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Non-fragile state estimation for reaction-diffusion genetic regulatory networks with mode-dependent time-varying delays

Jiarui Liu, Shuai Song, Yulong Song, Xiaona Song

School of Information Engineering, Henan University of Science and Technology, Luoyang 471023, Henan, China.

Correspondence to: Dr. Xiaona Song, School of Information Engineering, Henan University of Science and Technology, 263 Kaiyuan Avenue, Luoyang 471023, Henan, China. E-mail: xiaona_97@163.com; ORCID: 0000-0001-8476-5112

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Abstract

This paper investigates the problem of non-fragile state estimation for a class of reaction-diffusion genetic regulatory networks with mode-dependent time-varying delays and Markovian jump parameters. First, the Markov chain with partially unknown probabilities is used in this paper to describe the switching between system modes, which can make the model more generalizable. Moreover, considering the possible gain variations, we design a non-fragile state estimator that makes the estimation performance non-fragile to gain variations, thus guaranteeing the estimation performance. Sufficient conditions that ensure the asymptotic stability of the estimation error can be derived by using the Lyapunov stabilization theory and several inequality treatments. Finally, a simulation example is presented to demonstrate the effectiveness of the proposed estimator design scheme.

Keywords: Non-fragile state estimation, reaction-diffusion genetic regulatory networks, mode-dependent time-varying delays, partially unknown probabilities

1. INTRODUCTION

Recently, genetic regulatory networks (GRNs) have gained significant interest in the fields of biology, mathematics, and medicine[1–3]. This is because mathematical models of GRNs can describe the complex dynamic

behaviors and mutual regulation of mRNAs and proteins within genes during transcription and translation. The models describing GRNs are mainly classified into two types: Boolean models[4,5] and differential equation models[6,7]. In the Boolean model, the state of the gene is viewed as either on or off. The differential equation model can describe the regulatory relationship between mRNAs and proteins in detail and accurately. Therefore, differential equation models are more suitable for modeling GRNs than Boolean network models.

The diffusion phenomenon is widespread; e.g., molecules propagate in the air, and mRNAs and proteins diffuse orderly within genes. However, in[8,9], the authors default that mRNA and protein concentrations are uniformly distributed in space, which does not consider the existence of diffusion phenomena, making the constructed model inaccurate. Therefore, this paper introduces the reaction-diffusion term in the GRN model. During the modeling process, in addition to taking the diffusion phenomenon into account, it is crucial not to ignore the significance of time delay. This is because delay phenomena are present in the process of gene expression and demand attention. Based on the above analysis, we incorporate time delays in reaction-diffusion GRNs (RDGRNs)[10–13]. In[13], Zhang *et al*. constructed RDGRNs with constant delays. However, since constant delays cannot accurately describe the gene regulation process, many scholars investigate GRNs with time-varying delays[14–16]. Furthermore, compared with common time-varying delays[17], mode-dependent time-varying delays can reduce the conservativeness of the stability criterion. Thus, Tian *et al*. in[18] focused on the stability analysis of Markovian jump neural networks with mode-dependent time-varying delays. To the best of our knowledge, there has been no relevant research on RDGRNs with mode-dependent time-varying delays, which is one of the motivations for this paper.

For RDGRNs with time-varying delays, the mode of the system may switch when there are mutations in the gene sequence or changes in the external environment, and the probabilities of switching are described by the Markov chain[19–21]. Thus, the investigations of RDGRNs with Markovian jump parameters and time-varying delays have received much attention[22,23]. For example, in[23], Zou *et al*. studied the robust stochastic stability of delayed RDGRNs with Markovian jump parameters. However, it is known that the switching probability of the system is difficult to obtain. Yet, in[24] and[25], the authors have assumed that the switching probabilities are completely known, which is not realistic. Therefore, it is worthwhile for Markovian jump RDGRNs with partially unknown transfer probabilities[26–28] to be further scrutinized.

So far, for some applications of RDGRNs, it is a prerequisite to know the exact concentrations of mRNA and protein within genes. Nevertheless, due to the specificity of RDGRNs and the cost issue, the mRNA and protein concentrations are often only partially available, so it is extremely investigative to utilize the partially available state information to estimate the system state. In research on state estimation of RDGRNs[29–31], there are defaults that the estimator parameters can be executed exactly. However, in general, the parameters may be perturbed to some extent due to factors such as component aging and internal noise, leading to system vulnerability. Therefore, some scholars have begun to focus on the non-fragile state estimation and synchronization of GRNs, and several results have been achieved[32–34]. For example, Li *et al*. in[32] investigated the issue of non-fragile state estimation for delayed GRNs. Nevertheless, the design of non-fragile state estimators for RDGRNs has not been investigated, which is another research motivation for this paper.

In light of the above discussion, this work studies the non-fragile state estimation for Markovian jump RDGRNs with partially unknown transfer probabilities and mode-dependent time-varying delays. The main innovations of this paper are as follows: (1) The reaction-diffusion term, mode-dependent time-varying delays, and Markovian jump parameters are considered in the model, which makes this model investigated in this paper more generalized compared to the existing studies on GRNs[8]; (2) The partially unknown transfer probabilities[26] are used in this work, which is more realistic compared to Markovian switching, where the transfer probabilities[22] are fully known; and (3) The estimation performance of conventional state estimators[29] is

affected when facing cases such as component aging and internal noise, whereas the non-fragile state estimator designed in this paper for RDGRNs makes the estimation performance non-fragile to gain variations.

Notations: $\Re^{n\times n}$ denote the space of $n\times n$ real matrices, col$(\cdot)$ denotes the column vector, diag$(\cdot)$ indicates the diagonal matrix, and sym$\{\mathbb{M}\} = \mathbb{M} + \mathbb{M}^T$; $(g_{hq})_{n\times n}$ represents the $n\times n$ matrix consisting of the elements g_{hq}; Ω denotes the set $\{\nu_1 \le y \le \nu_2\}$. In addition, this paper uses the following abbreviations: $\tilde{m}_h \triangleq \tilde{m}_h(x,y)$, $\tilde{p}_h \triangleq \tilde{p}_h(x,y)$, $\bar{m} \triangleq \bar{m}(x,y)$, $\bar{p} \triangleq \bar{p}(x,y)$, $L_m \triangleq L_m(x,y)$, $L_p \triangleq L_p(x,y)$, $\hat{m} \triangleq \hat{m}(x,y)$, and $\hat{p} \triangleq \hat{p}(x,y)$.

2. NETWORK MODEL AND PRELIMINARIES

Consider the Markovian jump RDGRNs with mode-dependent time-varying delays as follows:

$$\begin{cases} \dfrac{\partial \tilde{m}_h}{\partial x} = e_h \dfrac{\partial^2 \tilde{m}_h}{\partial y^2} - b_{h,\varepsilon(x)}\tilde{m}_h + \displaystyle\sum_{q=1}^{n} g_{hq,\varepsilon(x)}\xi_q(\tilde{p}_q(x-\alpha_{\varepsilon(x)}(x),y)) + \delta_h, \\ \dfrac{\partial \tilde{p}_h}{\partial x} = \bar{e}_h \dfrac{\partial^2 \tilde{p}_h}{\partial y^2} - \bar{b}_{h,\varepsilon(x)}\tilde{p}_h + \bar{g}_{h,\varepsilon(x)}\tilde{m}_h(x-\beta_{\varepsilon(x)}(x),y), \end{cases} \tag{1}$$

where the border and initial conditions are given as follows:

