3,i}(\beta_{2,i}(t), \beta_{3,i}(t)) = -k_i^{-1}(\beta_{3,i}(t) + 0.5\nu_i^{-1}r_i^* s_i^* d_{ci}\beta_{2,i}^2(t) + \nu_i^{-1}h_{li}) - \nu_i^{-1}r_i^* s_i^* d_{ci}\beta_{2,i}(t)\beta_{3,i}(t),$$
$$\Psi_{3,i}(\beta_{2,i}(t)) = \frac{1}{\nu_i k_i},$$

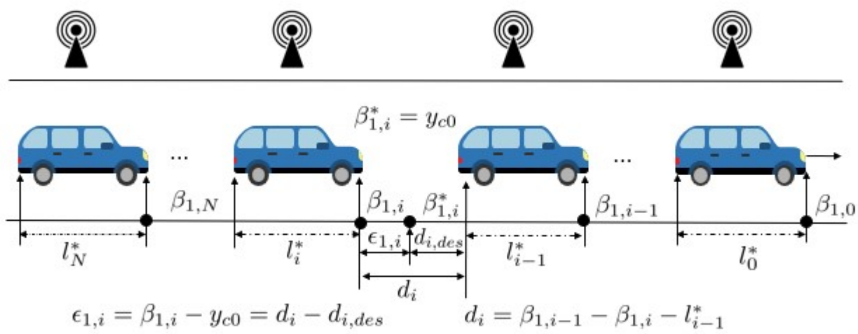

Figure 1. Longitudinal motion of the vehicle platoon.

where the physical meaning of parameters v_i, d_{ci}, s_i^* and r_i^* can be seen in [28].

Additionally, the engine input $\bar{a}_i$ is provided by

$$\bar{a}_i(t) = v_i u_i(t) + 0.5 r_i^* s_i^* d_{ci} \beta_{2,i}^2(t) + h_{li} + k_i r_i^* s_i^* d_{ci} \beta_{2,i}(t) \beta_{3,i}(t). \tag{2}$$

Then substituting (2) into (1), one has

$$\begin{cases} \dot{\beta}_{1,i}(t) = \beta_{2,i}(t) + \eta_{1,i}(\bar{\beta}_{1,i}(t)), \\ \dot{\beta}_{2,i}(t) = \beta_{3,i}(t) + \eta_{2,i}(\bar{\beta}_{2,i}(t)), \\ \dot{\beta}_{3,i}(t) = k_i^{-1}(-\beta_{3,i}(t) + u_i(t) + \tau_i(t) + \eta_{3,i}(\bar{\beta}_{3,i}(t)), \end{cases} \tag{3}$$

in which $\beta_{1,i}(t)$, $\beta_{2,i}(t)$ and $\beta_{3,i}(t)$ represent the vehicle position, the velocity and the acceleration, respectively. Based on safety considerations, we set the following constraints for the vehicular system (1)

$$|\beta_{1,i}(t)| < f_s(t), |\beta_{2,i}(t)| < f_v(t), |\beta_{3,i}(t)| < f_a(t), \tag{4}$$

where $f_s(t)$, $f_v(t)$ and $f_a(t)$ are time-varying constraints defined by designers.

The purpose of this paper is to construct an adaptive control scheme to achieve the following three parts:

(1) The closed-loop system is bounded.
(2) The error signals can converge to bounded compacts.
(3) The position, velocity, and acceleration of the platoon system are kept within time-varying constraints.

Accordingly, we make the following assumptions.

Assumption 1. *The reference signal y_{c0} and its i th derivative $y_{c0}^{(i)}(t)$ are bounded. There exist positive constants $Y_{c0}, Y_{c1}, \cdots, Y_{cn}$ such that*

$$|y_{c0}(t)| < Y_{c0} < f_s(t), 0 < |y_{c0}^{(i)}(t)| < Y_{ci}, i = 1, \cdots, n \tag{5}$$

where $f_s(t)$ is the time-varying constrained boundary.

Assumption 2. *For $f_s(t)$, $f_v(t)$ and $f_a(t)$, there exist positive constants $\bar{P}_{a,j}$, $\bar{P}_{b,j}$ and $\bar{P}_{c,j}$ such that*

$$|p_{ai}(t)| \le \bar{P}_{a,1} < f_s(t),\ |p_{bi}(t)| \le \bar{P}_{b,1} < f_v(t),\ |p_{ci}(t)| \le \bar{P}_{c,1} < f_a(t), \tag{6}$$

and

$$|p_{ai}^{(j)}(t)| \le \bar{P}_{a,j+1},\ |p_{bi}^{(j)}(t)| \le \bar{P}_{b,j+1},\ |p_{ci}^{(j)}(t)| \le \bar{P}_{c,j+1}, \tag{7}$$

where $i = 1, \cdots, n$, $j = 0, \cdots, n$, $p_{ai}^{(j)}(t)$, $p_{bi}^{(j)}(t)$ and $p_{ci}^{(j)}(t)$ denote the jth time derivative of time-varying functions $p_{ai}(t)$, $p_{bi}(t)$ and $p_{ci}(t)$, respectively.

Remark 1. *By Assumption 2, the relationship between time-varying functions, positive constants, and state boundaries can be obtained. Based on this assumption and the coordinate transformation between states, virtual controllers, and error signals, we will deduce the constraint boundaries of error signals. This will play a significant role in backstepping design and Lyapunov analysis, ensuring that the closed-loop system is bounded and error signals can converge to bounded compact sets.*

Assumption 3. *The external disturbance $\tau_i(t)$ satisfies $|\tau_i(t)| < \bar{\tau}_i$, where $\bar{\tau}_i$ is a positive constant.*

Lemma 1. [35] *For $\forall(A, B) \in R^n$, we have*

$$AB \leq \frac{\varepsilon^m}{m}|A|^m + \frac{1}{n\varepsilon^n}|B|^n,$$

where $\varepsilon > 0$, $m > 1$, $n > 1$, and $(m-1)(n-1) = 1$.

2.2. Radial basis function neural network

The radial basis function neural network (RBFNN) has been widely employed for approximating unknown nonlinear functions. Then $\chi(Z)$ can be approximated with

$$\chi(Z) = W^T S(Z) + \Lambda(Z), \tag{8}$$

where $\chi(Z)$ is defined on the compact set $\Omega_Z \subset \Re^n$; $\Lambda(Z)$ is the approximation error, which satisfies $|\Lambda(Z)| \leq \xi$ and ξ is an unknown positive constant; the ideal constant weight vector is defined as

$$W := \arg \min_{\hat{W} \in \Re^l} \{ \sup_{Z \in \Omega_Z} |\chi(Z) - S^T(Z)\hat{W}| \}, \tag{9}$$

where $\hat{W}$ is the estimate of W and $l > 1$ denotes NN nodes.