$$\begin{aligned} &\tilde{m}_h = 0,\ y \in \partial\Omega,\ x \in [-\beta,+\infty),\ \tilde{m}_h = \varphi(x,y),\ y \in \Omega,\ x \in [-\beta,0], \\ &\tilde{p}_h = 0,\ y \in \partial\Omega,\ x \in [-\alpha,+\infty),\ \tilde{p}_h = \theta(x,y),\ y \in \Omega,\ x \in [-\alpha,0], \end{aligned} \tag{2}$$

in which $h = 1,2,\ldots,n$; $\partial\Omega$ indicates the boundary of Ω, $\varphi(x,y)$ and $\theta(x,y)$ denote the initial conditions of $\tilde{m}_h$ and $\tilde{p}_h$, respectively. $\tilde{m}_h$ and $\tilde{p}_h$ are the mRNA and protein concentrations; x and $y \in \Omega$ denote time and space position, respectively; $\xi_q(\tilde{p}_q)$ denotes the monotonic feedback regulation function, which is described in the form $\xi_q(\tilde{p}_q) = \frac{(\frac{\tilde{p}_q}{\varrho_q})^{H_q}}{1+(\frac{\tilde{p}_q}{\varrho_q})^{H_q}}$, where H_q is the Hill coefficient representing the degree of cooperativity, ϱ_q is a positive constant, and $q = 1,\ldots,n$; the meaning and form of $\xi_q(\tilde{p}_q)$ are explained in detail by [35–37], so they are omitted here; $\delta_h = \sum_{q\in\mu_h} g^*_{hq,\varepsilon(x)}$, and μ_h is the set containing all the repressors of gene h; $\alpha_{\varepsilon(x)}(x)$ and $\beta_{\varepsilon(x)}(x)$ are time-varying delays that satisfy the following conditions

$$0 \le \alpha_{\varepsilon(x)}(x) \le \alpha_{\varepsilon(x)},\ \dot{\alpha}_{\varepsilon(x)}(x) \le \psi_{1,\varepsilon(x)} < 1,\ 0 \le \beta_{\varepsilon(x)}(x) \le \beta_{\varepsilon(x)},\ \dot{\beta}_{\varepsilon(x)}(x) \le \psi_{2,\varepsilon(x)} < 1, \tag{3}$$

with known scalars $\alpha_{\varepsilon(x)}$, $\beta_{\varepsilon(x)}$, $\psi_{1,\varepsilon(x)}$, and $\psi_{2,\varepsilon(x)}$. e_h and $\bar{e}_h$ represent the diffusion rate; $b_{h,\varepsilon(x)}$ and $\bar{b}_{h,\varepsilon(x)}$ denote the mRNA and protein degradation rate, respectively; $\bar{g}_{h,\varepsilon(x)}$ indicates the translation rate; $G_{\varepsilon(x)} = (g_{hq,\varepsilon(x)})_{n\times n}$ with

$$g_{hq,\varepsilon(x)} = \begin{cases} g^*_{hq,\varepsilon(x)}, & \text{if } q \text{ is an activator of gene } h, \\ 0, & \text{if there is no link between gene } q \text{ and } h, \\ -g^*_{hq,\varepsilon(x)}, & \text{if } q \text{ is a repressor of gene } h, \end{cases}$$

denotes the coupling matrix, and $g^*_{hq,\varepsilon(x)}$ is the dimensionless transcriptional rate.

Define $\varepsilon(x) \in \mathbb{U} \triangleq \{1,2,\ldots,U\}(t \ge 0)$ to be a Markov chain, the corresponding transfer probability matrix $\prod \triangleq (\pi_{ab})_{U\times U}$ is given as

$$\text{Prob}\{\varepsilon(t+\Delta t) = b \mid \varepsilon(x) = a\} = \begin{cases} \pi_{ab}\Delta t + 0(\Delta t), & a \ne b, \\ 1 + \pi_{ab}\Delta t + 0(\Delta t), & a = b, \end{cases}$$

where $\Delta t > 0$, $\lim\limits_{\Delta t \to 0} \frac{0(\Delta t)}{\Delta t} = 0$, π_{ab} is the transfer probability from mode a to b, and $\pi_{aa} = -\sum\limits_{a=1, a\neq b}^{U} \pi_{ab}$. Moreover, the transfer matrix with partially unknown probabilities can be expressed as

$$\begin{bmatrix} \pi_{11} & ? & \dots & ? \\ \pi_{21} & ? & \dots & \pi_{2U} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{U1} & ? & \dots & \pi_{UU} \end{bmatrix} \tag{4}$$

where "?" represents the unknown transfer probabilities. Define $\mathfrak{R}_r^a \triangleq \{b : \pi_{ab} \text{ is known}\}$, $\mathfrak{R}_{ur}^a \triangleq \{b : \pi_{ab} \text{ is unknown}\}$, and $\mathfrak{R} \triangleq \mathfrak{R}_r^a + \mathfrak{R}_{ur}^a$.

Let $\tilde{m}^*(y) = \text{col}\left(\tilde{m}_1^*(y), \tilde{m}_2^*(y), \dots, \tilde{m}_n^*(y)\right)$ and $\tilde{p}^*(y) = \text{col}\left(\tilde{p}_1^*(y), \tilde{p}_2^*(y), \dots, \tilde{p}_n^*(y)\right)$ be the unique equilibrium point of the system (1), that is

$$\begin{cases} 0 = e_h \dfrac{\partial^2 \tilde{m}_h^*(y)}{\partial y^2} - b_{h,\varepsilon(x)} \tilde{m}_h^*(y) + \sum\limits_{q=1}^{n} g_{hq,\varepsilon(x)} \xi_q(\tilde{p}_q^*(y)) + \delta_h, \\ 0 = \bar{e}_h \dfrac{\partial^2 \tilde{p}_h^*(y)}{\partial y^2} - \bar{b}_{h,\varepsilon(x)} \tilde{p}_h^*(y) + \bar{g}_{h,\varepsilon(x)} \tilde{m}_h^*(y). \end{cases}$$

Define $\bar{m}_h = \tilde{m}_h - \tilde{m}_h^*(y)$ and $\bar{p}_h = \tilde{p}_h - \tilde{p}_h^*(y)$, the system (1) is converted into

$$\begin{cases} \dfrac{\partial \bar{m}}{\partial x} = E \dfrac{\partial^2 \bar{m}}{\partial y^2} - B_{\varepsilon(x)} \bar{m} + G_{\varepsilon(x)} f(\bar{p}(x - \alpha_{\varepsilon(x)}(x), y)), \\ \dfrac{\partial \bar{p}}{\partial x} = \bar{E} \dfrac{\partial^2 \bar{p}}{\partial y^2} - \bar{B}_{\varepsilon(x)} \bar{p} + \bar{G}_{\varepsilon(x)} \bar{m}(x - \beta_{\varepsilon(x)}(x), y), \end{cases} \tag{5}$$

where

$$\begin{aligned} &\bar{m} = \text{col}(\bar{m}_1, \bar{m}_2, \dots, \bar{m}_n),\ E = \text{diag}(e_1, e_2, \dots, e_n),\ B_{\varepsilon(x)} = \text{diag}(b_{1,\varepsilon(x)}, b_{2,\varepsilon(x)}, \dots, b_{n,\varepsilon(x)}), \\ &\bar{p} = \text{col}(\bar{p}_1, \bar{p}_2, \dots, \bar{p}_n),\ \bar{E} = \text{diag}(\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n),\ \bar{B}_{\varepsilon(x)} = \text{diag}(\bar{b}_{1,\varepsilon(x)}, \bar{b}_{2,\varepsilon(x)}, \dots, \bar{b}_{n,\varepsilon(x)}), \\ &\bar{G}_{\varepsilon(x)} = \text{diag}(\bar{g}_{1,\varepsilon(x)}, \dots, \bar{g}_{n,\varepsilon(x)}),\ f_h(\bar{p}_h(x - \alpha_{\varepsilon(x)}(x), y)) = \xi_h(\bar{p}_h(x - \alpha_{\varepsilon(x)}(x), y) + \tilde{p}_h^*(y)) - \xi_h(\tilde{p}_h^*(y)), \\ &f(\bar{p}(x - \alpha_{\varepsilon(x)}(x), y)) = \text{col}(f_1(\bar{p}_1(x - \alpha_{\varepsilon(x)}(x), y)), f_2(\bar{p}_2(x - \alpha_{\varepsilon(x)}(x), y)), \dots, f_n(\bar{p}_n(x - \alpha_{\varepsilon(x)}(x), y))). \end{aligned}$$

Additionally, the measured outputs of (5) are

$$L_m = N\bar{m},\ \ L_p = \bar{N}\bar{p}, \tag{6}$$

where $N \in \mathfrak{R}^{n\times n}$ and $\bar{N} \in \mathfrak{R}^{n\times n}$ are known matrices; L_m and L_p represent the measured outputs of mRNA and protein, respectively.

To estimate the states of (5) by using the measured outputs in (6), one can construct the estimator as follows:

$$\begin{cases} \dfrac{\partial \hat{m}}{\partial x} = E\dfrac{\partial^2 \hat{m}}{\partial y^2} - B_{\varepsilon(x)}\hat{m} + G_{\varepsilon(x)} f(\hat{p}(x-\alpha_{\varepsilon(x)}(x), y)) + (K_{1,\varepsilon(x)} + \Delta K_{1,\varepsilon(x)})(L_m - N\hat{m}), \\ \dfrac{\partial \hat{p}}{\partial x} = \bar{E}\dfrac{\partial^2 \hat{p}}{\partial y^2} - \bar{B}_{\varepsilon(x)}\hat{p} + \bar{G}_{\varepsilon(x)}\hat{m}(x-\beta_{\varepsilon(x)}(x), y) + (K_{2,\varepsilon(x)} + \Delta K_{2,\varepsilon(x)})(L_p - \bar{N}\hat{p}), \end{cases} \tag{7}$$

where $\hat{m}$ and $\hat{p}$ represent the estimations of $\bar{m}$ and $\bar{p}$, $K_{1,\varepsilon(x)}$ and $K_{2,\varepsilon(x)}$ are the estimator gain matrices, and $\Delta K_{1,\varepsilon(x)}$ and $\Delta K_{2,\varepsilon(x)}$ denote the gain perturbation, which takes the following form:

$$\Delta K_{1,\varepsilon(x)} = A_{1,\varepsilon(x)}\Lambda_{1,\varepsilon(x)}(x)C_{1,\varepsilon(x)},\ \Delta K_{2,\varepsilon(x)} = A_{2,\varepsilon(x)}\Lambda_{2,\varepsilon(x)}(x)C_{2,\varepsilon(x)}, \tag{8}$$

where $A_{1,\varepsilon(x)}$, $C_{1,\varepsilon(x)}$, $A_{2,\varepsilon(x)}$, and $C_{2,\varepsilon(x)}$ stand for the given matrices; $\Lambda_{1,\varepsilon(x)}(x)$ and $\Lambda_{2,\varepsilon(x)}(x)$ are unknown matrices satisfying $\Lambda^{\mathrm{T}}_{1,\varepsilon(x)}(x)\Lambda_{1,\varepsilon(x)}(x) \le I$ and $\Lambda^{\mathrm{T}}_{2,\varepsilon(x)}(x)\Lambda_{2,\varepsilon(x)}(x) \le I$, respectively.

Set $\varepsilon(x) = a$, we denote $B_a \triangleq B_{\varepsilon(x)}, G_a \triangleq G_{\varepsilon(x)}, \alpha_a \triangleq \alpha_{\varepsilon(x)}, \bar{B}_a \triangleq \bar{B}_{\varepsilon(x)}, \bar{G}_a \triangleq \bar{G}_{\varepsilon(x)}, \beta_a \triangleq \beta_{\varepsilon(x)}, K_{da} \triangleq K_{d,\varepsilon(x)}$, $\Delta K_{da} \triangleq \Delta K_{d,\varepsilon(x)}$, $A_{da} \triangleq A_{d,\varepsilon(x)}$, $\Lambda_{da} \triangleq \Lambda_{d,\varepsilon(x)}$, $C_{da} \triangleq C_{d,\varepsilon(x)}$, $(d = 1, 2)$. Define $\phi_m = \bar{m} - \hat{m}$ and $\phi_p = \bar{p} - \hat{p}$ to be the error state vectors; based on (5)–(7), the following error system is derived,

$$\begin{cases} \dfrac{\partial \phi_m}{\partial x} = E\dfrac{\partial^2 \phi_m}{\partial y^2} - B_a\phi_m + G_a f(\phi_p(x-\alpha_a(x), y)) - (K_{1a} + \Delta K_{1a})W\phi_m, \\ \dfrac{\partial \phi_p}{\partial x} = \bar{E}\dfrac{\partial^2 \phi_p}{\partial y^2} - \bar{B}_a\phi_p + \bar{G}_a\phi_m(x-\beta_a(x), y) - (K_{2a} + \Delta K_{2a})\bar{W}\phi_p, \end{cases} \tag{9}$$

where $f(\phi_p(x-\alpha_a(x), y)) = f(\bar{p}(x-\alpha_a(x), y)) - f(\hat{p}(x-\alpha_a(x), y))$, $\phi_m(x-\beta_a(x), y) = \bar{m}(x-\beta_a(x), y) - \hat{m}(x-\beta_a(x), y)$, $\Delta K_{1a} = A_{1a}\Lambda_{1a}(x)C_{1a}$, and $\Delta K_{2a} = A_{2a}\Lambda_{2a}(x)C_{2a}$.
Because $f_h(\cdot)$ is a monotonically increasing function, which satisfies the following inequality

$$0 \le \frac{f_h(z)}{z} \le \vartheta_h,\ \forall z \ne 0;\ h = 1, \ldots, n, \tag{10}$$

where ϑ_h is a known positive real number.

The following lemmas are important tools for obtaining the principal results of this article.

Lemma 1.[38] Assume $\hbar \ge 0 \in \Re^{n\times n}$ and the function $\ell \in \mathcal{H}_l^n([r_1, r_2])$ satisfying $\ell(r_1) = 0$ or $\ell(r_2) = 0$, then

$$\int_{r_1}^{r_2} \ell^T(y)\hbar\ell(y)dy \le \frac{4(r_2-r_1)^2}{\pi^2}\int_{r_1}^{r_2} \dot{\ell}(y)^T\hbar\dot{\ell}(y)dy.$$

Further, if $\ell(r_1) = \ell(r_2) = 0$, one has

$$\int_{r_1}^{r_2} \ell^T(y)\hbar\ell(y)dy \le \frac{(r_2-r_1)^2}{\pi^2}\int_{r_1}^{r_2} \dot{\ell}(y)^T\hbar\dot{\ell}(y)dy.$$

Lemma 2.[39] With real matrices H and J of appropriate dimensions, the inequality satisfies

$$HJ^T + JH^T \le \gamma^{-1}HH^T + \gamma JJ^T,$$

where γ is a positive scalar.