Besides, the basis function vector is $S(Z) = [s_1(Z), \cdots, s_l(Z)]^T$ and for $i = 1, \cdots, l$

$$s_i(Z) = \exp\left[\frac{-(Z - q_i)^T (Z - q_i)}{b_i^2}\right]. \tag{10}$$

Here, $q_i = [q_{i1}, \cdots, q_{in}]^T$ is the center of the receptive field, while b_i is the width of Gaussian function.

Remark 2. *In this paper, the RBFNN technique (8) is employed to approximate unknown functions. Although the approximation error inherently exists, it satisfies $|\Lambda(Z)| \leq \xi$, $\xi > 0$, which indicates that the error signal can remain strictly confined within bounded limits and exhibits no tendency toward unbounded divergence. In addition, by combining tan-BLF, backstepping, and inequality scaling, we can further reduce the adverse effects of approximation errors and ultimately achieve boundedness of the closed-loop system.*

Remark 3. *For smoothing functions existing in the system, some articles made them known and provided a constant to constrain the boundary, such as Assumption 2 in reference[29]. Unfortunately, this method relies too heavily on precise system models. Although literature[32] sets boundaries for the coupling of known functions and system states for unknown smoothing functions in Assumption 1, this method requires the design of appropriate basis functions. If the proposed assumption cannot characterize the nonlinear characteristics of the unknown function, this will result in errors and lead to a decrease in controller performance. Therefore, we remove these excessive assumptions and not only utilize the RBFNN to approximate any continuous nonlinearities, but also improve the dynamic adaptability of the controller.*

3. ADAPTIVE NEURAL CONTROL

In this section, we will design an adaptive NN controller to ensure the boundedness of the vehicular system with unknown functions and full state constraints. To proceed, given the following coordinate transformation

$$\begin{cases} \epsilon_{1,i}(t) = \beta_{1,i}(t) - y_{c0}(t), \\ \epsilon_{2,i}(t) = \beta_{2,i}(t) - \omega_{1,i}(t), \\ \epsilon_{3,i}(t) = \beta_{3,i}(t) - \omega_{2,i}(t), \end{cases} \tag{11}$$

where $\omega_{1,i}(t)$ and $\omega_{2,i}(t)$ are virtual controllers, and $\epsilon_{j,i}(t)$, $j = 1, 2, 3$ are error variables.

Step 1. Based on (1) and (11), one has

$$\begin{aligned} \dot{\epsilon}_{1,i}(t) &= \dot{\beta}_{1,i}(t) - \dot{y}_{c0}(t) \\ &= \epsilon_{2,i}(t) + \omega_{1,i}(t) + \eta_{1,i}(\bar{\beta}_{1,i}(t)) - \dot{y}_{c0}(t). \end{aligned} \tag{12}$$

By the RBFNN technique, unknown functions can be approximated by

$$\begin{aligned} \chi_{1,i}(Z_{i,1}) &= \eta_{1,i}(\bar{\beta}_{1,i}(t)) - \dot{y}_{c0}(t) \\ &= W_{1,i}^T S_{1,i}(Z_{1,i}) + \Lambda_{1,i}(Z_{1,i}), \end{aligned} \tag{13}$$

where $Z_{i,1} = [\bar{\beta}_{1,i}(t), y_{c0}(t), \dot{y}_{c0}(t)]^T$, $W_{1,i}$ is the NN weight vector, $S_{1,i}(Z_{1,i})$ is the basis function vector, and $\Lambda_{1,i}(Z_{1,i})$ satisfies $|\Lambda_{1,i}(Z_{1,i})| \leq \xi_{1,i}$ with $\xi_{1,i} > 0$.

Next, we select the following tan-BLF

$$Q_{1,i}(t) = \frac{p_{ai}^2(t)}{\pi} \tan\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) + \frac{1}{2}\tilde{W}_{1,i}^T \delta_{1,i}^{-1} \tilde{W}_{1,i}, \tag{14}$$

where $\delta_{1,i} > 0$ represents the design parameter, $\tilde{W}_{1,i} = W_{1,i} - \hat{W}_{1,i}$ is the estimation error and $\hat{W}_{1,i}$ is the evaluation of $W_{1,i}$. Besides, according to Assumption 2, we can obtain that $|\epsilon_{1,i}(t)| \leq p_{ai}(t)$, where $p_{ai}(t) = f_s(t) - B_1$ and B_1 is a positive constant.

Then we further get

$$\begin{aligned} \dot{Q}_{1,i}(t) =& \frac{2p_{ai}(t)\dot{p}_{ai}(t)}{\pi} \tan\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) - \tilde{W}_{1,i}^T \delta_{1,i}^{-1} \dot{\hat{W}}_{1,i} \\ &+ \sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) \epsilon_{1,i}(t) \left(\epsilon_{2,i}(t) + \omega_{1,i}(t) + \chi_{1,i}(Z_{1,i})\right) \\ &- \frac{\dot{p}_{ai}(t)}{p_{ai}(t)} \epsilon_{1,i}^2(t) \sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right). \end{aligned} \tag{15}$$

In view of $\chi_{1,i}(Z_{i,1}) = W_{1,i}^T S_{1,i}(Z_{1,i}) + \Lambda_{1,i}(Z_{1,i})$, by Young's inequality, one has

$$\sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) \epsilon_{1,i}(t)\Lambda_{1,i}(Z_{1,i}) \leq \frac{1}{2}\sec^4\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) \epsilon_{1,i}^2(t) + \frac{1}{2}\xi_{1,i}^2. \tag{16}$$

Substituting (16) into (15), we can obtain

$$\begin{aligned}\dot{Q}_{1,i}(t) \leq & \frac{2p_{ai}(t)\dot{p}_{ai}(t)}{\pi}\tan\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) - \tilde{W}_{1,i}^T\delta_{1,i}^{-1}\dot{\hat{W}}_{1,i} + \frac{1}{2}\xi_{1,i}^2 \\ & + \sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right)\epsilon_{1,i}(t)\left(\epsilon_{2,i}(t) + \omega_{1,i}(t) + W_{1,i}^T S_{1,i}(Z_{1,i})\right) \\ & + \frac{1}{2}\sec^4\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right)\epsilon_{1,i}^2(t) - \frac{\dot{p}_{ai}(t)}{p_{ai}(t)}\epsilon_{1,i}^2(t)\sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right).\end{aligned} \tag{17}$$

The virtual controller and the adaptive law are designed as

$$\begin{aligned}\omega_{1,i}(t) = & -(\alpha_{1,i} + 2\Gamma_{1,i}(t))\frac{p_{ai}^2(t)}{\pi\epsilon_{1,i}(t)}\sin\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right)\cos\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) \\ & - \Gamma_{1,i}(t)\epsilon_{1,i}(t) - \hat{W}_{1,i}^T S_{1,i}(Z_{1,i}) - \frac{1}{2}\sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right)\epsilon_{1,i}(t),\end{aligned} \tag{18}$$

$$\dot{\hat{W}}_{1,i} = \delta_{1,i}\left[-\rho_{1,i}\hat{W}_{1,i} + \sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right)\epsilon_{1,i}(t)S_{1,i}(Z_{1,i})\right], \tag{19}$$

where $\alpha_{1,i} > 0$ and $\rho_{1,i} > 0$ are tuning parameters. Besides, $\Gamma_{1,i}(t)$ is defined as

$$\Gamma_{1,i}(t) = \sqrt{\left(\frac{\dot{p}_{ai}(t)}{p_{ai}(t)}\right)^2 + \theta_{1,i}}, \tag{20}$$

in which $\Gamma_{1,i}(t) > 0$ and $\theta_{1,i} > 0$.