Lemma 3.[40] The existence of matrices $\Psi = \Psi^{\mathrm{T}}$, H and J of appropriate dimensions, then

$$\Psi + H\Lambda(x)J + H^{\mathrm{T}}\Lambda^{\mathrm{T}}(x)J^{\mathrm{T}} < 0,$$

for all satisfying $\Lambda^{\mathrm{T}}(x)\Lambda(x) \leq I$, the sufficient condition for $\Lambda(x)$ to hold is the existence of a positive constant γ such that the following equation holds:

$$\Psi + \gamma^{-1}HH^{\mathrm{T}} + \gamma J^{\mathrm{T}}J < 0.$$

3. MAIN RESULTS

For this section, establishing new criteria by choosing appropriate Lyapunov-Krasovskii functionals (LKFs), which assures that system (9) is asymptotically stable.

Theorem 1. Given constants $\psi_{1a}, \psi_{2a}, \varpi$, matrix $\mathbb{N}$, and estimator gain matrices K_{1a} and K_{2a}, the system (9) is asymptotically stable if there exist positive matrices P_{la}, F_l, Q_l, $(l = 1, 2)$ with suitable dimensions, symmetric matrices $W_a \in \mathfrak{R}^{n\times n}$ and $\bar{W}_a \in \mathfrak{R}^{n\times n}$, and the diagonal matrix $R_1 > 0 \in \mathfrak{R}^{n\times n}$, which makes the below linear matrix inequalities (LMIs) feasible:

$$\Psi_1 + \Psi_2 + \Psi_3 \leq 0, \tag{11}$$

$$P_{1b} + W_a \leq 0,\ P_{2b} + \bar{W}_a \leq 0,\ \forall b \in \mathfrak{R}^a_{ur},\ a \neq b, \tag{12}$$

where

$$\begin{aligned}
\Psi_1 =& \mathrm{sym}\{\zeta_1^T P_{1a}\zeta_6 + \zeta_2^T P_{2a}\zeta_7\} + \zeta_1^T \sum_{b=\mathfrak{R}_r^a} \pi_{ab}(P_{1b} + W_a)\zeta_1 + \zeta_2^T \sum_{b=\mathfrak{R}_r^a} \pi_{ab}(P_{2b} + \bar{W}_a)\zeta_2 + \zeta_1^T F_1\zeta_1 + \zeta_2^T F_2\zeta_2 \\
&- (1-\psi_{2a})\zeta_3^T F_1\zeta_3 - (1-\psi_{1a})\zeta_4^T F_2\zeta_4 + \mathrm{sym}\{\zeta_5^T R_1(\mathbb{N}\zeta_4 - \zeta_5)\}, \\
\Psi_2 =& -\mathrm{sym}\{\zeta_1^T Q_1 B_a\zeta_1\} - \mathrm{sym}\{\zeta_1^T \frac{\pi^2}{\varpi^2} Q_1 E\zeta_1\} - \mathrm{sym}\{\zeta_1^T Q_1 B_a\zeta_6\} + \mathrm{sym}\{\zeta_1^T Q_1 G_a\zeta_5\} - \mathrm{sym}\{\zeta_1^T Q_1\zeta_6\} \\
&+ \mathrm{sym}\{\zeta_5^T Q_1 G_a\zeta_6\} - \mathrm{sym}\{\zeta_6^T Q_1\zeta_6\} - \mathrm{sym}\{\zeta_1^T Q_1 K_{1a}N\zeta_1\} - \mathrm{sym}\{\zeta_1^T Q_1\Delta K_{1a}N\zeta_1\} - \mathrm{sym}\{\zeta_1^T Q_1 K_{1a}N\zeta_6\} \\
&- \mathrm{sym}\{\zeta_1^T Q_1\Delta K_{1a}N\zeta_6\}, \\
\Psi_3 =& -\mathrm{sym}\{\zeta_2^T Q_2\bar{B}_a\zeta_2\} - \mathrm{sym}\{\zeta_2^T \frac{\pi^2}{\varpi^2} Q_2\bar{E}\zeta_2\} - \mathrm{sym}\{\zeta_2^T Q_2\bar{B}_a\zeta_7\} + \mathrm{sym}\{\zeta_2^T Q_2\bar{G}_a\zeta_3\} - \mathrm{sym}\{\zeta_2^T Q_2\zeta_7\} \\
&+ \mathrm{sym}\{\zeta_3^T Q_2\bar{G}_a\zeta_7\} - \mathrm{sym}\{\zeta_7^T Q_2\zeta_7\} - \mathrm{sym}\{\zeta_2^T Q_2 K_{2a}\bar{N}\zeta_2\} - \mathrm{sym}\{\zeta_2^T Q_2\Delta K_{2a}\bar{N}\zeta_2\} - \mathrm{sym}\{\zeta_2^T Q_2 K_{2a}\bar{N}\zeta_7\} \\
&- \mathrm{sym}\{\zeta_2^T Q_2\Delta K_{2a}\bar{N}\zeta_7\},
\end{aligned}$$

where $\psi_{1a} = \psi_{1,\varepsilon(x)}$, $\psi_{2a} = \psi_{2,\varepsilon(x)}$, $\mathbb{N} = \mathrm{diag}(\mathbb{N}_1, \mathbb{N}_2, \ldots, \mathbb{N}_n)$, $\varpi = \nu_2 - \nu_1$, and $\zeta_\upsilon = \left[0_{n,(\upsilon-1)n}\ I_n\ 0_{n,(7-\upsilon)n}\right]$, $(\upsilon = 1, \ldots, 7)$.

Proof. Construct the LKFs for the system (9) as follows:

$$V(x) = V_1(x) + V_2(x) + V_3(x), \tag{13}$$

where

$$V_1(x) = \int_\Omega \phi_m^T P_{1a}\phi_m dy + \int_\Omega \phi_p^T P_{2a}\phi_p dy,$$
$$V_2(x) = \int_\Omega \frac{\partial \phi_m^T}{\partial y} Q_1 E \frac{\partial \phi_m}{\partial y} dy + \int_\Omega \frac{\partial \phi_p^T}{\partial y} Q_2 \bar{E} \frac{\partial \phi_p}{\partial y} dy,$$
$$V_3(x) = \int_\Omega \int_{x-\beta_a(x)}^{x} \phi_m^T(z,y) F_1 \phi_m(z,y) dz dy + \int_\Omega \int_{x-\alpha_a(x)}^{x} \phi_p^T(z,y) F_2 \phi_p(z,y) dz dy.$$

The time derivative of $V(x)$ is calculated as follows:

$$\dot{V}_1(x) = 2\int_\Omega \phi_m^T P_{1a} \frac{\partial \phi_m}{\partial x} dy + \int_\Omega \sum_{b=1}^{U} \pi_{ab} \phi_m^T P_{1b} \phi_m dy + 2\int_\Omega \phi_p^T P_{2a} \frac{\partial \phi_p}{\partial x} dy + \int_\Omega \sum_{b=1}^{U} \pi_{ab} \phi_p^T P_{2b} \phi_p dy, \tag{14}$$

$$\dot{V}_2(x) = 2\int_\Omega \frac{\partial^2 \phi_m^T}{\partial y \partial x} Q_1 E \frac{\partial \phi_m}{\partial y} dy + 2\int_\Omega \frac{\partial^2 \phi_p^T}{\partial y \partial x} Q_2 \bar{E} \frac{\partial \phi_p}{\partial y} dy, \tag{15}$$

$$\begin{aligned}
\dot{V}_3(x) &= \int_\Omega \phi_m^T F_1 \phi_m dy - (1-\dot{\beta}_a(x)) \int_\Omega \phi_m^T(x-\beta_a(x),y) F_1 \phi_m(x-\beta_a(x),y) dy \\
&\quad + \int_\Omega \phi_p^T F_2 \phi_p dy - (1-\dot{\alpha}_a(x)) \int_\Omega \phi_p^T(x-\alpha_a(x),y) F_2 \phi_p(x-\alpha_a(x),y) dy \\
&\le \int_\Omega \phi_m^T F_1 \phi_m dy - (1-\psi_{2a}) \int_\Omega \phi_m^T(x-\beta_a(x),y) F_1 \phi_m(x-\beta_a(x),y) dy \\
&\quad + \int_\Omega \phi_p^T F_2 \phi_p dy - (1-\psi_{1a}) \int_\Omega \phi_p^T(x-\alpha_a(x),y) F_2 \phi_p(x-\alpha_a(x),y) dy.
\end{aligned} \tag{16}$$

According to (9), the following is obviously true

$$\begin{aligned}
0 &= 2\int_\Omega \left(\phi_m + \frac{\partial \phi_m}{\partial x}\right)^T Q_1 \left[-\frac{\partial \phi_m}{\partial x} + E\frac{\partial^2 \phi_m}{\partial y^2} - B_a \phi_m + G_a f(\phi_p(x-\alpha_a(x),y)) - (K_{1a}+\Delta K_{1a}) N \phi_m \right] dy, \\
0 &= 2\int_\Omega \left(\phi_p + \frac{\partial \phi_p}{\partial x}\right)^T Q_2 \left[-\frac{\partial \phi_p}{\partial x} + \bar{E}\frac{\partial^2 \phi_p}{\partial y^2} - \bar{B}_a \phi_p + \bar{G}_a \phi_m(x-\beta_a(x),y) - (K_{2a}+\Delta K_{2a}) \bar{N} \phi_p \right] dy.
\end{aligned} \tag{17}$$

According to the definition of the transfer probability matrix (4), for symmetric matrices $W_a \in \mathfrak{R}^{n\times n}$ and $\bar{W}_a \in \mathfrak{R}^{n\times n}$, one gets

$$0 = \sum_{b=1}^{U} \pi_{ab} \phi_m^T W_a \phi_m, \; 0 = \sum_{b=1}^{U} \pi_{ab} \phi_p^T \bar{W}_a \phi_p, \; \forall a \in U. \tag{18}$$

Based on (17) and the boundary conditions (2), using the integration by parts, one can get

$$2\int_\Omega \phi_m^T Q_1 E \frac{\partial^2 \phi_m}{\partial y^2} dy = -2\int_\Omega \frac{\partial \phi_m^T}{\partial y} Q_1 E \frac{\partial \phi_m}{\partial y} dy, \tag{19}$$

and

$$2\int_\Omega \phi_p^T Q_2 \bar{E} \frac{\partial^2 \phi_p}{\partial y^2} dy = -2\int_\Omega \frac{\partial \phi_p^T}{\partial y} Q_2 \bar{E} \frac{\partial \phi_p}{\partial y} dy. \tag{20}$$

Following (19) and (20) and applying Lemma 1, we can derive that

$$-2\int_\Omega \frac{\partial \phi_m^T}{\partial y} Q_1 E \frac{\partial \phi_m}{\partial y} + \frac{\partial \phi_p^T}{\partial y} Q_2 \bar{E} \frac{\partial \phi_p}{\partial y} dy \le -2\frac{\pi^2}{\varpi^2} \int_\Omega \phi_m^T Q_1 E \phi_m + \phi_p^T Q_2 \bar{E} \phi_p dy. \tag{21}$$

From (17) and the boundary conditions (2), we have that

$$2\int_{\Omega}\frac{\partial\phi_m^T}{\partial x}Q_1E\frac{\partial^2\phi_m}{\partial y^2}dy=-2\int_{\Omega}\frac{\partial\phi_m^T}{\partial y}Q_1E\frac{\partial^2\phi_m}{\partial y\partial x}dy, \tag{22}$$

and

$$2\int_{\Omega}\frac{\partial\phi_p^T}{\partial x}Q_2\bar{E}\frac{\partial^2\phi_p}{\partial y^2}dy=-2\int_{\Omega}\frac{\partial\phi_p^T}{\partial y}Q_2\bar{E}\frac{\partial^2\phi_p}{\partial y\partial x}dy. \tag{23}$$

Based on (10), for arbitrary diagonal matrix $R_1>0$, one obtains

$$0\le 2f^T(\phi_p(x-\alpha_a(x),y))R_1(\mathbb{N}\phi_p-f(\phi_p(x-\alpha_a(x),y))). \tag{24}$$

By combining (14)–(24), it can be proved that

$$\begin{aligned}\dot{V}(x)\le&\int_{\Omega}\ell^T(x,y)(\Psi_1+\Psi_2+\Psi_3)\ell(x,y)dy\\&+\int_{\Omega}\sum_{b=\Re_{ur}^a}\pi_{ab}\phi_m^T(P_{1b}+W_a)\phi_m+\sum_{b=\Re_{ur}^a}\pi_{ab}\phi_p^T(P_{2b}+\bar{W}_a)\phi_p dy,\end{aligned} \tag{25}$$

where $\ell(x,y)=\mathrm{col}\left\{\phi_m,\phi_p,\phi_m(x-\beta_a(x),y),\phi_p(x-\alpha_a(x),y),f(\phi_p(x-\alpha_a(x),y)),\frac{\partial\phi_m}{\partial x},\frac{\partial\phi_p}{\partial x}\right\}$.
Therefore, combining the above analysis, one can conclude that the error system (9) is asymptotically stable. The proof is complete. ■

Having performed the stability analysis in Theorem 1, we will deal with the design of a non-fragile estimator. The following theorem gives a solution for designing a non-fragile estimator.

Theorem 2. Given positive constants ψ_{1a}, ψ_{2a}, ϖ, γ_1, γ_2, and matrix $\mathbb{N}$, the error system (9) is asymptotically stable if there exist positive matrices P_{la}, F_l, Q_l, Γ_{la}, $(l=1,2)$ with suitable dimensions, symmetric matrices $W_a\in\Re^{n\times n}$ and $\bar{W}_a\in\Re^{n\times n}$, and the diagonal matrix $R_1>0\in\Re^{n\times n}$, which makes the below LMIs feasible:

$$\begin{bmatrix}\hat{\Psi}&\gamma_1\mathbb{A}_{1a}&\mathbb{C}_{1a}^T&\gamma_2\mathbb{A}_{2a}&\mathbb{C}_{2a}^T\\*&-\gamma_1 I&0&0&0\\*&*&-\gamma_1 I&0&0\\*&*&*&-\gamma_2 I&0\\*&*&*&*&-\gamma_2 I\end{bmatrix}\le 0, \tag{26}$$

where $\hat{\Psi}=\Psi_1+\hat{\Psi}_1+\hat{\Psi}_2$, $\mathbb{A}_{1a}=\Big[-A_{1a}^TQ_1^T\quad\underbrace{0\quad0\quad\dots\quad0}_{6\text{ times}}\Big]^T$, $\mathbb{C}_{1a}=\Big[C_{1a}N\quad\underbrace{0\quad0\quad\dots\quad0}_{4\text{ times}}\quad C_{1a}N\quad0\Big]$,

$$\mathbb{A}_{2a} = \begin{bmatrix} 0 & -A_{2a}^T Q_2^T & \underbrace{0 \quad 0 \quad \dots \quad 0}_{5 \text{ times}} \end{bmatrix}^T, \mathbb{C}_{2a} = \begin{bmatrix} 0 & C_{2a}\bar{N} & \underbrace{0 \quad 0 \quad \dots \quad 0}_{4 \text{ times}} & C_{2a}\bar{N} \end{bmatrix}, \text{and}$$