Accordingly, it is obvious to deduce that

$$\Gamma_{1,i}(t) + \frac{\dot{p}_{ai}(t)}{p_{ai}(t)} \geq 0. \tag{21}$$

Furthermore, we substitute (18), (19) and (21) into (17), then

$$\begin{aligned}\dot{Q}_{1,i}(t) \leq & -\alpha_{1,i}\frac{p_{ai}(t)}{\pi}\tan\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) + \frac{1}{2}\xi_{1,i}^2 + \frac{1}{2}\rho_{1,i}\|W_{1,i}\|^2 \\ & - \frac{1}{2}\rho_{1,i}\|\tilde{W}_{1,i}\|^2 + \sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right)\epsilon_{1,i}(t)\epsilon_{2,i}(t).\end{aligned} \tag{22}$$

Finally, by defining $\Pi_{1,i} = \min\{\alpha_{1,i}, \rho_{1,i}, \delta_{1,i}\}$, (22) can be rewritten as

$$\dot{Q}_{1,i}(t) \leq -\Pi_{1,i}Q_{1,i}(t) + \sec^2\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right)\epsilon_{1,i}(t)\epsilon_{2,i}(t) + \frac{1}{2}\xi_{1,i}^2 + \frac{1}{2}\rho_{1,i}\|W_{1,i}\|^2. \tag{23}$$

Remark 4. *In step 1, the virtual controller (18) is ultimately derived through coordinate transformation (11), tan-BLF (14), and the RBFNN technique. The basic design principle is to construct a suitable Lyapunov function, ensuring that its derivative is negative definite or semi-negative definite. Meanwhile, the design of (18) should also incorporate the adaptive law (19) to ensure the convergence of parameter estimation errors and state errors. Therefore, the role of each virtual controller is to stabilize the current subsystem and provide reference inputs for the next subsystem, ultimately achieving the stability of the entire system.*

Remark 5. *Compared to log-BLF, tan-BLF grows faster when the state approaches the constraint boundary, making the gradient of the Lyapunov function steeper and actively preventing constraint violations. This plays a significant role in systems with full state constraints. In addition, the structure of tan-BLF is more suitable for time-varying constraints because its trigonometric properties can simplify the adaptation to changing boundaries, which is beneficial in constraining dynamic environments that evolve over time.*

Remark 6. *During the design process, the singularity of the virtual controller has been avoided. When* $\epsilon_{1,i}(t) \to 0$*, we use the L'Hôpital's rule to acquire*

$$\lim_{\epsilon_{1,i}(t)\to 0} \frac{(\alpha_{1,i} + 2\Gamma_{1,i}(t))}{\pi\epsilon_{1,i}(t)} p_{ai}^2(t) \sin\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) \cos\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) = 0. \tag{24}$$

Therefore, the singularity problem will not happen in (18).

Step 2. Through (1) and (11), one has

$$\begin{aligned}\dot{\epsilon}_{2,i}(t) &= \dot{\beta}_{2,i}(t) - \dot{\omega}_{1,i}(t) \\ &= \epsilon_{3,i}(t) + \omega_{2,i}(t) + \eta_{2,i}(\bar{\beta}_{2,i}(t)) - \dot{\omega}_{1,i}(t).\end{aligned} \tag{25}$$

By the RBFNN technique, unknown functions can be approximated by

$$\begin{aligned}\chi_{2,i}(Z_{2,i}) &= \eta_{2,i}(\bar{\beta}_{2,i}(t)) - \dot{\omega}_{1,i}(t) \\ &= W_{2,i}^T S_{2,i}(Z_{2,i}) + \Lambda_{2,i}(Z_{2,i}),\end{aligned} \tag{26}$$

where $Z_{2,i} = [\beta_{1,i}(t), \beta_{2,i}(t), p_{ai}(t), \dot{p}_{ai}(t), \ddot{p}_{ai}(t), y_{c0}(t), \dot{y}_{c0}(t), \hat{W}_{1,i}]^T$, $W_{2,i}$ is the NN weight vector, $S_{2,i}(Z_{2,i})$ is the basis function vector, and $\Lambda_{2,i}(Z_{2,i})$ satisfies $|\Lambda_{2,i}(Z_{2,i})| \leq \xi_{2,i}$ with $\xi_{2,i} > 0$.

Then we select the following tan-BLF

$$Q_{2,i}(t) = Q_{1,i}(t) + \frac{p_{bi}^2(t)}{\pi} \tan\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right) + \frac{1}{2}\tilde{W}_{2,i}^T \delta_{2,i}^{-1} \tilde{W}_{2,i}, \tag{27}$$

in which $\delta_{2,i} > 0$ is the design parameter, $\tilde{W}_{2,i} = W_{2,i} - \hat{W}_{2,i}$ is the estimation error and $\hat{W}_{2,i}$ is the evaluation of $W_{2,i}$. From Assumption 2, we have $|\epsilon_{2,i}(t)| \leq p_{bi}(t)$, where $p_{bi}(t) = f_v(t) - B_2$ and B_2 is a positive constant.