$$\begin{aligned}
\Psi_1 =& \text{sym}\{\zeta_1^T P_{1a}\zeta_6 + \zeta_2^T P_{2a}\zeta_7\} + \zeta_1^T \sum_{b=\mathfrak{R}_r^a} \pi_{ab}(P_{1b} + W_a)\zeta_1 + \zeta_2^T \sum_{b=\mathfrak{R}_r^a} \pi_{ab}(P_{2b} + \bar{W}_a)\zeta_2 + \zeta_1^T F_1\zeta_1 + \zeta_2^T F_2\zeta_2 \\
& - (1-\psi_{2a})\zeta_3^T F_1\zeta_3 - (1-\psi_{1a})\zeta_4^T F_2\zeta_4 + \text{sym}\{\zeta_5^T R_1(\mathbb{N}\zeta_4 - \zeta_5)\}, \\
\hat{\Psi}_1 =& -\text{sym}\{\zeta_1^T Q_1 B_a\zeta_1\} - \text{sym}\{\zeta_1^T \frac{\pi^2}{\varpi^2} Q_1 E\zeta_1\} - \text{sym}\{\zeta_1^T Q_1 B_a\zeta_6\} + \text{sym}\{\zeta_1^T Q_1 G_a\zeta_5\} - \text{sym}\{\zeta_1^T Q_1\zeta_6\} \\
& + \text{sym}\{\zeta_5^T Q_1 G_a\zeta_6\} - \text{sym}\{\zeta_6^T Q_1\zeta_6\} - \text{sym}\{\zeta_1^T \Gamma_{1a} N\zeta_1\} - \text{sym}\{\zeta_1^T \Gamma_{1a} N\zeta_6\}, \\
\hat{\Psi}_2 =& -\text{sym}\{\zeta_2^T Q_2 \bar{B}_a\zeta_2\} - \text{sym}\{\zeta_2^T \frac{\pi^2}{\varpi^2} Q_2 \bar{E}\zeta_2\} - \text{sym}\{\zeta_2^T Q_2 \bar{B}_a\zeta_7\} + \text{sym}\{\zeta_2^T Q_2 \bar{G}_a\zeta_3\} - \text{sym}\{\zeta_2^T Q_2\zeta_7\} \\
& + \text{sym}\{\zeta_3^T Q_2 \bar{G}_a\zeta_7\} - \text{sym}\{\zeta_7^T Q_2\zeta_7\} - \text{sym}\{\zeta_2^T \Gamma_{2a}\bar{N}\zeta_2\} - \text{sym}\{\zeta_2^T \Gamma_{2a}\bar{N}\zeta_7\}.
\end{aligned}$$

Furthermore, the other parameters are consistent with Theorem 1, and the estimator gain matrices are obtained by $K_{1a} = \bar{Q}_1^{-1}\Gamma_{1a}$ and $K_{2a} = \bar{Q}_2^{-1}\Gamma_{2a}$.

Proof. Based on (8) and the proof procedure of Theorem 1, one obtains that

$$\Theta = \hat{\Psi} + \mathbb{A}_{1a}\Lambda_{1a}\mathbb{C}_{1a} + \mathbb{C}_{1a}^{\mathrm{T}}\Lambda_{1a}^{\mathrm{T}}\mathbb{A}_{1a}^{\mathrm{T}} + \mathbb{A}_{2a}\Lambda_{2a}\mathbb{C}_{2a} + \mathbb{C}_{2a}^{\mathrm{T}}\Lambda_{2a}^{\mathrm{T}}\mathbb{A}_{2a}^{\mathrm{T}} \leq 0. \tag{27}$$

Since $\Lambda_{1a}^{\mathrm{T}}(x)\Lambda_{1a}(x) \leq I$ and $\Lambda_{2a}^{\mathrm{T}}(x)\Lambda_{2a}(x) \leq I$, by using Lemma 2, one obtains

$$\mathbb{A}_{1a}\Lambda_{1a}\mathbb{C}_{1a} + \mathbb{C}_{1a}^{\mathrm{T}}\Lambda_{1a}^{\mathrm{T}}\mathbb{A}_{1a}^{\mathrm{T}} \leq \gamma_1\mathbb{A}_{1a}\mathbb{A}_{1a}^{\mathrm{T}} + \gamma_1^{-1}\mathbb{C}_{1a}^{\mathrm{T}}\mathbb{C}_{1a}, \tag{28}$$

and

$$\mathbb{A}_{2a}\Lambda_{2a}\mathbb{C}_{2a} + \mathbb{C}_{2a}^{\mathrm{T}}\Lambda_{2a}^{\mathrm{T}}\mathbb{A}_{2a}^{\mathrm{T}} \leq \gamma_2\mathbb{A}_{2a}\mathbb{A}_{2a}^{\mathrm{T}} + \gamma_2^{-1}\mathbb{C}_{2a}^{\mathrm{T}}\mathbb{C}_{2a}. \tag{29}$$

According to (27), (28), and (29), by using Lemma 3, we can get

$$\Theta \leq \hat{\Psi} + \gamma_1\mathbb{A}_{1a}\mathbb{A}_{1a}^{\mathrm{T}} + \gamma_1^{-1}\mathbb{C}_{1a}\mathbb{C}_{1a}^{\mathrm{T}} + \gamma_2\mathbb{A}_{2a}\mathbb{A}_{2a}^{\mathrm{T}} + \gamma_2^{-1}\mathbb{C}_{2a}\mathbb{C}_{2a}^{\mathrm{T}} \leq 0, \tag{30}$$

where $\mathbb{A}_{1a}$, $\mathbb{A}_{2a}$, $\mathbb{C}_{1a}$, and $\mathbb{C}_{2a}$ are already given in the statement of Theorem 2.

By combining (14)–(24) and (27)–(30), it can be proved that

$$\begin{aligned}
\dot{V}(x) \leq & \int_\Omega \ell^T(x,y)(\hat{\Psi} + \gamma_1\mathbb{A}_{1a}\mathbb{A}_{1a}^T + \gamma_1^{-1}\mathbb{C}_{1a}\mathbb{C}_{1a}^T + \gamma_2\mathbb{A}_{2a}\mathbb{A}_{2a}^T + \gamma_2^{-1}\mathbb{C}_{2a}\mathbb{C}_{2a}^T)\ell(x,y)dy \\
& + \int_\Omega \sum_{b=\mathfrak{R}_{ur}^a} \pi_{ab}\phi_m^T(P_{1b} + W_a)\phi_m + \sum_{b=\mathfrak{R}_{ur}^a} \pi_{ab}\phi_p^T(P_{2b} + \bar{W}_a)\phi_p dy,
\end{aligned} \tag{31}$$

Table 1. The mode-dependent time-varying delays and related parameters

	$\alpha_a(x)$	$\beta_a(x)$	α_a	β_a	ψ_{1a}	ψ_{2a}
$a=1$	$0.3+0.3\sin(x)$	$0.25+0.25\sin(x)$	0.6	0.5	0.3	0.25
$a=2$	$0.4+0.4\sin(x)$	$0.35+0.35\sin(x)$	0.8	0.7	0.4	0.35
$a=3$	$0.5+0.5\sin(x)$	$0.45+0.45\sin(x)$	1	0.9	0.5	0.45

where $\ell(x,y) = \mathrm{col}\left(\phi_m, \phi_p, \phi_m(x-\beta_a(x),y), \phi_p(x-\alpha_a(x),y), f(\phi_p(x-\alpha_a(x),y)), \frac{\partial\phi_m}{\partial x}, \frac{\partial\phi_p}{\partial x}\right)$, we employ Schur's complement, which can convert the inequality (30) into (26).

Combining the above analysis, we can conclude that the system (9) is asymptotically stable when the inequalities (12) and (26) hold. The proof is complete. ■

4. SIMULATION EXAMPLE

In this section, a numerical simulation is provided to illustrate the validity of the theoretical results in this paper. The corresponding parameters of the system (1) are selected as:

$$B_1 = \begin{bmatrix} 0.74 & 0 & 0 \\ 0 & 0.72 & 0 \\ 0 & 0 & 0.65 \end{bmatrix}, B_2 = \begin{bmatrix} 0.69 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0 & 0.71 \end{bmatrix}, B_3 = \begin{bmatrix} 0.71 & 0 & 0 \\ 0 & 0.74 & 0 \\ 0 & 0 & 0.67 \end{bmatrix}, N = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1.1 \end{bmatrix},$$

$$\bar{B}_1 = \begin{bmatrix} 0.75 & 0 & 0 \\ 0 & 0.69 & 0 \\ 0 & 0 & 0.62 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0.77 & 0 & 0 \\ 0 & 0.71 & 0 \\ 0 & 0 & 0.68 \end{bmatrix}, \bar{B}_3 = \begin{bmatrix} 0.72 & 0 & 0 \\ 0 & 0.76 & 0 \\ 0 & 0 & 0.72 \end{bmatrix}, \bar{N} = \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 1.3 & 0 \\ 0 & 0 & 0.9 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.49 & -0.29 & 0.43 \\ -0.52 & 0.91 & 0.63 \\ 0.51 & 0.34 & -0.62 \end{bmatrix}, G_2 = \begin{bmatrix} 0.53 & -0.34 & 0.42 \\ -0.52 & 0.94 & 0.61 \\ 0.53 & 0.31 & -0.63 \end{bmatrix}, G_3 = \begin{bmatrix} 0.55 & -0.36 & 0.41 \\ -0.53 & 0.97 & 0.67 \\ 0.56 & 0.37 & -0.68 \end{bmatrix},$$

$$\bar{G}_1 = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}, \bar{G}_2 = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}, \bar{G}_3 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, E = \begin{bmatrix} 0.15 & 0 & 0 \\ 0 & 0.16 & 0 \\ 0 & 0 & 0.13 \end{bmatrix},$$

$$\bar{E} = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.21 & 0 \\ 0 & 0 & 0.19 \end{bmatrix}.$$

The selection of the transfer matrix with partially unknown probabilities is shown below:

$$\Pi = \begin{bmatrix} -0.8 & 0.5 & 0.3 \\ ? & -1.0 & ? \\ 0.8 & ? & ? \end{bmatrix};$$

based on the above transfer matrix, the process of switching between modes is shown in Figure 1.

The mode-dependent time-varying delays and related parameters are shown in Table 1.

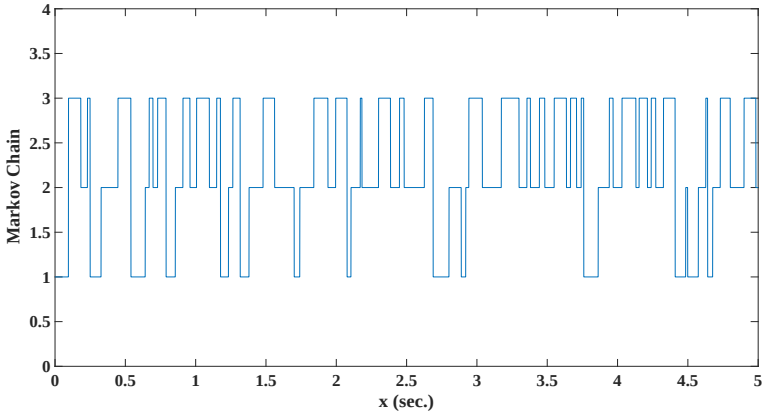

Figure 1. Switching Modes of the system.

The parameters of the estimator gain perturbation are chosen as:

$$A_{11} = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.16 & 0 \\ 0 & 0 & 0.19 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.31 & 0 & 0 \\ 0 & 0.18 & 0 \\ 0 & 0 & 0.23 \end{bmatrix}, A_{13} = \begin{bmatrix} 0.35 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0.33 & 0 & 0 \\ 0 & 0.26 & 0 \\ 0 & 0 & 0.18 \end{bmatrix}, A_{22} = \begin{bmatrix} 0.28 & 0 & 0 \\ 0 & 0.22 & 0 \\ 0 & 0 & 0.17 \end{bmatrix}, A_{23} = \begin{bmatrix} 0.26 & 0 & 0 \\ 0 & 0.19 & 0 \\ 0 & 0 & 0.23 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 0.55 & 0 & 0 \\ 0 & 0.45 & 0 \\ 0 & 0 & 0.45 \end{bmatrix}, C_{12} = \begin{bmatrix} 0.35 & 0 & 0 \\ 0 & 0.31 & 0 \\ 0 & 0 & 0.31 \end{bmatrix}, C_{13} = \begin{bmatrix} 0.42 & 0 & 0 \\ 0 & 0.33 & 0 \\ 0 & 0 & 0.35 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 0.45 & 0 & 0 \\ 0 & 0.35 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}, C_{22} = \begin{bmatrix} 0.38 & 0 & 0 \\ 0 & 0.32 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}, C_{23} = \begin{bmatrix} 0.51 & 0 & 0 \\ 0 & 0.43 & 0 \\ 0 & 0 & 0.45 \end{bmatrix},$$

$$\Lambda_{1a}(x) = \Lambda_{2a}(x) = \begin{bmatrix} \sin x & 0 & 0 \\ 0 & \sin x & 0 \\ 0 & 0 & \sin x \end{bmatrix}, a = 1, 2, 3.$$

Let $\gamma_1 = \gamma_2 = 0.5$, $\varpi = 1$, $\mathbb{N} = 0.65 I_3$, and regulation function $f(\epsilon) = \frac{\epsilon^2}{1+\epsilon^2}$, by solving the LMIs in Theorem 2, it can prove that the LMIs (12) and (26) of this paper are feasible; the estimator gain matrices are derived:

$$K_{11} = \begin{bmatrix} 0.9024 & -0.4021 & -0.1306 \\ -0.3080 & 2.1696 & -0.2721 \\ -0.1623 & -0.3567 & 1.1212 \end{bmatrix}, K_{21} = \begin{bmatrix} 0.0671 & -9.0706 & -5.7745 \\ -8.5686 & 0.0830 & -4.1928 \\ -5.7549 & -5.9144 & 0.0639 \end{bmatrix},$$

$$K_{12} = \begin{bmatrix} 0.9036 & -0.4430 & -0.1300 \\ -0.2972 & 2.2427 & -0.2630 \\ -0.1473 & -0.3743 & 1.0542 \end{bmatrix}, K_{22} = \begin{bmatrix} 0.0703 & -7.1541 & -3.5354 \\ -7.5068 & 0.1065 & -3.5237 \\ -3.7345 & -5.1003 & 0.1021 \end{bmatrix},$$

$$K_{13} = \begin{bmatrix} 0.5541 & -0.4042 & -0.1278 \\ -0.2766 & 1.6326 & -0.2426 \\ -0.1444 & -0.3375 & 0.7699 \end{bmatrix}, K_{23} = \begin{bmatrix} 0.0110 & -6.1614 & -4.1572 \\ -6.2536 & 0.0032 & -1.0914 \\ -5.7068 & -2.1649 & -0.0302 \end{bmatrix}.$$

Based on the obtained estimator gain matrix, Figures 2-5 illustrate the simulation results we obtained. Figures 2 and 3 demonstrate the estimation errors for mRNA and protein, respectively. To observe the trend of