Correspondingly, we further get

$$\begin{aligned}\dot{Q}_{2,i}(t) =& \dot{Q}_{1,i}(t) + \frac{2p_{bi}(t)\dot{p}_{bi}(t)}{\pi} \tan(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}) - \tilde{W}_{2,i}^T \delta_{2,i}^{-1} \dot{\hat{W}}_{2,i} \\ &+ \sec^2(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)})\epsilon_{2,i}(t)(\epsilon_{3,i}(t) + \omega_{2,i}(t) + \chi_{2,i}(Z_{2,i}) \\ &- \frac{\dot{p}_{bi}(t)}{p_{bi}(t)}\epsilon_{2,i}^2(t) \sec^2(\frac{\pi\epsilon_{2,i}^2(t)}{2p_b^2(t)}).\end{aligned} \tag{28}$$

In view of $\chi_{2,i}(Z_{2,i})= W_{2,i}^T S_{2,i}(Z_{2,i}) + \Lambda_{2,i}(Z_{2,i})$, by Young's inequality, one has

$$\sec^2\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}(t)\Lambda_{2,i}(Z_{2,i}) \leq \frac{1}{2}\sec^4\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}^2(t) + \frac{1}{2}\xi_{2,i}^2. \tag{29}$$

By (29) and (28), one can deduce that

$$\begin{aligned}\dot{Q}_{2,i}(t) \leq & \dot{Q}_{1,i}(t) + \frac{2p_{bi}(t)\dot{p}_{bi}(t)}{\pi} \tan\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right) + \frac{1}{2}\xi_{2,i}^2 \\ & + \sec^2\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}(t)\tilde{W}_{2,i}^T S_{2,i}(Z_{2,i}) - \tilde{W}_{2,i}^T \delta_{2,i}^{-1}\dot{\hat{W}}_{2,i} \\ & + \sec^2\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}(t)\left(\epsilon_{3,i}(t) + \omega_{2,i}(t) + \hat{W}_{2,i}^T S_{2,i}(Z_{2,i})\right) \\ & + \frac{1}{2}\sec^4\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}^2(t) - \frac{\dot{p}_{bi}(t)}{p_{bi}(t)}\epsilon_{2,i}^2(t)\sec^2\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right).\end{aligned} \tag{30}$$

Besides, the virtual controller and the adaptive law are constructed as follows

$$\begin{aligned}\omega_{2,i}(t) = & -(\alpha_{2,i} + 2\Gamma_{2,i}(t))\frac{p_{bi}^2(t)}{\pi\epsilon_{2,i}(t)}\sin\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\cos\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right) \\ & - \Gamma_{2,i}(t)\epsilon_{2,i}(t) - \hat{W}_{2,i}^T S_{2,i}(Z_{2,i}) - \frac{1}{2}\sec^2\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}(t) \\ & - \sec^2\left(\frac{\pi\epsilon_{1,i}(t)}{2p_{ai}^2(t)}\right)\epsilon_{1,i}(t) \Big/ \sec^2\left(\frac{\pi\epsilon_{2,i}(t)}{2p_{bi}^2(t)}\right),\end{aligned} \tag{31}$$

$$\dot{\hat{W}}_{2,i} = \delta_{2,i}\left[-\rho_{2,i}\hat{W}_{2,i} + \sec^2\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}(t)S_{2,i}(Z_{2,i})\right], \tag{32}$$

in which $\alpha_{2,i} > 0$ and $\rho_{2,i} > 0$ are tuning parameters. In addition, $\Gamma_{2,i}(t)$ is

$$\Gamma_{2,i}(t) = \sqrt{\left(\frac{\dot{p}_{bi}(t)}{p_{bi}(t)}\right)^2 + \theta_{2,i}}, \tag{33}$$

where $\Gamma_{2,i}(t) > 0$ and $\theta_{2,i} > 0$.

Naturally, we can obtain

$$\Gamma_{2,i}(t) + \frac{\dot{p}_{bi}(t)}{p_{bi}(t)} \geq 0. \tag{34}$$

Substituting (23), (31), (32) and (34) into (30) will yield

$$\begin{aligned}\dot{Q}_{2,i}(t) \leq & -\Pi_{1,i}Q_{1,i}(t) + \frac{1}{2}\xi_{1,i}^2 + \frac{1}{2}\rho_{1,i}\|W_{1,i}\|^2 + \frac{1}{2}\xi_{2,i}^2 \\ & + \frac{1}{2}\rho_{2,i}\|W_{2,i}\|^2 - \alpha_{2,i}\frac{p_{bi}(t)}{\pi}\tan\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right) \\ & - \frac{1}{2}\rho_{2,i}\|\tilde{W}_{2,i}\|^2 + \sec^2\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}(t)\epsilon_{3,i}(t).\end{aligned} \tag{35}$$

We define $\Pi_{2,i} = \min\{\Pi_{1,i}, \alpha_{2,i}, \rho_{2,i}, \delta_{2,i}\}$, (35) can be changed to

$$\begin{aligned}\dot{Q}_{2,i}(t) \leq &-\Pi_{2,i}Q_{2,i}(t) + \sec^2\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}(t)\epsilon_{3,i}(t)\\ &+\frac{1}{2}\xi_{1,i}^2 + \frac{1}{2}\xi_{2,i}^2 + \frac{1}{2}\rho_{1,i}\|W_{1,i}\|^2 + \frac{1}{2}\rho_{2,i}\|W_{2,i}\|^2.\end{aligned} \tag{36}$$

Step 3. Based on (1) and (11), we have

$$\begin{aligned}\dot{\epsilon}_{3,i}(t) &= \dot{\beta}_{3,i}(t) - \dot{\omega}_{2,i}(t)\\ &= k_i^{-1}(-\beta_{3,i}(t) + u_i(t) + \tau_i(t) + \eta_{3,i}(\bar{\beta}_{3,i}(t)) - \dot{\omega}_{2,i}(t).\end{aligned} \tag{37}$$

Unknown functions can be approximated by the following RBFNN technique

$$\begin{aligned}\chi_{3,i}(Z_{2,i}) &= k_i^{-1}(-\beta_{3,i}(t) + \eta_{3,i}(\bar{\beta}_{3,i}(t))) - \dot{\omega}_{2,i}(t)\\ &= W_{3,i}^T S_{3,i}(Z_{3,i}) + \Lambda_{3,i}(Z_{3,i}),\end{aligned} \tag{38}$$

where $Z_{3,i} = [\beta_{1,i}(t), \beta_{2,i}(t), \beta_{3,i}(t), p_{ai}(t), \dot{p}_{ai}(t), \ddot{p}_{ai}(t), \dddot{p}_{ai}(t), y_{c0}(t), \dot{y}_{c0}(t), \ddot{y}_{c0}(t), p_{bi}(t), \dot{p}_{bi}(t), \ddot{p}_{bi}(t), \hat{W}_{1,i}, \hat{W}_{2,i}]^T$, $W_{3,i}$ is the NN weight vector, $S_{3,i}(Z_{3,i})$ is the basis function vector, and $\Lambda_{3,i}(Z_{3,i})$ satisfies $|\Lambda_{3,i}(Z_{3,i})| \leq \xi_{3,i}$ with $\xi_{3,i} > 0$.

Consider the following tan-BLF

$$Q_{3,i}(t) = Q_{2,i}(t) + \frac{p_{ci}^2(t)}{\pi}\tan\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right) + \frac{1}{2}\tilde{W}_{3,i}^T\delta_{3,i}^{-1}\tilde{W}_{3,i}, \tag{39}$$

in which $\delta_{3,i} > 0$ is the design parameter, $\tilde{W}_{3,i} = W_{3,i} - \hat{W}_{3,i}$ is the estimation error and $\hat{W}_{3,i}$ is the evaluation of $W_{3,i}$. By Assumption 2, we can obtain that $|\epsilon_{3,i}(t)| \leq p_{ci}(t)$, where $p_{ci}(t) = f_v(t) - B_3$ and B_3 is a positive constant.