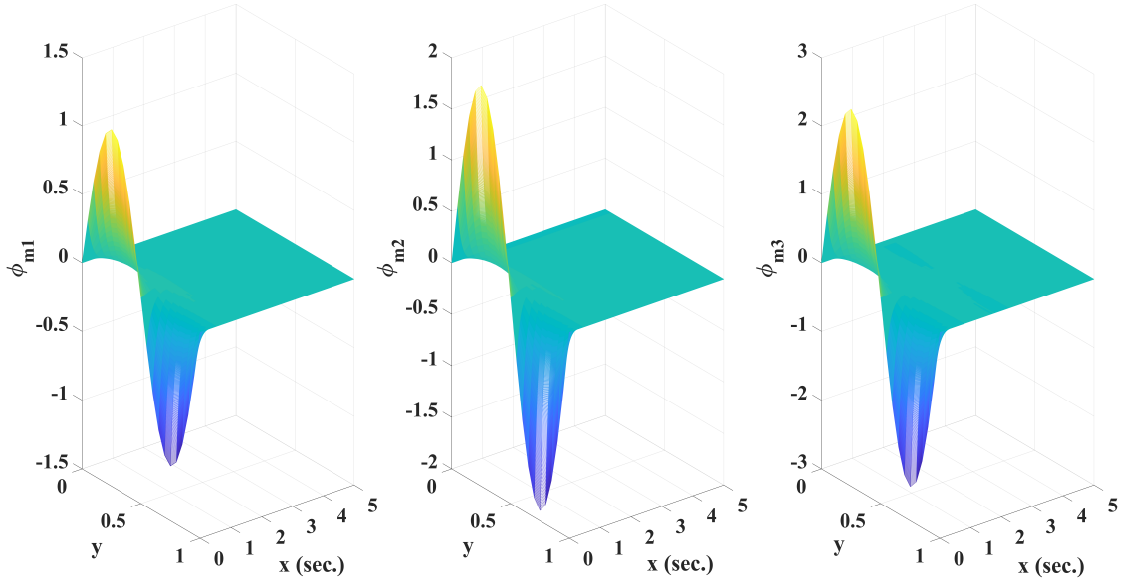

Figure 2. The evolution of mRNA estimation errors.

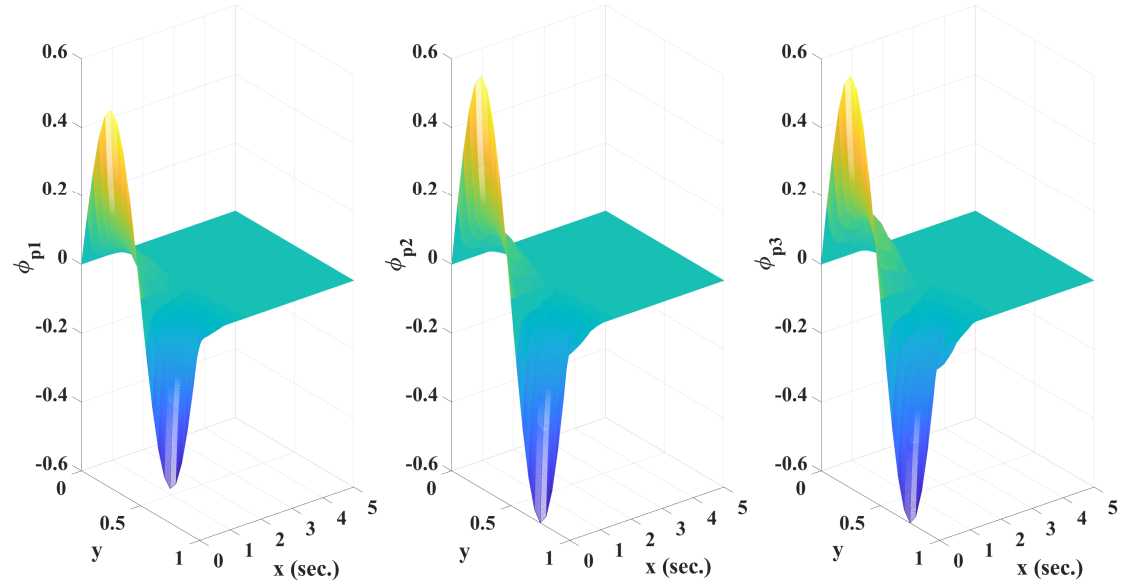

Figure 3. The evolution of protein estimation errors.

the estimation errors more clearly, we take the spatial point at $y = 0.2$ as the example and display the trajectory plots of the estimation errors of mRNA and protein in Figures 4 and 5, respectively. The above simulation results show that the estimation errors are asymptotically stable, thus proving the effectiveness of the non-fragile state estimator designed in this paper.

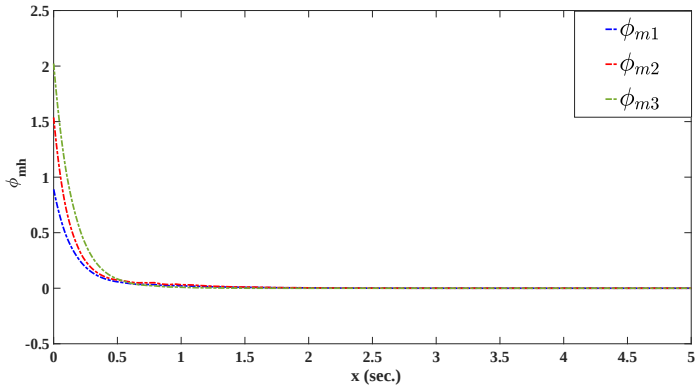

Figure 4. The trajectories of mRNA estimation errors ϕ_{mh}, $h = 1, 2, 3$.

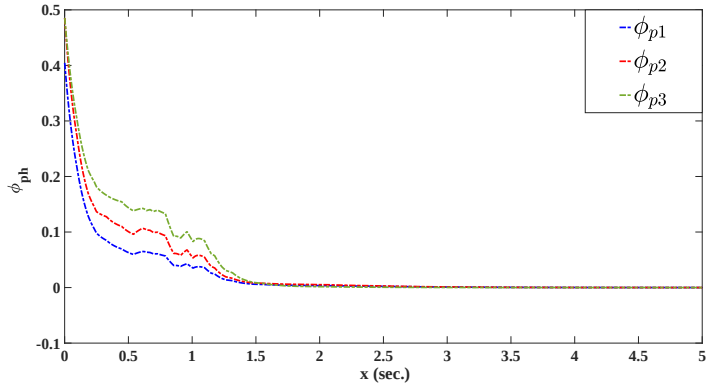

Figure 5. The trajectories of protein estimation errors ϕ_{ph}, $h = 1, 2, 3$.

5. CONCLUSIONS

This paper has investigated the non-fragile state estimation for Markovian jump RDGRNs with mode-dependent time-varying delays. Moreover, the switching topology of the system satisfies the Markov chain with partial unknown transfer probabilities. By utilizing the suitable LKFs, sufficient conditions that guarantee the asymptotic stability of the error system were established. Ultimately, a numerical example was used to demonstrate the validity of the designed non-fragile state estimator. Nonetheless, some limitations of this paper still exist. For example, the measurement methods adopted in this paper are costly and challenging to implement. Therefore, we will employ point measurements to reduce the measurement cost in future work.

DECLARATIONS

Authors' contributions

Made significant contributions in manuscript editing: Liu J
Made substantial contributions to the conception and design of the study: Song S
Made substantial contributions to manuscript revision: Song Y
Methodology validation: Song X

Availability of data and materials

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Conflicts of interest
All authors declared that there are no conflicts of interest.

Ethical approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

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