Hence, the derivative of $Q_{3,i}(t)$ is

$$\begin{aligned}\dot{Q}_{3,i}(t) =&\dot{Q}_{2,i}(t) + \frac{2p_{ci}(t)\dot{p}_{ci}(t)}{\pi}\tan\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right) - \tilde{W}_{3,i}^T\delta_{3,i}^{-1}\dot{\hat{W}}_{3,i}\\ &+\sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}(t)\left(k_i^{-1}u(t) + k_i^{-1}\tau_i(t) + \chi_{3,i}(Z_{3,i})\right)\\ &-\frac{\dot{p}_{ci}(t)}{p_{ci}(t)}\epsilon_{3,i}^2(t)\sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right).\end{aligned} \tag{40}$$

According to $\chi_{3,i}(Z_{3,i}) = W_{3,i}^T S_{3,i}(Z_{3,i}) + \Lambda_{3,i}(Z_{3,i})$, Assumption 3 and Young's inequality, then we have

$$\sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}(t)k_i^{-1}\tau_i \leq \frac{1}{2k_i}\sec^4\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}^2(t) + \frac{1}{2k_i}\bar{\tau}_i^2, \tag{41}$$

$$\sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}(t)\Lambda_{3,i}(Z_{3,i}) \leq \frac{1}{2}\sec^4\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}^2(t) + \frac{1}{2}\xi_{3,i}^2. \tag{42}$$

Based on (41), (42) and (40), $\dot{Q}_{3,i}(t)$ satisfies

$$\begin{aligned}\dot{Q}_{3,i}(t) \leq & \dot{Q}_{2,i}(t) + \frac{2p_{ci}(t)\dot{p}_{ci}(t)}{\pi}\tan\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right) + \frac{1}{2}\xi_{3,i}^2 + \frac{1}{2k_i}\bar{\tau}_i^2 \\ & + \sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}(t)\tilde{W}_{3,i}^T S_{3,i}(Z_{3,i}) - \tilde{W}_{3,i}^T\delta_{3,i}^{-1}\dot{\hat{W}}_{3,i} \\ & + \sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}(t)\left(k_i^{-1}u_i(t) + \hat{W}_{3,i}^T S_{3,i}(Z_{3,i})\right) \\ & + \left(\frac{1+k_i}{2k_i}\right)\sec^4\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}^2(t) - \frac{\dot{p}_{ci}(t)}{p_{ci}(t)}\epsilon_{3,i}^2(t)\sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right). \end{aligned} \tag{43}$$

Subsequently, we construct the virtual controller and the adaptive law as

$$\begin{aligned} u_i(t) = & k_i\Bigg[-(\alpha_{3,i} + 2\Gamma_{3,i}(t))\frac{p_{ci}^2(t)}{\pi\epsilon_{3,i}(t)}\sin\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\cos\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right) \\ & - \Gamma_{3,i}(t)\epsilon_{3,i}(t) - \hat{W}_{3,i}^T S_{3,i}(Z_{3,i}) - \left(\frac{1+k_i}{2k_i}\right)\sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}(t) \\ & - \sec^2\left(\frac{\pi\epsilon_{2,i}(t)}{2p_{bi}^2(t)}\right)\epsilon_{2,i}(t)\Big/\sec^2\left(\frac{\pi\epsilon_{3,i}(t)}{2p_{ci}^2(t)}\right)\Bigg], \end{aligned} \tag{44}$$

$$\dot{\hat{W}}_{3,i} = \delta_{3,i}\left[-\rho_{3,i}\hat{W}_{3,i} + \sec^2\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right)\epsilon_{3,i}(t)S_{3,i}(Z_{3,i})\right], \tag{45}$$

in which $\alpha_{3,i} > 0$ and $\rho_{3,i} > 0$ are tuning parameters. In addition, $\Gamma_{3,i}(t)$ is

$$\Gamma_{3,i}(t) = \sqrt{\left(\frac{\dot{p}_{ci}(t)}{p_{ci}(t)}\right)^2 + \theta_{3,i}}, \tag{46}$$

where $\Gamma_{3,i}(t) > 0$ and $\theta_{3,i} > 0$.

Besides, the following inequality is valid

$$\Gamma_{3,i}(t) + \frac{\dot{p}_{ci}(t)}{p_{ci}(t)} \geq 0. \tag{47}$$

When we substitute (36), (44), (45) and (47) into (43), then

$$\begin{aligned}\dot{Q}_{3,i}(t) \leq & -\Pi_{2,i}Q_{2,i}(t) + \frac{1}{2k_i}\bar{\tau}_i^2 + \frac{1}{2}\xi_{1,i}^2 + \frac{1}{2}\xi_{2,i}^2 + \frac{1}{2}\xi_{3,i}^2 \\ & + \frac{1}{2}\rho_{1,i}\|W_{1,i}\|^2 + \frac{1}{2}\rho_{2,i}\|W_{2,i}\|^2 + \frac{1}{2}\rho_{3,i}\|W_{3,i}\|^2 \\ & - \frac{1}{2}\rho_{3,i}\|\tilde{W}_{3,i}\|^2 - \alpha_{3,i}\frac{p_{ci}(t)}{\pi}\tan\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right). \end{aligned} \tag{48}$$

4. STABILITY ANALYSIS

In this section, we summarize the main results in the following Theorem 1 and prove the stability of the closed-loop system.

Theorem 1. *Under Assumptions 1-3, we consider the plant (1), virtual controllers (18), (31), adaptive laws (19), (32), (45) and the controller (44). If the initial conditions satisfy* $|\beta_{1,i}(0)| < f_s(0)$, $|\beta_{2,i}(0)| < f_v(0)$ *and* $|\beta_{3,i}(0)| < f_a(0)$, *the designed control scheme can ensure that*
(i) All closed-loop signals are bounded.
(ii) Error signals are kept in bounded sets Ω_ϵ.
(iii) System states remain within constrained boundaries.

Proof. (i) Firstly, we select the Lyapunov function as $Q(t) = Q_{3,i}(t)$. So the derivative of $Q(t)$ satisfies

$$\begin{aligned}\dot{Q}(t) \leq & -\Pi_{2,i}Q_{2,i}(t) + \frac{1}{2k_i}\bar{\tau}_i^2 + \frac{1}{2}\xi_{1,i}^2 + \frac{1}{2}\xi_{2,i}^2 + \frac{1}{2}\xi_{3,i}^2 \\ & + \frac{1}{2}\rho_{1,i}\|W_{1,i}\|^2 + \frac{1}{2}\rho_{2,i}\|W_{2,i}\|^2 + \frac{1}{2}\rho_{3,i}\|W_{3,i}\|^2 \\ & - \frac{1}{2}\rho_{3,i}\|\tilde{W}_{3,i}\|^2 - \alpha_{3,i}\frac{p_{ci}(t)}{\pi}\tan\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right).\end{aligned} \tag{49}$$

By defining $\Pi = \min\{\Pi_{2,i}, \alpha_{3,i}, \rho_{3,i}, \delta_{3,i}, i = 1, \cdots, n\}$ and $\Xi = \sum_{j=1}^{3}\frac{1}{2}\left(\rho_{j,i}\|W_{j,i}\|^2 + \xi_{j,i}^2\right)$, then (49) can ultimately be written as

$$\dot{Q}(t) \leq -\Pi Q(t) + \Xi. \tag{50}$$

Next we multiply both sides of (50) by $e^{\Pi t}$, and integrate it over $[0, t]$, then

$$Q(t)e^{\Pi t} \leq Q(0) - \frac{\Xi}{\Pi} + \frac{\Xi}{\Pi}e^{\Pi t}. \tag{51}$$

Moreover, the following inequality can also be obtained

$$Q(t) \leq \left(Q(0) - \frac{\Xi}{\Pi}\right)e^{-\Pi t} + \frac{\Xi}{\Pi} \leq Q(0)e^{-\Pi t} + \frac{\Xi}{\Pi}. \tag{52}$$

Based on the definition of $Q(t)$, we can conclude that all signals are bounded.

(ii) According to (52), one has

$$\frac{p_{ai}^2(t)}{\pi}\tan\left(\frac{\pi\epsilon_{1,i}^2(t)}{2p_{ai}^2(t)}\right) \leq Q(t) \leq Q(0)e^{-\Pi t} + \frac{\Xi}{\Pi}, \tag{53}$$

$$\frac{p_{bi}^2(t)}{\pi}\tan\left(\frac{\pi\epsilon_{2,i}^2(t)}{2p_{bi}^2(t)}\right) \leq Q(t) \leq Q(0)e^{-\Pi t} + \frac{\Xi}{\Pi}, \tag{54}$$

$$\frac{p_{ci}^2(t)}{\pi}\tan\left(\frac{\pi\epsilon_{3,i}^2(t)}{2p_{ci}^2(t)}\right) \leq Q(t) \leq Q(0)e^{-\Pi t} + \frac{\Xi}{\Pi}. \tag{55}$$

Therefore, tracking errors are maintained in the following sets

$$\Omega_{\epsilon_{1,i}}(t) = \left\{ \epsilon_{1,i} \in R \middle| |\epsilon_{1,i}(t)| \le |p_{ai}(t)| \sqrt{\frac{2}{\pi} \arctan\left(\frac{\Xi\pi}{\Pi p_{ai}^2(t)}\right)} \right\},$$

$$\Omega_{\epsilon_{2,i}}(t) = \left\{ \epsilon_{2,i} \in R \middle| |\epsilon_{2,i}(t)| \le |p_{bi}(t)| \sqrt{\frac{2}{\pi} \arctan\left(\frac{\Xi\pi}{\Pi p_{bi}^2(t)}\right)} \right\},$$

$$\Omega_{\epsilon_{3,i}}(t) = \left\{ \epsilon_{3,i} \in R \middle| |\epsilon_{3,i}(t)| \le |p_{ci}(t)| \sqrt{\frac{2}{\pi} \arctan\left(\frac{\Xi\pi}{\Pi p_{ci}^2(t)}\right)} \right\}.$$

(iii) According to Assumption 2, one gets that

$$|\epsilon_{1,i}(0)| < p_{ai}(0), \ |\epsilon_{2,i}(0)| < p_{bi}(0), \ |\epsilon_{3,i}(0)| < p_{ci}(0).$$

Recalling the transformation (11), we have

$$\begin{cases} \beta_{1,i}(t) = \epsilon_{1,i}(t) + y_{c0}(t), \\ \beta_{2,i}(t) = \epsilon_{2,i}(t) + \omega_{1,i}(t), \\ \beta_{3,i}(t) = \epsilon_{3,i}(t) + \omega_{2,i}(t), \end{cases}$$

Based on the boundedness of $\epsilon_{1,i}(t)$, $\epsilon_{2,i}(t)$ and $\epsilon_{3,i}(t)$ and Assumption 1, we further get

$$\begin{cases} |\beta_{1,i}(t)| < \Omega_{\epsilon_{1,i}}(t) + Y_{c0} < f_s(t), \\ |\beta_{2,i}(t)| < \Omega_{\epsilon_{2,i}}(t) + \bar{\omega}_{1,i} < f_v(t), \\ |\beta_{3,i}(t)| < \Omega_{\epsilon_{3,i}}(t) + \bar{\omega}_{2,i} < f_a(t), \end{cases}$$

where $\bar{\omega}_{1,i}$ and $\bar{\omega}_{2,i}$ are the upper boundaries of virtual controllers. Hence, system states $\beta_{1,i}(t)$, $\beta_{2,i}(t)$ and $\beta_{3,i}(t)$ are bounded. This ends the proof. □

Remark 7. *Reference[23] studied the CAV platoon with unknown dynamics and communication resource limitations. In contrast, we consider the situation where unknown functions and full state constraints coexist, and design a control scheme based on the RBFNN technology, tan-BLF, and backstepping method. Compared with[25], we investigate more complex third-order vehicle platoons with unknown functions, which makes the proposed control strategy more applicable. In addition, through Lyapunov analysis, it can be proven that both error signals and system states are constrained within the set range, ensuring the safe driving of vehicles.*

Remark 8. *This paper investigates the vehicle platoon under full state constraints. Firstly, in (4), we provide positive time-varying constraints on states and propose Assumptions 1-2 to ensure the relationship between the constraint boundaries and tracking signals and error signals. Subsequently, suitable tan-BLFs are defined in the backstepping design. In the proof of Theorem 1, we first demonstrate the boundedness of the closed-loop system, and then prove that error signals can converge to bounded compact sets. Finally, based on Assumptions 1-2, the coordinate transformation (11), and bounded compact sets of error signals, it can be concluded that system states always remain within predefined boundaries.*

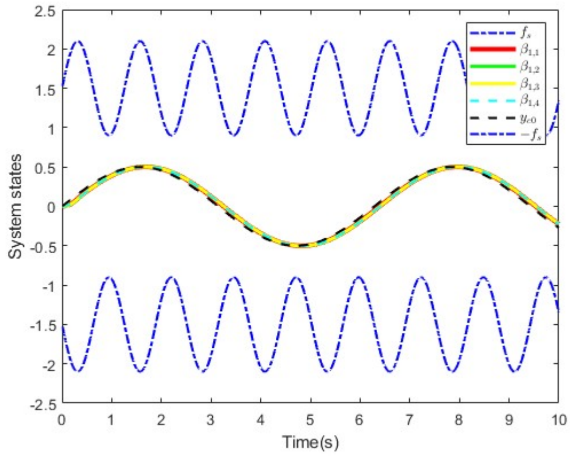

Figure 2. Trajectories of the position $\beta_{1,i}$.

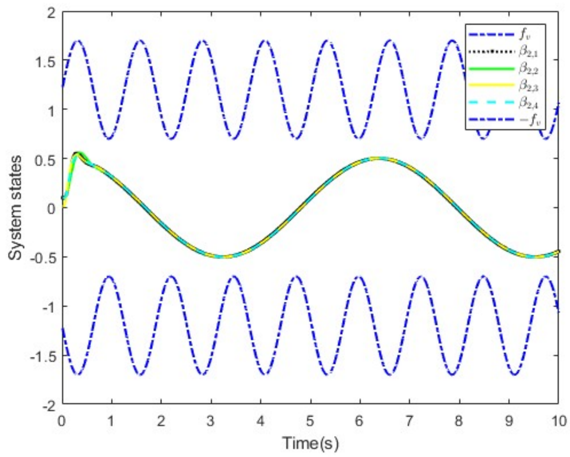

Figure 3. Trajectories of the velocity $\beta_{2,i}$.

5. SIMULATION

In this section, we will employ an example to illustrate the effectiveness of our control strategy. Next, consider the following vehicular model referred to in [28].

$$\begin{cases} \dot{\beta}_{1,i}(t) = \beta_{2,i}(t) + 0.01\beta_{1,i}(t)e^{-0.5\beta_{1,i}(t)} \\ \dot{\beta}_{2,i}(t) = \beta_{3,i}(t) + 0.02\beta_{1,i}(t)\sin(\beta_{2,i}(t)), \\ \dot{\beta}_{3,i}(t) = k_i^{-1}(-\beta_{3,i}(t) + u_i(t) + \tau_i(t) + 0.015\beta_{3,i}(t)\sin(\beta_{2,i}(t))e^{-0.5\beta_{1,i}(t)}. \end{cases} \tag{56}$$

where the external disturbance is $\tau_i(t) = 0.8e^{-0.5t}$, and the reference signal is $y_{c0}(t) = 0.5\sin(t)$.

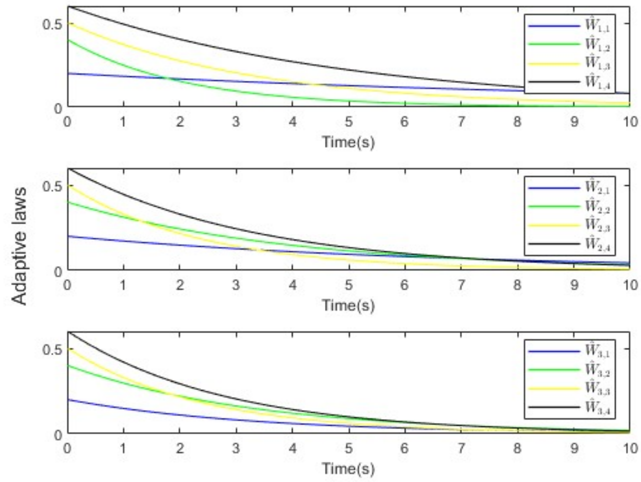

Figure 5. Trajectories of the error $\epsilon_{1,i}$.

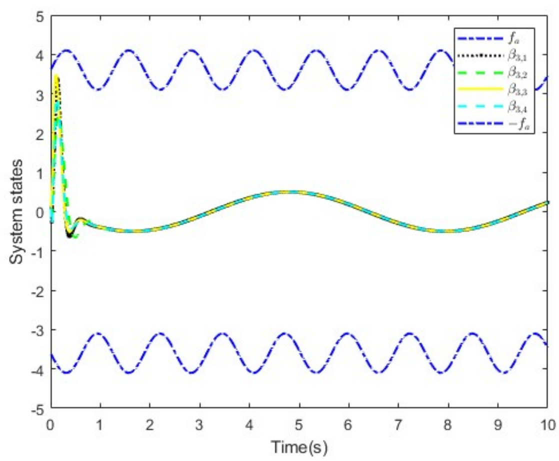

Figure 4. Trajectories of the acceleration $\beta_{3,i}$.

The time-varying boundaries of system states are presented as

$$\begin{cases} f_s(t) = 1.5 + 0.6\sin(5t), \\ f_v(t) = 1.2 + 0.5\sin(5t), \\ f_a(t) = 3.6 + 0.5\sin(5t). \end{cases}$$

Besides, the bounds of error signals are

$$\begin{cases} p_{ai}(t) = 1 + 0.6\sin(5t), \\ p_{bi}(t) = 0.8 + 0.5\sin(5t), \\ p_{ci}(t) = 1.4 + 0.5\sin(5t). \end{cases}$$

The parameters of controllers are selected as: $\alpha_{11} = 16$, $\alpha_{12} = \alpha_{13} = \alpha_{14} = 15$, $\alpha_{22} = 20$, $\alpha_{21} = \alpha_{23} = \alpha_{24} = 30$, $\alpha_{31} = \alpha_{33} = 100$, and $\alpha_{32} = \alpha_{34} = 120$. The parameters related to adaptive law are $\delta_{11} = \delta_{21} = 0.3$, $\delta_{14} = 0.4$,

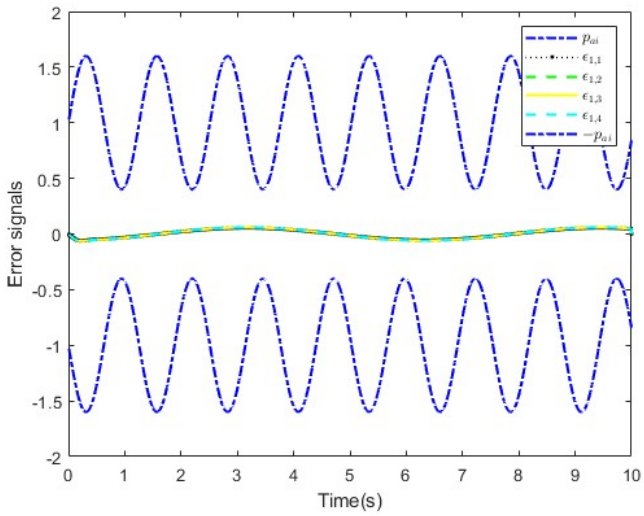

Figure 6. Trajectories of the error $\epsilon_{2,i}$.

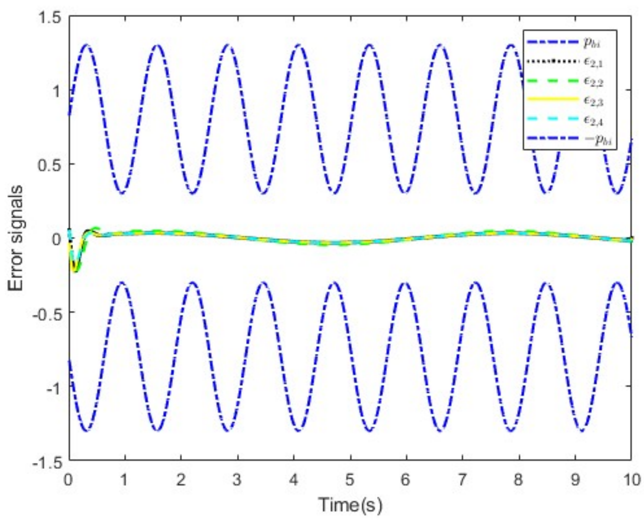

Figure 7. Trajectories of the error $\epsilon_{3,i}$.

$\delta_{13} = \delta_{22} = \delta_{31} = \delta_{32} = 0.5$, $\delta_{12} = \delta_{33} = \delta_{34} = 0.6$, $\delta_{23} = 0.7$, $\rho_{11} = 0.3$, $\rho_{12} = 0.8$, $\rho_{14} = \rho_{21} = \rho_{22} = 0.5$, $\rho_{13} = \rho_{23} = \rho_{24} = \rho_{31} = \rho_{32} = \rho_{34} = 0.6$ and $\rho_{33} = 0.7$. Initial values are $\beta_{1i}(0) = 0 (i = 1, 2, 3, 4)$, $\beta_{21}(0) = \beta_{22}(0) = \beta_{24}(0) = \beta_{31}(0) = \beta_{33}(0) = \beta_{34}(0) = 0.1$, $\beta_{23}(0) = 0.01$, $\beta_{32} = 0.2$, $\hat{W}_{j,1} = 0.2$, $\hat{W}_{j,2} = 0.4$, $\hat{W}_{j,3} = 0.5$, and $\hat{W}_{j,4} = 0.4$ with $j = 1, 2, 3$.

Figure 2-Figure 9 demonstrate the simulation results. Specifically, Figure 2-Figure 4 show the trajectories of the position, velocity and acceleration of the vehicle platoon, which also indicate that system signal can track the reference signal well and the states are kept in the designed bounds. Figure 5-Figure 7 imply that the error signals have not exceeded the boundaries. Figure 8 shows the trajectories of adaptive laws, while Figure 9 depicts the control inputs. Notably, it can be observed from these results that all signals are bounded and control inputs have good convergence performance. Therefore, this bidirectional communication topology enables vehicles to wirelessly exchange information about their position, velocity and acceleration, reducing reliance on a single preceding vehicle and avoiding instability of the platoon caused by a single point of failure.

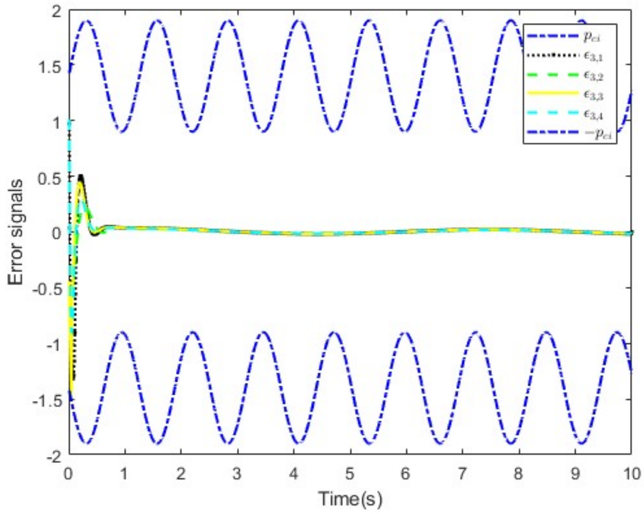

Figure 8. Adaptive laws of $\hat{W}_{1i}$, $\hat{W}_{2i}$, and $\hat{W}_{3i}$.

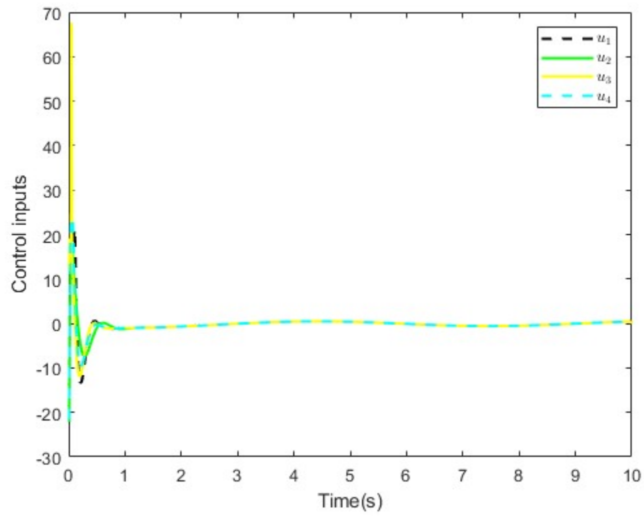

Figure 9. Control inputs of u_1, u_2, u_3, and u_4.

In addition, by fusing the information of the front and rear vehicles, disturbance propagation can be suppressed more quickly. This provides a suitable scheme design and safety considerations for practical vehicle research.

6. CONCLUSIONS

This paper proposes an adaptive neural control strategy for a third-order vehicle platoon with unknown functions and full-state constraints. For safety considerations, we impose time-varying constraints on the position, velocity, and acceleration of the vehicle. Meanwhile, the redundant terms caused by constraints can be eliminated by selecting appropriate tan-BLFs and the singularity issue can be avoided. Besides, we use the NN technique to approximate unknown nonlinearities instead of making too many assumptions, which also simplifies the design process. Finally, an example is utilized to verify the feasibility of the proposed control scheme. Notably, this article does not consider the optimal control scheme and the need to reduce the burden on the platoon communication network. In future research, we will introduce a dynamic programming and event-triggering mechanism to further improve the adaptability of the control algorithm.

DECLARATIONS

Authors' contributions

Made substantial contributions to conception and design of the study and performed data analysis and interpretation: Ma, X.; Di, J.; Tan, C.;
Contributed to approach validation, software simulation, and writing:Ma, X.; Tan, C.; Liu, S.
Contributed to the investigation, supervision, and writing-review and preparation: Di, J.; Liu, S.

Availability of data and materials

Not applicable.

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Conflicts of interest

Tan, C. is the Guest Editor of the special issue "Adaptive Event-Triggered Control and Optimization for Complex Systems". He is not involved in any steps of editorial processing, notably including reviewers' selection, manuscript handling and decision-making, while the other authors have declared that they have no conflicts of interest.

Ethical approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

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